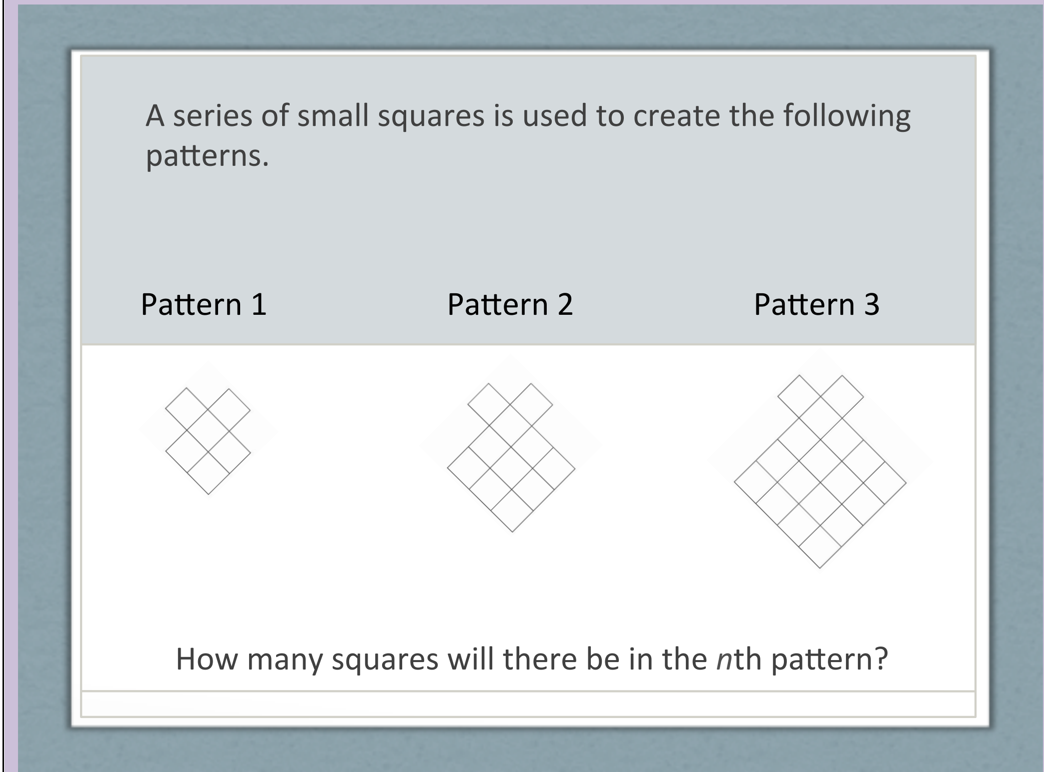
Stimulus question 36

Linear and quadratic sequences

Teaching notes

Key question / task



Resources:

- 3 or 4 copies of the first sequence of visual patterns, and colours to explain the different ways of 'dividing' them.
- PowerPoint presentation.

Reasoning: questions to discuss and explore

- Before finding the *n*th term, can you describe to someone else how to make the next pattern?
- When finding the *n*th term, do the different methods suggested give the same answers for the sequence?
- Which is the most interesting or surprising method?
- Can you think of different methods of looking at the patterns of squares?

Consolidation

- Recognising a simple quadratic sequence from a simple pattern, e.g. *n*th term = $n^2 + 2$;
- Construct a sequence of visual patterns from a simple quadratic rule, e.g. *n*th term = $n^2 + 2$.

Possible extensions

- Create a different sequence of visual patterns using the same rule (*n*th term = $n^2 + 2n + 3$).
- Create a different sequence of visual patterns for other sequences (linear or quadratic).

Commentary / notes:

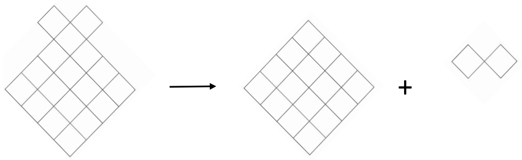
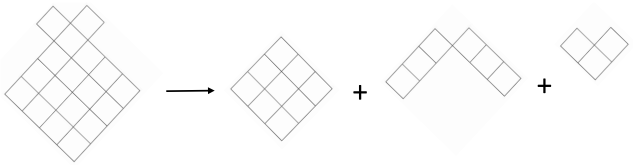
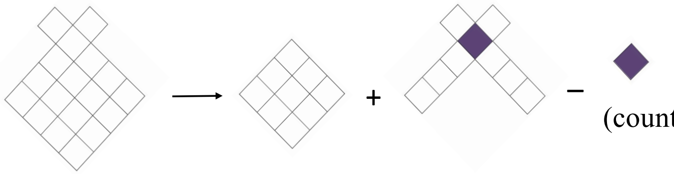
The first PowrPoint slide provides an opportunity to revise the method of discovering the nth term of a linear sequence.

Solutions are: $4n$; $3n + 5$; $10n - 3$; $14 - 2n$

The second slide revises the method of discovering the nth term of a simple quadratic sequence, i.e. it should be possible to make the connection with the first sequence immediately (nth term = n^2) and write the rule each time.

Solutions are: n^2; $n^2 + 2$; $n^2 - 1$; $(n - 1)^2$; $(n + 2)^2$; $2n^2$

Slides 3 to 6 require learners to interpret a visual pattern step by step. Before finding the nth term, discuss how to construct the next pattern, i.e. what should be added to the last pattern? Then discuss the different possible ways of interpreting the whole pattern (more than are shown here). The third pattern is used as an example, so learners should remember that we are dealing with the case n = 3.

	$(3 + 1)^2 + 2 \longrightarrow (n + 1)^2 + 2$
	$3^2 + (2 \times 3) + 3 \longrightarrow n^2 + 2n + 3$
 (counted twice, see PowerPoint)	$3^2 + 2(3 + 2) - 1 \longrightarrow n^2 + 2(n + 2) - 1$

Check that all answers are equivalent.

The nth term can be found by listing the numbers of small squares, and using a 'difference method' for the sequence 6, 11, 18. However, the main purpose of this task is to explore visual patterns.

The final slide contains further questions, and once again there are many ways of dividing up the pattern each time. Number of coloured squares = n^2 Number of white squares = $6n + 7$ Total number of squares = $n^2 + 6n + 7$

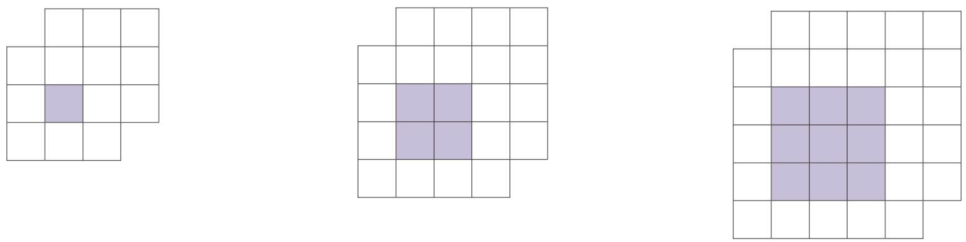
GCSE Subject Content: Note that underlined type indicates intermediate level		
Foundation	**Intermediate**	**Higher**
Finding the *n*th term of a sequence where the rule is linear or quadratic.		

Learner Outcomes and Assessment (*to aid comment-only marking*)	
Reasoning strand – Learners are able to:	**Assessment guidance – Can learners:**
• Select appropriate mathematics and techniques to use; • Explain results and procedures precisely using appropriate mathematical language.	• Express the *n*th term of a linear sequence? • Express the *n*th term of a quadratic sequence? • Interpret a visual pattern to find the *n*th term? • Explain how to divide a visual pattern to discover the *n*th term of a sequence?
Algebra strand – Learners are able to:	
• Express the *n*th term rule using algebra; • Produce non-linear sequences knowing the *n*th term rule.	

Extension slide

A series of small squares is used to create the following patterns.

Pattern 1 Pattern 2 Pattern 3



How many coloured squares, how many white squares and how many squares in total will there be in the *n*th pattern?