Contextualised task 37

Deepest Wales

Teaching Notes

This task allows students to apply straight edge and compass constructions to the problem of finding the point in a country furthest from its borders. This could also be thought of as the point deepest within the country. There are several different ways to define the centre of a country, and there are also several ways to define the centre of a triangle as the first part of the activity demonstrates. Having solved this problem, students then explore the notion of a 'coast to area ratio'.

Task A: The centres of a triangle
Students are introduced to the intriguing fact that there are several ways to define the centre of a triangle. They use some of the ideas to locate the point that is furthest from the Welsh border. This requires them to apply straight edge and compass constructions.

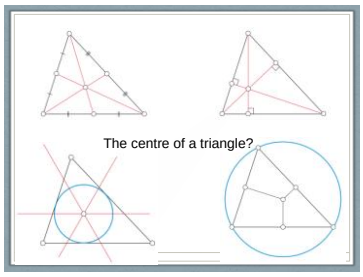
You will need:
- Teachers' script
- PowerPoint
- Task A: Question sheet
- Task A: Wales
- Task A: England (optional)
- Task A: Scotland (optional)
- Mark scheme

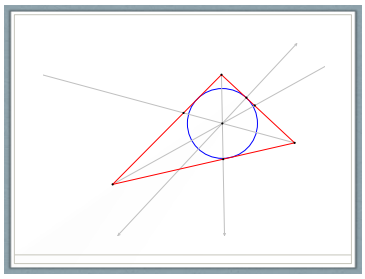
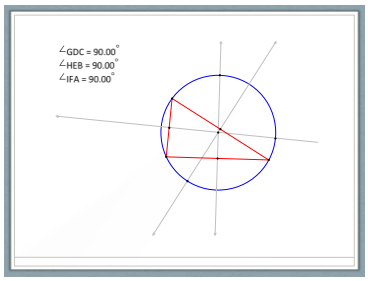
Task B: The wiggliness ratio
Students now move their attention from the border of a country as a whole, to its coastline only. They calculate the coast to area ratio, which provides a measure of a population's access to the sea. Note that the concept of a coastline length is surprisingly complex: some sources give the UK coastline length as low as 12,500 km. The figure used here (31,368 km), includes all the islands of the UK as at mean high tide. 'How Long is The Coast Of Britain?' was an early mathematical publication on fractals by Benoit Mandelbrot.

You will need:
- Question sheet
- Mark scheme

Task A: Teachers' script for PowerPoint presentation

The text in the right-hand boxes provides a possible script to be read to students. However, it is probably preferable to use your own words and elaboration. When questions are asked, time for discussion in pairs / groups should be provided. Ensure that students are given opportunity to explain their reasoning in response to these questions. All students need to understand the concepts in order to make progress with the task.

Slide 1		*Keep this slide on the screen until you are ready to start the presentation*
Slide 2		The centre of a circle is easy to picture. And so is the centre of a square. But the centre of a triangle is not at all obvious. There are many different ways to define the centre of a triangle. The first is called the 'centroid'. *Advance one click* Look at this diagram. How has the centroid been found? *(The midpoint of each side is joined to the opposite corner. All three lines cross at a single point.)* Another way to define the centre of a triangle is called the 'orthocentre'. *Advance one click* Look at this diagram. How has the orthocentre been found? *(A line through each corner that is perpendicular to the opposite side. All three lines cross at a single point.)* *Advance one click* Two other centres of a triangle are defined as the 'incentre' … *Advance one click* … and the 'circumcentre'. *Advance one click* Let's look at these in more detail.

Slide 3		Here is a diagram showing a circle inscribed in a triangle. The circle touches the triangle at three points. The centre of this circle is also the 'incentre' of the triangle. How could you find the incentre if you were given any triangle? *(Bisect each of the three angles. The three lines cross at a single point.)*
Slide 4		This diagram shows a circle drawn so that it touches all three corners of a triangle. The centre of this circle is also the 'circumcentre' of the triangle. If you had a triangle, how could you find its circumcentre? *(Find the perpendicular bisector of each side. The three lines cross at a single point.)* (Sometimes, the circumcentre of a triangle is outside the triangle!)
Slide 5		Here is a map of the UK. *Invite students to come and point to the place that they think is furthest from the sea.* The point furthest from the sea is the centre of the largest circle that can be inscribed within the map. *Advance one click* The circle touches at these three points *Advance three clicks* The centre of the circle … *Advance two clicks* … is Coton in the Elms in Derbyshire
Slide 6		To find the centre of this circle we only need to consider the three points where it touches the coastline. *Advance one click to remove circle* We need to draw a triangle using these points. *Advance one click to draw triangle* The circumcentre of the triangle is where the three perpendicular bisectors cross. *Advance three clicks* Each of the three original points is equidistant from the circumcentre. *Advance one click to draw circle again.*

Slide 7		Here is a familiar map. Where would you visualise the largest circle that can be inscribed within Wales? *Invite students to come and point to the place that they think is furthest from the boundary, and therefore 'deepest' within Wales.*

Deepest Wales

Task A: Question

You have seen how to find the point in the UK furthest from the sea.

Use the map of Wales to find the point deepest within Wales; i.e. furthest from its border.

There are maps of Scotland and England available too if more practice is wanted.

Remember

- Inscribe the largest possible circle
- Find three points where the circle touches the border. Join to make a triangle.
- Construct the perpendicular bisector of each side of the triangle

Task A: Wales

Task A: Scotland

Task A: England

Task A: Mark scheme

The information below is intended as a guide only

Full credit

Inscribes a circle within the map as shown below – centre just to the east of Llandovery

Draws a triangle based on three points of contact with the boundary of the map
Constructs the perpendicular bisector of at least two of the sides:

- Correct construction arcs are visible
- Pencil lines are crisp and clear
- Angle 89° to 91°
- Lines bisected ±1 mm

Perpendicular bisectors intersect at a point

Partial credit

Provides a solution as detailed above but with one of the following:

- No visible construction arcs
- Construction lines are not crisp and clear
- 'Perpendicular' bisector at an angle of 88° to 92°
- Perpendicular bisector 'bisects' a line ±2 mm

OR

Provides an otherwise correct solution based on an initial circle in a different position

Limited credit

Demonstrates ability to construct the perpendicular bisector of a line segment

No credit

Any other response.

Deepest Wales

Task B: Question sheet

The area of Wales is 20,779 km^2. The length of its coastline is 2,700 km.

The coast to area ratio (m/km^2) for Wales is 2,700,000 : 20,779.

This ratio can be written as 129.9 : 1. So there are about 129.9 metres of coastline for every square kilometre of land area. This is a measure of how accessible the coast is for people who live in Wales.

The table shows the twenty countries with the longest coastlines in the world.

1) Compare the coast to area ratio for Wales with the coast to area ratio for the UK as a whole.

2) Calculate the coast to area ratios for each country in the table. How does Wales compare with these countries?

Country	**Coastline (km)**	**Area (km^2)**	
Australia	25,675	7,686,850	
Brazil	7,461	8,515,767	
Canada	202,080	9,984,671	
Chile	6,435	756,096	
China	14,500	9,569,901	
Croatia	5,835	56,594	
Denmark	7,314	42,916	
Greece	13,676	131,957	
Greenland	44,087	2,166,086	
India	7,517	3,166,414	
Indonesia	54,716	1,811,569	
Italy	7,600	301,338	
Japan	29,751	377,944	
Mexico	9,330	1,972,550	
Micronesia	6,112	702	
New Zealand	15,134	268,021	
Norway	25,148	385,199	
Philippines	36,289	298,170	
Russia	37,653	17,098,242	
Turkey	7,200	783,562	
United Kingdom	31,368	243,610	
United States of America	20,083	9,857,306	

Task B: Mark scheme

The information below is intended as a guide only.

Full credit

Calculates the coast to area ratio for the UK as 128.8 (to one decimal place)

AND

Observes that the figure is very similar to that of Wales alone

AND

Finds all coast to area ratios as stated in the table.

Country	**Coastline (km)**	**Area (km^2)**	**Coast to area ratio (m/km^2)**
Australia	25,675	7,686,850	3.4
Brazil	7,461	8,515,767	0.9
Canada	202,080	9,984,671	20.2
Chile	6,435	756,096	8.5
China	14,500	9,569,901	1.5
Croatia	5,835	56,594	103.1
Denmark	7,314	42,916	170.4
Greece	13,676	131,957	103.6
Greenland	44,087	2,166,086	20.4
India	7,517	3,166,414	2.4
Indonesia	54,716	1,811,569	30.2
Italy	7,600	301,338	25.2
Japan	29,751	377,944	78.7
Mexico	9,330	1,972,550	4.7
Micronesia	6,112	702	8706.6
New Zealand	15,134	268,021	56.5
Norway	25,148	385,199	65.3
Philippines	36,289	298,170	121.7
Russia	37,653	17,098,242	2.2
Turkey	7,200	783,562	9.2
United Kingdom	31,368	243,610	128.8
United States of America	20,083	9,857,306	2
Wales	**2,700**	**20,779**	**129.9**

AND

Observes that Wales has a very high coast to area ratio compared with the countries in this list.

Partial credit

Rounds incorrectly throughout an otherwise correct solution

OR

Calculates at least 15 coast to area ratios correctly

AND

Makes the required observations.

Limited credit

Calculates the required ratios but does not interpret the information correctly

OR

Calculates between 10 and 15 coast to area ratios correctly and comments on the results.

No credit

Any other response.

Progression in reasoning			
Identify processes and connections • Transfer mathematical skills across the curriculum in a variety of contexts and everyday situations	Apply skills within familiar contexts *e.g. constructs a perpendicular bisector of a line segment*	Identify, perhaps with some guidance, the skills needed within increasingly complex and unfamiliar contexts *e.g. constructs two or three intersecting perpendicular bisectors as directed*	Identify independently the skills needed within increasingly complex and unfamiliar contexts *e.g. identifies how each of the triangle centres are defined*
Represent and communicate • Use appropriate notation, symbols and units of measurement, including compound measures	*e.g. uses the symbols m, km, km^2 correctly*	*e.g. correctly interprets the compound measure m/km^2*	*e.g. converts between compound measures such as km/m^2 and m/km^2*
Review • Interpret answers within the context of the problem and consider whether answers, including calculator, analogue and digital displays, are sensible	*e.g. rounds the coast to area ratio to a sensible degree of accuracy*	*e.g. considers the reasonableness of solutions when finding coast to area ratios and double-checks that the result for Micronesia is correct*	*e.g. recognises the need to construct a third perpendicular bisector and ensure that it crosses the other two at their point of intersection*

GCSE Content	
GCSE Mathematics – Numeracy and GCSE Mathematics	**GCSE Mathematics only**
Understanding number and place value • Rounding decimals to the nearest whole number or a given number of decimal places. • Ordering and comparing whole numbers, decimals, fractions and percentages.	
Understanding number relationships and methods of calculation • Calculating using ratios in a variety of situations; proportional division.	
Solving numerical problems • <u>Rounding an answer to a reasonable degree of accuracy in the light of the context.</u>	
Understanding and using properties of shape • The geometrical terms: point, line, right angle, perpendicular, edge and vertex. • Vocabulary of triangles and circles: isosceles, equilateral, scalene, radius, diameter, circumference and arc. • Bisecting a given line, bisecting a given angle. • <u>Constructing the perpendicular from a point to a line.</u>	• Use of ruler and pair of compasses to do constructions.
Understanding and using properties of position, movement and transformation • <u>Constructing the locus of a point which moves such that it satisfies certain conditions, for example, (i) a given distance from a fixed point or line, (ii) equidistant from two fixed points or lines.</u> • <u>Solving problems involving intersecting loci in two dimensions</u>	
Understanding and using measures • Using compound measures such as m/s, km/h, mph, mpg, <u>kg/m^3, g/cm^3, population per km^2</u>	

<u>Key</u>
Foundation tier content is in standard text.
Intermediate tier content which is in addition to foundation tier content is in <u>underlined text</u>.
Higher tier content which is in addition to intermediate tier content is in **bold text**.