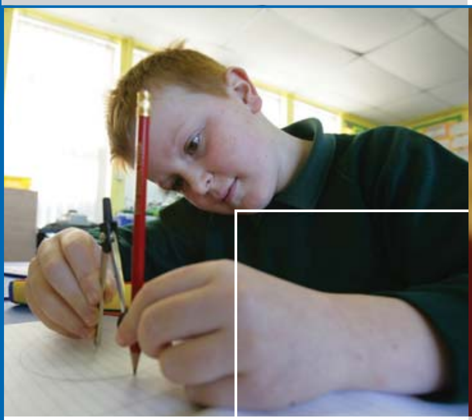
Mathematics

Guidance for Key Stages 2 and 3

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Llywodraeth Cynulliad Cymru
Welsh Assembly Government

Mathematics

Guidance for Key Stages 2 and 3

Audience	Teachers at Key Stages 2 and 3; local authorities; regional consortia; tutors in initial teacher training; and others with an interest in continuing professional development.
Overview	These materials provide key messages for planning learning and teaching in mathematics. They include profiles of learners' work to exemplify the standards set out in the level descriptions and illustrate how to use level descriptions to make best-fit judgements at the end of Key Stages 2 and 3.
Action required	To review learning plans and activities and to prepare to make judgements at the end of Key Stages 2 and 3.
Further information	Enquiries about this document should be directed to: Curriculum Division The Education Directorate Welsh Assembly Government Cathays Park Cardiff CF10 3NQ e-mail: curriculumdivision@wales.gsi.gov.uk
Additional copies	This document can be accessed from the Learning Wales website at gov.wales/learning
Related documents	*Mathematics in the National Curriculum for Wales; Skills framework for 3 to 19-year-olds in Wales; Making the most of learning: Implementing the revised curriculum; Ensuring consistency in teacher assessment: Guidance for Key Stages 2 and 3* (Welsh Assembly Government, 2008) This guidance is also available in Welsh.

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Contents

Introduction

The programmes of study set out the opportunities that learners should be given at each key stage and provide the basis from which you, as a teacher, can plan learning and teaching. They are divided into two sections, Skills and Range. The Skills section lists the skills to be developed in a subject and the Range section comprises the opportunities and contexts through which these skills should be developed and consolidated.

Ongoing formative assessment – assessment **for** learning – lies at the heart of good teaching. Through the assessments that you make in the course of your teaching, you will build up an extensive knowledge of your learners' strengths, as well as the areas that need further development, and you will use this knowledge to help you plan for the next steps in their learning. Learners will also gain understanding of specific learning goals and the associated success criteria so that, supported by you, they can develop their capacity for self-assessment and peer assessment. In this way, they can establish their current position, set and move towards targets, and discover if and when the targets have been reached. Individual targets are linked to improving the quality of a learner's work, as highlighted through formative feedback, and are therefore linked to success criteria for specific tasks. Level descriptions do not make effective targets as these describe attainment across the breadth of the programme of study at the end of a key stage.

Level descriptions can help to inform your planning, teaching and assessment at Key Stages 2 and 3 by indicating expectations at particular levels and progression in the subject. Evidence from assessment for learning will indicate where more time is needed to consolidate learning and when learners are ready to move on. You may wish to keep some evidence so that you can discuss a learner's work and progress with them and/or with colleagues or parents/guardians. However, there is no statutory requirement to keep unnecessarily complex records or detailed evidence on every learner.

The essential function of level descriptions is to help you make rounded summative judgements at the end of Key Stages 2 and 3 about a learner's overall performance. Level descriptions are designed neither to be used to 'level' individual pieces of work nor for the production of half-termly or termly data. It is only by the end of the key stage that you will have built up sufficient knowledge about a learner's performance across a range of work, and in a variety of contexts, to enable you to make a judgement in relation to the level descriptions.

It may be that some learners will be more advanced in some aspects of the work than in others, and that no one level description provides an exact fit. That is to be expected, and the range of individual learners' work included in these materials illustrates the making of best-fit judgements under those circumstances. Many schools/departments have found it helpful to develop their own learner profiles to support moderation of end of key stage judgements. These profiles also help to maintain a common understanding of standards when they are reviewed annually and refreshed when necessary.

When making judgements at the end of Key Stages 2 and 3, you should decide which level description **best fits** a learner's performance. The aim is for a rounded judgement that:

- is based on your knowledge of how the learner performs across a range of contexts
- takes into account different strengths and areas for development in that learner's performance
- is checked against adjacent level descriptions to ensure that the level judged to be the most appropriate is the closest overall match to the learner's performance in the attainment target.

National curriculum outcomes have been written for learners working below Level 1. These are non-statutory and guidance on their use is planned.

Using these materials

This booklet is divided into three sections.

Section 1	highlights key messages for learning and teaching in mathematics.
Section 2	highlights expectations and progression in mathematics.
Section 3	contains a series of learner profiles. These are designed to show the use of the level descriptions in coming to judgements about a learner's overall performance at the end of Key Stages 2 and 3.

This booklet is for reference when you wish to:

- review your learning plans and activities
- consider the standards set out in the revised mathematics Order
- work with other teachers to reach a shared understanding of the level descriptions
- prepare to make judgements at the end of the key stage
- develop your own learner profiles
- support transition from Key Stage 2 to Key Stage 3.

For ease of reference, the level descriptions are included in a leaflet with this booklet.

A CD-ROM is also included with this booklet. It contains a PDF version of *Mathematics in the National Curriculum for Wales, Skills framework for 3 to 19-year-olds in Wales* and this guidance.

This guidance is part of a series of materials that will help teachers at Key Stages 2 and 3 to implement the revised curriculum and its associated assessment arrangements. The series includes:

- *Making the most of learning: Implementing the revised curriculum* – overview guidance on implementing the new curriculum
- *Skills framework for 3 to 19-year-olds in Wales* – which includes guidance about progression in skills
- *Ensuring consistency in teacher assessment: Guidance for Key Stages 2 and 3*
- *A curriculum for all learners: Guidance to support teachers of learners with additional learning needs*
- specific guidance for all national curriculum subjects, personal and social education, careers and the world of work, and religious education.

Section 1

Key messages for learning and teaching in mathematics

Planning the learning

Skills are the essence of mathematics – Solving mathematical problems, Communicating mathematically and Reasoning mathematically. These are the higher order mathematics skills, and they relate closely to much of the Developing thinking across the curriculum and the Developing communication across the curriculum sections in the *Skills framework for 3 to 19-years-old in Wales*.

The Range provides the context and techniques for developing these higher order skills, and is described in terms of four/five different strands of mathematics (Number; Measures and money; Algebra (KS3 and KS4 only); Shape, position and movement; Handling data). Many of the techniques in the Range relate closely to the Developing number section of the skills framework.

Effective teaching integrates the Skills and Range seamlessly; the mathematical Skills should be infused throughout the Range. In practice, in anything other than a short, specific task, learners' work may well involve a number of strands simultaneously. Number concepts are used extensively in Measures and money, and Handling data; Algebra is often used in Shape when considering sequences of geometrical patterns; work in Handling data may well use the context of Money.

For example, an activity for Key Stage 2 learners in which they plan and compare trips from one part of Wales to another could include elements from all three strands of the mathematical skills, across the Number, Measures and money, and Handling data strands of the Range.

The Range also provides the opportunity to emphasise the importance of mathematics in real life, including contexts such as:

- financial literacy, e.g. budgeting, saving, hire purchase and mobile phone rates
- bilingualism and the Curriculum Cymreig, e.g. using real-life data with a Wales context
- valuing diversity, e.g. contexts could include examples from a range of cultures, including Welsh, Celtic and Indian patterns; and interpreting and discussing demographic data about life expectancies and infant mortality rates around the world

- twenty-first century issues, e.g. mathematics in modern industry, medicine, research, science and technology
- cross-curricular links, e.g. the presentation of results in science and geography, the mathematics of spreadsheets in ICT, and measurement and dimension in design and technology.

There are several examples in Section 3 of the same activity attempted by Key Stage 2 learners who are working at different levels, and that show differentiation by outcome.

Michelle and Emyr have both completed ‘Boxes’ and explained why they placed the digits as they did. Michelle’s explanation was in terms purely of the size of the numbers, but Emyr considered the chance of getting higher or lower numbers, and so gave a more mathematical explanation than Michelle did.

Michelle and Emyr also both completed ‘Building triangles’. Both completed a table to show the relationship between the number of triangles made and the number of matchsticks needed. Michelle noticed that the number of matchsticks ‘goes up in twos’ and built up the table to find how many matchsticks would be needed to make nine triangles. Emyr, on the other hand, found that ‘you need to double the number of triangles and add one,’ and could use this result to ‘predict the number of sticks needed for any number of triangles’.

Michelle, Emyr and Marya each attempted ‘Consecutive numbers’. The outcomes of these tasks show clear differentiation. Michelle noticed that adding two consecutive numbers always gave an odd number, then tried to work her way through the integers from one up to 32, using from two to seven consecutive numbers as necessary. Emyr started by working systematically with two consecutive numbers and was able to find a pattern: ‘If you add two consecutive numbers they go up in twos and are all odd’. He then moved on to add three consecutive numbers: ‘When you add three consecutive numbers they go up in threes’, and five consecutive numbers: ‘the answers are multiples of five’. Marya was able to give reasons for her results: ‘Adding three consecutive numbers makes a number in the three times table. I think this is because 19 + 20 + 21 = 20 – 1 + 20 + 20 + 1 = 3 x 20.’

Mathematics and skills across the curriculum

A non-statutory *Skills framework for 3 to 19-year-olds in Wales* has been developed in order to provide guidance about continuity and progression in developing thinking, communication, ICT and number for learners from 3 to 19.

At Key Stages 2 and 3, learners should be given opportunities to build on skills they have started to acquire and develop during the Foundation Phase. Learners should continue to acquire, develop, practise, apply and refine these skills through group and individual tasks in a variety of contexts across the curriculum. Progress can be seen in terms of the refinement of these skills and by their application to tasks that move from: concrete to abstract; simple to complex; personal to the ‘big picture’; familiar to unfamiliar; and supported to independent and interdependent.

Icons have been used in all subject Orders to signal explicit requirements for the development of skills across the curriculum. In mathematics, the four icons appear on all double-page spreads of the Programmes of Study for Key Stages 2, 3 and 4, signifying that skills from all four sections of the skills framework should be integrated throughout the programmes of study. Though there is no expectation that all of the statements in the skills framework will apply equally in mathematics, there will be many opportunities to include these skills when planning the learning in mathematics.

This is emphasised in the samples of work in Section 3, where some of the mathematical skills from the programmes of study and skills from the skills framework are listed as opportunities the task provides for learners to use and develop these skills. There is no expectation that all learners will address all the skills listed, nor that these are the only skills that could be in evidence in their work.

Developing thinking

Learners develop their thinking across the curriculum through the processes of **planning**, **developing** and **reflecting**.

In mathematics, learners ask questions, explore alternative ideas and make links with previous learning in order to develop strategies to solve problems. They gather, select, organise and use information, and identify patterns and relationships. They predict outcomes, make and test hypotheses, reason mathematically when investigating, and analyse and interpret mathematical information. They describe what they have learned, reflect on their work by evaluating their results in line with the original problem, and justify their conclusions and generalisations.

This section of the skills framework is closely reflected in the mathematical skills of Solve mathematical problems and Reason mathematically.

There are several examples throughout the learner profiles that reflect one or more of these characteristics, though those described below are not the only ones.

Ask questions, explore alternative ideas and make links with previous learning in order to develop strategies to solve problems	Adam (KS3), in his work on ‘Half-time scores’, posed his own question about the different ways of getting half-time draw scores. Sarah (KS3), in ‘Hidden faces’, extended her previous work on cubes to consider the results for cuboids. Adam also explored alternative ideas in ‘Beat that’. Marya (KS2), in ‘Match up’, made links with previous work on converting metric measures in order to simplify her calculations.
Gather, select, organise and use information, and identify patterns and relationships	Michelle (KS2), in ‘Our class’, Emyr (KS2), in ‘Using the library’, Oliver (KS3), in ‘Healthy living’, Adam, in ‘Five a day’, and Sarah, in ‘Calls and texts’, all gathered, selected, organised and used information. Michelle, Emyr, Marya and Oliver identified patterns and relationships, in ‘Consecutive numbers’, as did Michelle and Emyr, in ‘Building triangles’, and Marya, in ‘Frogs and toads’.

Predict outcomes, make and test hypotheses, reason mathematically when investigating, and analyse and interpret mathematical information	Emyr, in ‘Using the library’ and ‘Building triangles’, Marya, in ‘Frogs and toads’, and Sarah, in ‘Calls and texts’, predicted outcomes, made and tested hypotheses, and reasoned mathematically. Michelle, in ‘Brecon Beacons’, Emyr, in ‘Design a bungalow’, and Adam, in ‘Chances are’, analysed and interpreted mathematical information. Oliver, in ‘Fraction triangles’, and Adam, in ‘Beat that’, both reasoned mathematically, as did Michelle and Emyr in ‘Boxes’.
Describe what they have learned, reflect on their work by evaluating their results in line with the original problem, and justify their conclusions and generalisations	Marya, in ‘Pentominoes’, and Sarah, in ‘Growing shapes’, described what they had learned. Oliver, in ‘Healthy living’, and Adam, in ‘Five a day’, evaluated their results in line with the original problem. Michelle gave a simple justification of her results in ‘Building triangles’, as did Marya in ‘Consecutive numbers.’

Developing communication

Learners develop their communication skills across the curriculum through the skills of **oracy**, **reading**, **writing** and **wider communication**.

In mathematics, learners listen and respond to others. They discuss their work with others using appropriate mathematical language. They read and extract information from mathematical texts. When solving problems, they present their findings and reasoning orally and in writing, using symbols, diagrams, tables and graphs as appropriate.

This section of the skills framework is similarly reflected in the mathematical skills of Communicate mathematically.

Again, there are several examples throughout the learner profiles where the work shows some of these characteristics.

Listen and respond to others	Several of the activities start with a class discussion of different strategies for the activity. Many of the learners' comments during the activities are in response to comments or questions from the teacher or other learners. In 'Probability game', Sarah worked with two of her classmates to decide upon the basis of a game of chance. They discussed various suggestions amongst themselves before deciding upon the final version.
Discuss their work with others using appropriate mathematical language	Marya, in 'Consecutive numbers', described her work and correctly used the terms odd, even, and multiple. In several of the examples, learners use appropriate mathematical terms to explain their work.
Read and extract information from mathematical texts	Michelle, in 'Brecon Beacons', Adam, in 'Chances are', Marya, in 'Network Q Rally', and Emyr, in 'Designing a bungalow', read and extracted information from mathematical texts.
Present their findings and reasoning orally and in writing, using symbols, diagrams, tables and graphs as appropriate	There are numerous examples where the learners have presented their findings orally and/or in writing. Emyr, in 'Building triangles', Marya, in 'Frogs and toads', and Adam, in 'Beat that', used diagrams or symbols to present their work. Michelle and Emyr, in 'Building triangles', Emyr, in 'Using the library', Marya, in 'Frogs and toads' and 'Network Q Rally', Oliver, in 'The chessboard' and 'Healthy living', Adam, in 'Five a day', and Sarah, in 'Growing shapes', all presented their work using tables. Michelle, in 'Our class', Oliver, in 'Healthy living', and Sarah, in 'Calls and texts', used graphs to present their work.

Developing ICT

Learners develop their ICT skills across the curriculum by **finding**, **developing**, **creating and presenting information and ideas** and by using a wide range of equipment and software.

In mathematics, learners use a variety of ICT resources to find, select, organise and interpret information, including real-life data, to explore relationships and patterns in mathematics, to make and test hypotheses and predictions, to create and transform shapes, and to present their findings using text, tables and graphs.

Learners should be given opportunities to use a variety of ICT resources, including simple and graphic calculators, presentation software, databases and spreadsheets, the internet, digital instruments and programmable toys as tools to help develop their mathematical skills and understanding.

There is, therefore, an expectation that the use of ICT will have a central place in activities provided for learners.

Adam, in 'Half-time scores', and Sarah, in 'Probability game', have both used ICT to produce posters of their findings. Adam has set out his findings as tables, and explained his work using formulae and text. Sarah has used text-boxes and tables to set out her work and used text to explain it. Marya, in 'Frogs and toads', has used an investigative programme from the NGfL Cymru website to continue her investigation and check her formula.

Developing number

Learners develop their number skills across the curriculum by **using mathematical information**, **calculating**, **and interpreting and presenting findings**.

In mathematics, learners use their number skills throughout the programme of study when solving problems in a variety of practical and relevant contexts, and when investigating within mathematics itself.

Developing number is clearly an essential ingredient throughout mathematics.

The examples of work in Section 3 include a variety of activities from contexts across the Range. All except the brief examples in 'Bits and bobs' are preceded by a list of some of the mathematical skills and a separate list of some strands from the skills framework, that can be developed through the activity. Strands from Developing number are included in all the skills framework lists.

Mathematics and learning across the curriculum

At Key Stages 2 and 3, learners should be given opportunities to build on the experiences gained during the Foundation Phase, and to promote their knowledge and understanding of Wales, their personal and social development and well-being, and their awareness of the world of work.

Icons have been used in all the subject Orders to indicate specific requirements for aspects of learning across the curriculum. There are no specific requirements in the mathematics Order relating to either the Curriculum Cymreig or personal and social education, so these icons are not used. However, there will be many opportunities to plan mathematics tasks using relevant contexts.

Curriculum Cymreig

Learners should be given opportunities to develop and apply knowledge and understanding of the cultural, economic, environmental, historical and linguistic characteristics of Wales.

Mathematics contributes to the Curriculum Cymreig by offering learners the opportunity to learn and apply mathematics in the context of data from their own local community, from the local and national environment, and from current issues related to Wales. The traditional Welsh vocabulary for some numbers and Welsh quilt and Celtic patterns provide investigative opportunities to contribute to learners' development of a sense of Welsh identity.

Michelle's work on the heights of summits in 'Brecon Beacons' uses a local context for work involving ordering numbers; 'Network Q Rally' provides a Wales context for Marya's calculations involving money.

Personal and social education

Learners should be given opportunities to promote their health and emotional well-being and moral and spiritual development; to become active citizens and promote sustainable development and global citizenship; and to prepare for lifelong learning.

Mathematics contributes to learners' personal and social education by providing opportunities to apply mathematics to real-life problems. It helps them to analyse and interpret information presented to them on environmental and other twenty-first century issues, and to develop an informed and challenging attitude to real-life information, questioning its validity and recognising its implications for their world.

Michelle, in 'Our class', considers some of the environmental issues that the families of her classmates address in their day-to-day lives. In 'Five a day', Adam compares how many items of fruit or vegetables both he and a friend eat over the course of two weeks. Oliver, in 'Healthy living', analyses the results of a survey of his classmates into healthy eating and exercise.

Careers and the world of work

Learners should be given opportunities to develop their awareness of careers and the world of work and how their studies contribute to their readiness for a working life.

Mathematics contributes to learners' awareness of careers and the world of work by providing opportunities to apply mathematics in the context of financial awareness of employment, budgeting, saving and spending.

In the mathematics Order, the icon for careers and the world of work is included with specific references to financial awareness, but there will be other opportunities also to choose a context relating to careers and the world of work.

Marya, in 'Network Q Rally', uses a context of spending, while Adam, in 'Five a day', shows that he is aware of a link between his mother's shopping day and his daily diet. Sarah, in 'Probability game', shows an element of entrepreneurship in her attempts to improve the profitability of her game.

Section 2

Expectations and progression in mathematics

The Curriculum 2008 builds on Mathematical Development in the Foundation Phase; both are written in the format of Skills and Range with the same strand headings. Levels 1, 2 and 3 in the attainment target for mathematics match closely the Foundation Phase Outcomes 4, 5 and 6 for Mathematical Development.

The programmes of study are set out in such a way that they identify overall progression through Key Stage 2 and Key Stage 3. Progression in the development of mathematical skills across the two key stages is shown across the following pages.

1. Solve mathematical problems

Key Stage 2	Key Stage 3
Pupils should be given opportunities to:	
select and use the appropriate mathematics, materials, units of measure and resources to solve problems in a variety of contexts	select, organise and use the mathematics, resources, measuring instruments, units of measure, sequences of operation and methods of computation needed to solve problems
identify, obtain and process information needed to carry out the work	identify what further information or data may be required in order to pursue a particular line of enquiry; formulate questions and identify sources of information
develop their own mathematical strategies and ideas and consider those of others	develop and use their own mathematical strategies and ideas and consider those of others
try different approaches; use a variety of strategies, sequences of operation and methods of calculating	select, trial and evaluate a variety of possible approaches; break complex problems into a series of tasks
use their prior knowledge to find mathematical facts that they have not learned, and to solve numerical problems	use their knowledge of mathematical relationships and structure to derive facts that they have not yet learned, and to solve numerical problems
use flexible and effective methods of computation and recording	use a range of mental, written and calculator computational strategies
estimate solutions to calculations; use alternative strategies to check the accuracy of answers	use a variety of checking strategies, including mental estimation, approximation and inverse operations
appreciate the continuous nature of measures, and that measurement is approximate; estimate measures, and measure to an appropriate degree of accuracy in a range of contexts.	develop their skills of estimating and measuring; recognise limitations on the accuracy of data and measurement; select an appropriate degree of accuracy.

2. Communicate mathematically

Key Stage 2	Key Stage 3
Pupils should be given opportunities to:	
use correct mathematical language, notation, symbols and conventions to talk about or to represent their work to others	use a wide range of mathematical language, notation, symbols and conventions to explain and communicate their work to others
recognise, and generalise in words, patterns that arise in numerical, spatial or practical situations	generalise and explain patterns and relationships in words and symbols; express simple functions in words and symbolically
visualise and describe shapes, movements and transformations	visualise, describe and represent shapes, movements and transformations, using related mathematical language
read information from charts, diagrams, graphs and texts	read mathematical forms of communication, including tables, diagrams, graphs, mathematical texts and ICT
use a variety of methods to represent data	present work clearly, using diagrams, labelled graphs and symbols
devise and refine their own ways of recording	evaluate different forms of recording and presenting information, taking account of the context and audience
explain strategies, methods, choices and conclusions to others in a variety of ways – verbally, graphically, using informal written methods.	explain strategies, methods, choices, conclusions and reasoning to others in a variety of ways, including orally, graphically and in writing.

3. Reason mathematically

Key Stage 2	Key Stage 3
Pupils should be given opportunities to:	
develop a variety of mental and written strategies of computation	extend mental methods of computation to consolidate a range of non-calculator methods
check results and interpret solutions to calculations, including calculator displays; check against the context of the problem that solutions are reasonable	justify how they arrived at a conclusion to a problem; give solutions in the context of the problem; confirm that results are of the right order of magnitude
develop early ideas of algebra and mathematical structure by exploring number sequences and relationships; explain and predict subsequent terms	interpret and use simple algebraic relationships and functions; predict subsequent terms or patterns in number or geometric sequences
investigate and generalise repeating patterns and relationships; search for pattern in their own results	understand general algebraic statements; make and test generalisations; recognise particular examples of a general statement
present and interpret a wide range of graphs and diagrams that represent data; draw conclusions from this data; recognise that some conclusions can be uncertain or misleading	interpret mathematical information presented in a variety of forms; draw inferences from graphs, diagrams and statistics; recognise that some conclusions and graphical representations of data can be misleading; examine critically, improve and justify their choice of mathematical presentation
make and investigate mathematical hypotheses, predictions and conjectures.	explain, follow and compare lines of mathematical argument; make conjectures and hypotheses, design methods to test them, and analyse results to see whether they are valid; appreciate the difference between mathematical explanation and experimental evidence; recognise inconsistencies and bias
	evaluate results by relating them to the initial question or problem; develop an understanding of the reliability of results; recognise that inferences drawn from data analysis may suggest the need for further investigation.

The level descriptions describe attainment in relation to the Skills and the Range. Many of the individual statements in the level descriptions relate to both Skills and Range, reflecting the fact that Skills and Range should be integrated throughout the curriculum – that the Range provides the context for the development of the Skills.

For example:

Level 4: *They draw and interpret frequency diagrams and construct and interpret simple line graphs.*

In Key Stage 2, this relates to *present and interpret a wide range of graphs and diagrams that represent data* in Reason mathematically, and to *use and present data in a variety of ways* in Handling data: Collect, represent and interpret data.

In Key Stage 3, it relates to *interpret mathematical information presented in a variety of forms; draw inferences from graphs, diagrams and statistics* in Reason mathematically, and to *construct appropriate diagrams and graphs to represent discrete and continuous data* and *interpret information given in a wide range of graphs* in Handling data: Collect, represent, analyse and interpret data.

Some of the statements also apply to more than one of the skills.

For example:

Level 5: *They describe situations mathematically using symbols, words and diagrams and draw their own conclusions, explaining their reasoning.*

This relates to both Communicate mathematically and Reason mathematically.

For planning purposes, it might be helpful to trace broad expectations and progression in terms of the mathematical skills through the level descriptions. The following tables show one way of doing this. However, the level at which a learner demonstrates a skill will depend to a large extent on the nature of the task – on whether or not the task provides sufficient challenge for learners to develop their skills to higher levels. In practice, also, learners may be using two or more different skills simultaneously. For example, more often than not learners need to reason mathematically when solving problems, and when they are working in pairs or groups they could well be reasoning mathematically and communicating their ideas to their peers simultaneously. In what follows, particular references to Range have been included in brackets where necessary to give a context to the Skill level.

1. Solve mathematical problems

Level 1	Pupils use mathematics as an integral part of classroom activities.
Level 2	Pupils choose the appropriate operation (when solving addition or subtraction problems).
Level 3	Pupils organise their work, check results and try different approaches.
Level 4	Pupils develop their own strategies for solving problems. Pupils choose and use suitable units and instruments.
Level 5	Pupils identify and obtain information to solve problems. Pupils make sensible estimates (of a range of everyday measures).
Level 6	Pupils solve complex problems by breaking them down into smaller tasks.
Level 7	Pupils consider alternative approaches.
Level 8	Pupils develop and follow alternative approaches, reflecting on their own lines of enquiry and using a range of mathematical techniques.
Exceptional Performance	Pupils solve problems (using intersections and gradients of graphs, Pythagoras' theorem and trigonometric ratios).

2. Communicate mathematically	
Level 1	Pupils represent their work with objects or pictures and discuss it. Pupils use everyday language (to compare and describe positions and properties of regular shapes).
Level 2	Pupils talk about their work using familiar mathematical language, and represent it using symbols and simple diagrams.
Level 3	Pupils talk about and explain their work. Pupils interpret mathematical symbols and diagrams.
Level 4	Pupils present information and results systematically. Pupils describe (number) patterns and relationships, and use simple formulae expressed in words.
Level 5	Pupils describe situations mathematically using symbols, words and diagrams.
Level 6	Pupils discuss information presented in a variety of mathematical forms. Pupils describe in words the rule for the next term or the nth term of a sequence (where the rule is linear). Pupils represent mappings expressed algebraically.
Level 7	Pupils describe in symbols the next term or the nth term of a sequence (with a quadratic rule).
Level 8	Pupils convey mathematical or statistical meaning through precise and consistent use of symbols.
Exceptional Performance	Pupils use mathematical language and symbols effectively in presenting a reasoned argument. Pupils express general laws in symbolic form.

3. Reason mathematically	
Level 1	Pupils measure and order objects using direct comparison. Pupils recognise, use and make repeating patterns. Pupils sort and classify objects, demonstrating the criterion they have used.
Level 2	Pupils sort objects and classify them using more than one criterion. Pupils record their results (in simple lists, tables, diagrams and block graphs).
Level 3	Pupils find particular examples that satisfy a general statement. Pupils classify (shapes) in various ways. Pupils interpret information (simple tables and lists, bar charts and pictograms).
Level 4	Pupils search for a solution by trying out ideas of their own. Pupils check their results are reasonable (by considering the context or the size of the numbers). Pupils interpret (frequency diagrams and simple line graphs).
Level 5	Pupils check whether their results are sensible in the context of the problem. Pupils draw their own conclusions, explaining their reasoning. Pupils make general statements of their own, based on available evidence. Pupils appreciate that different outcomes may result from repeating an experiment.
Level 6	Pupils give some mathematical justifications to support their methods, arguments or conclusions. Pupils interpret (frequency diagrams, pie charts and scatter diagrams).
Level 7	Pupils justify their generalisations, arguments or solutions. Pupils appreciate the difference between mathematical explanation and experimental evidence. Pupils specify and test hypotheses, taking account of bias.
Level 8	Pupils examine and discuss generalisations or solutions they have reached. Pupils interpret (graphs of linear, quadratic, cubic and reciprocal functions, and graphs that model real situations, cumulative frequency tables and diagrams). Pupils compare distributions and make inferences.
Exceptional Performance	Pupils give reasons for the choices they make when investigating within mathematics. Pupils interpret (graphs based on trigonometric functions, histograms). Pupils present a convincing reasoned argument, including mathematical justification.

However, although it may be useful to separate out the different aspects of the level descriptions in order to see how attainment is characterised as learners' progress through the levels, it cannot be over-emphasised that effective planning and learning in mathematics bring together work on these different aspects. When judgements are made about a learner's attainment at the end of Key Stages 2 and 3, it is important to consider each level description as a whole. This ties in with reporting a single outcome for Mathematical Development at the end of the Foundation Phase, and a single grade for mathematics at GCSE.

Section 3

Making judgements at the end of Key Stages 2 and 3

This section shows how level descriptions can be used when making judgements about which level best describes a learner's overall performance at the end of Key Stages 2 and 3.

You may find the following points useful when considering the profiles in this section.

- The learner profiles are not presented as a model for how you should collect evidence about your learners. Although you will want to be able to explain why you have awarded a particular level to a learner at the end of the key stage, there is no requirement for judgements to be explained in this way or supported by detailed collections of evidence on each learner. Decisions about collecting evidence, and about its purpose and use, are matters for teachers working within an agreed school policy.
- The commentaries on the pieces of work have been written to explain the judgement made about a learner's performance. They are not intended as an example of a report to parents/guardians.
- The materials in each learner profile can only represent a small part of the information and experiences that make up a teacher's knowledge of each learner. They do not reflect the extent of the knowledge of each learner that you will have built up over time across a range of different contexts. You will use this knowledge to make a rounded judgement about the level that best fits each learner's performance.
- You will arrive at judgements by taking into account strengths and weaknesses in performance across a range of contexts and over a period of time. Opportunities will need to be provided for learners to demonstrate attainment in all aspects of the level descriptions.
- Some of your learners may need to use a range of alternative forms of communication to show what they know, what they understand and what they can do.

Several of the activities in the examples that follow are based on ideas from the *Optional Assessment Materials for Mathematics at Key Stage 2* and *Optional Assessment Materials for Mathematics at Key Stage 3*, produced by ACCAC in 2003.

In the examples, the two tables list mathematical skills and strands from the skills framework that can be developed within each activity. However, there is no expectation that the work of all learners doing these activities would show evidence of all of these skills, nor that these are the only skills that may be in evidence.

Key Stage 2

Michelle

Level 3

Michelle is an 11-year-old learner in Key Stage 2.

Her teacher knows much more about Michelle's performance than can be included here. However, this profile has been selected to illustrate characteristic features of Michelle's work across a range of activities. Each example is accompanied by a brief commentary to provide a context and indicate particular qualities in the work.

Michelle's teacher judges that her performance in mathematics is best described as Level 3.

Activity | Boxes

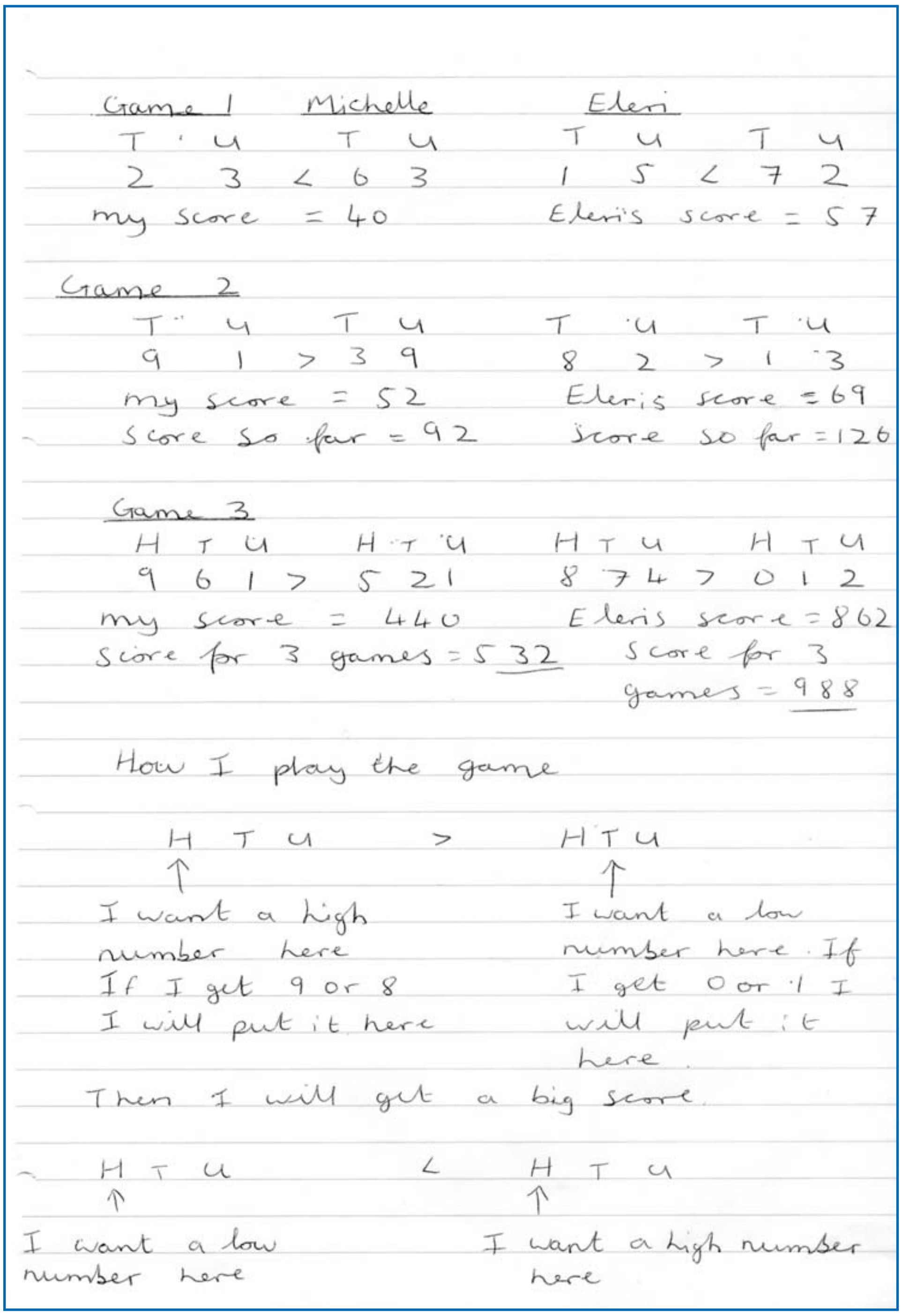
The focus of this activity is a game using place value and subtraction with numbers up to 1000. The game is played in pairs where pupils take turns to generate single digits using a set of cards. As each digit is generated, the player decides where to place it to form a valid inequality using either two-digit or three-digit numbers, and then replaces the card. The winner of the game is the player whose pair of numbers has the greater difference. In playing the game, pupils develop a strategy for placing a digit by considering the likelihood of getting a higher or lower number, and explain their strategy.

There are opportunities for pupils to:

- develop their own mathematical strategies and ideas and consider those of others
- use flexible and effective methods of computation
- explain strategies, methods and choices
- use correct mathematical language, notation and symbols to talk about or represent their work.

Pupils are also:

- activating prior skills, knowledge and understanding
- thinking about cause and effect
- using numbers.



Game 1 Michelle Eleri

T U T U T U T U

2 3 < 6 3 1 5 < 7 2

my score = 40 Eleri's score = 57

Game 2

T U T U T U T U

9 1 > 3 9 8 2 > 1 3

my score = 52 Eleri's score = 69

Score so far = 92 Score so far = 126

Game 3

H T U H T U H T U H T U

9 6 1 > 5 2 1 8 7 4 > 0 1 2

my score = 440 Eleris score = 862

Score for 3 games = 532 Score for 3 games = 988

How I play the game

H T U > H T U

↑ ↑

I want a high number here If I get 9 or 8 I will put it here

I want a low number here. If I get 0 or 1 I will put it here.

Then I will get a big score.

H T U < H T U

↑ ↑

I want a low number here

I want a high number here

Commentary

Michelle interpreted correctly the mathematical symbols of ‹ and › and placed the digits appropriately in the H, T and U columns to create inequalities using two-digit or three-digit numbers. She considered the value of the digit before placing it to make either the higher or the lower number. She calculated the score correctly using subtraction, and, in consultation with her partner, checked her working to decide who had won the game. She developed and explained her strategy for placing the digits in terms of 'high' and 'low' numbers. Her work shows that she can use place value in numbers up to 1000.

Activity | Brecon Beacons

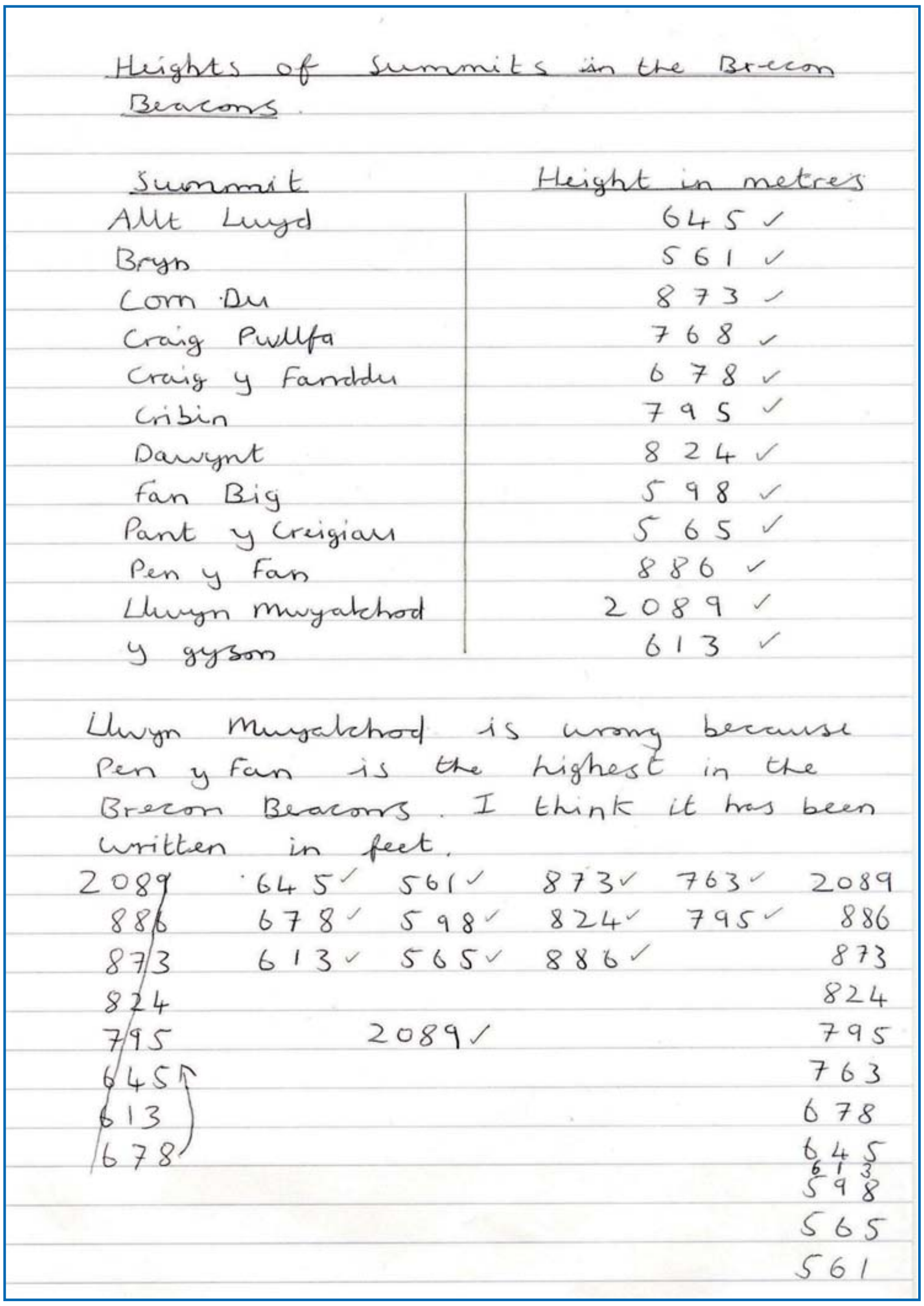
The focus of this activity is ordering numbers up to 1000. Pupils are presented with a table of the heights of mountains in the Brecon Beacons, within which there is a deliberate error. They are asked to identify the error, to explain how they identified it and to correct it, and then to reorganise the data, starting with the highest and finishing with the lowest summit height.

There are opportunities for pupils to:

- select and use the appropriate mathematics
- identify, obtain and process information
- read information from charts
- devise and refine their own ways of recording.

Pupils are also:

- determining the process/method
- thinking logically
- monitoring progress
- using the number system
- interpreting data and presenting findings.



Heights of Summits in the Brecon Beacons.

Summit	Height in metres
Allt Lwyd	645 ✓
Bryn	561 ✓
Corn Du	873 ✓
Craig Pwllfa	768 ✓
Craig y Fanddu	678 ✓
Cribin	795 ✓
Dawynt	824 ✓
Fan Big	598 ✓
Pant y Creigiau	565 ✓
Pen y Fan	886 ✓
Llwyn mwyalchod	2089 ✓
y gyson	613 ✓

Llwyn Mwyalchod is wrong because Pen y Fan is the highest in the Brecon Beacons. I think it has been written in feet.

2089	645✓	561✓	873✓	763✓	2089
886	678✓	598✓	824✓	795✓	886
873	613✓	565✓	886✓		873
824					824
795		2089✓			795
645					763
613					678
678					645
					613
					598
					565
					561

Commentary

Michelle correctly identified the error, based on her local geographical knowledge rather than on mathematical understanding. She was able to suggest a reason for the error, but did not correct the error in her subsequent work. She began to list mountain heights in decreasing order, using a strategy based on the hundreds digit, and adjusted her system when she spotted that she had made a mistake. In her second attempt, she reorganised the mountain heights systematically into groups depending on the hundreds digit, and monitored her progress by ticking off all the numbers as she used them. This strategy helped her to overcome her difficulty and she was then able to order the heights correctly.

Activity | Consecutive numbers

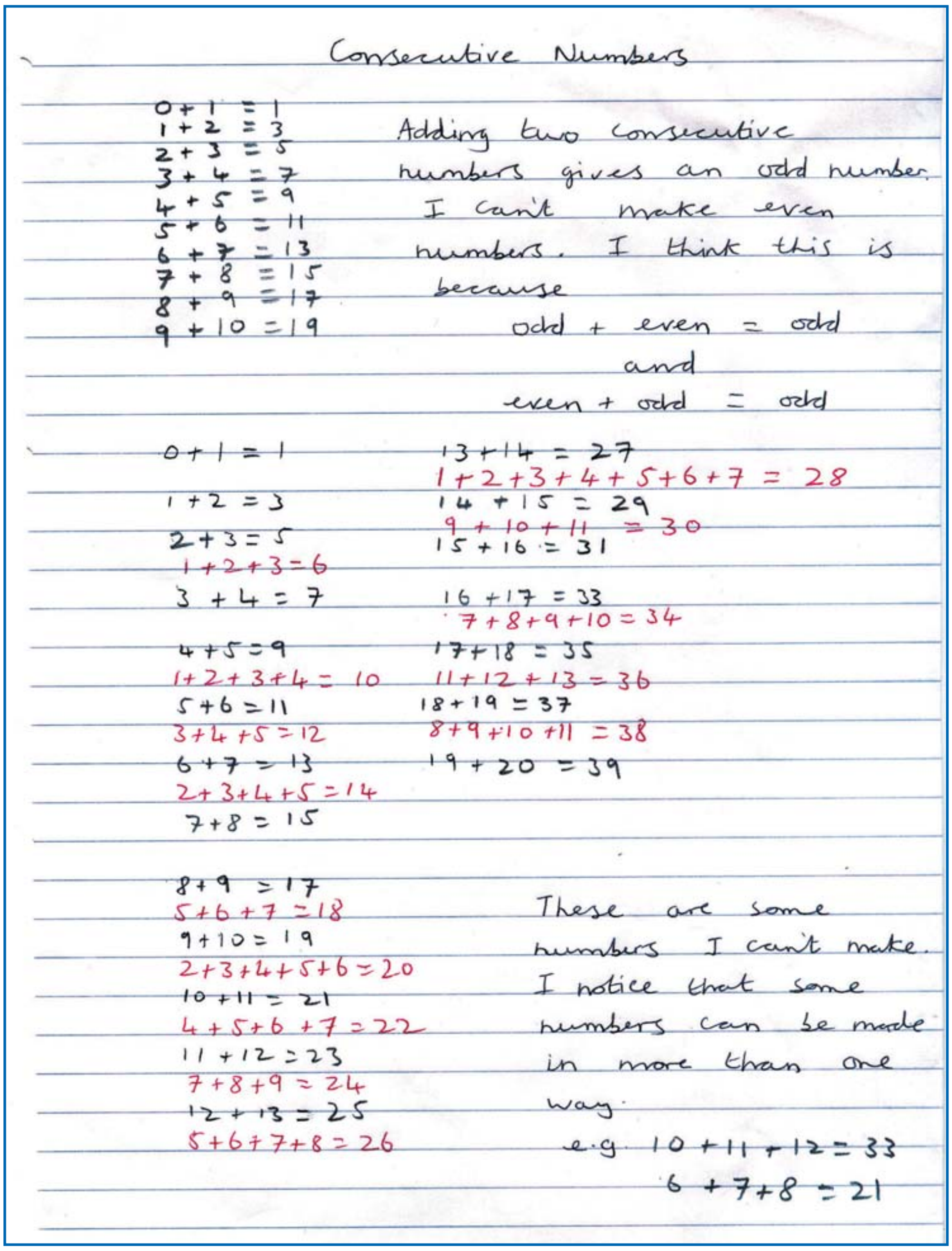
The focus of this activity is to identify and explain patterns that arise when adding together consecutive numbers.

There are opportunities for pupils to:

- develop their own mathematical strategies and ideas
- devise and refine their own ways of recording
- use flexible and effective methods of computation and recording
- estimate solutions to calculations; use alternative strategies to check the accuracy of answers
- recognise, and generalise in words, patterns that arise
- use their prior knowledge to find mathematical facts that they have not learned.

Pupils are also:

- determining the process/method and strategy
- thinking logically and seeking patterns
- communicating information
- talking about and explaining work
- recording data and presenting findings.



Consecutive Numbers

0 + 1 = 1
1 + 2 = 3
2 + 3 = 5
3 + 4 = 7
4 + 5 = 9
5 + 6 = 11
6 + 7 = 13
7 + 8 = 15
8 + 9 = 17
9 + 10 = 19

Adding two consecutive numbers gives an odd number. I can't make even numbers. I think this is because

odd + even = odd
and
even + odd = odd

0 + 1 = 1
1 + 2 = 3
2 + 3 = 5
1 + 2 + 3 = 6
3 + 4 = 7
4 + 5 = 9
1 + 2 + 3 + 4 = 10
5 + 6 = 11
3 + 4 + 5 = 12
6 + 7 = 13
2 + 3 + 4 + 5 = 14
7 + 8 = 15

13 + 14 = 27
1 + 2 + 3 + 4 + 5 + 6 + 7 = 28
14 + 15 = 29
9 + 10 + 11 = 30
15 + 16 = 31
16 + 17 = 33
7 + 8 + 9 + 10 = 34
17 + 18 = 35
11 + 12 + 13 = 36
18 + 19 = 37
8 + 9 + 10 + 11 = 38
19 + 20 = 39

8 + 9 = 17
5 + 6 + 7 = 18
9 + 10 = 19
2 + 3 + 4 + 5 + 6 = 20
10 + 11 = 21
4 + 5 + 6 + 7 = 22
11 + 12 = 23
7 + 8 + 9 = 24
12 + 13 = 25
5 + 6 + 7 + 8 = 26

These are some numbers I can't make. I notice that some numbers can be made in more than one way.

e.g. 10 + 11 + 12 = 33
6 + 7 + 8 = 21

Commentary

Michelle found a variety of ways of adding two or more consecutive numbers to try to make totals for numbers 1 to 31. She worked systematically, first adding two consecutive numbers when she noticed that this always gave an odd number. She recorded these on alternate lines on her page, and then tried to fill in the gaps by adding three or more consecutive numbers. She recorded some of her additions separately on rough paper; this showed that she used a variety of ways of adding the numbers, including 'near doubles' (14 + 15 = 14 + 14 add 1), adding three or more numbers two at a time (8 + 9 + 10 = 17 + 10), and 'splitting' the tens and units (14 + 15 = 10 + 4 + 10 + 5). She continued to fill in most of the gaps successfully using four or more consecutive numbers, accepting that there were some that she could not complete.

Activity | Building triangles

In this activity, pupils aim to identify patterns in a sequence, find a rule in words for the pattern, use this rule to predict later entries in the pattern, and prove their rule for a specific entry. They use matchsticks or straws to develop patterns of increasing numbers of equilateral triangles laid edge-to-edge in a straight line, then attempt the same activity using squares.

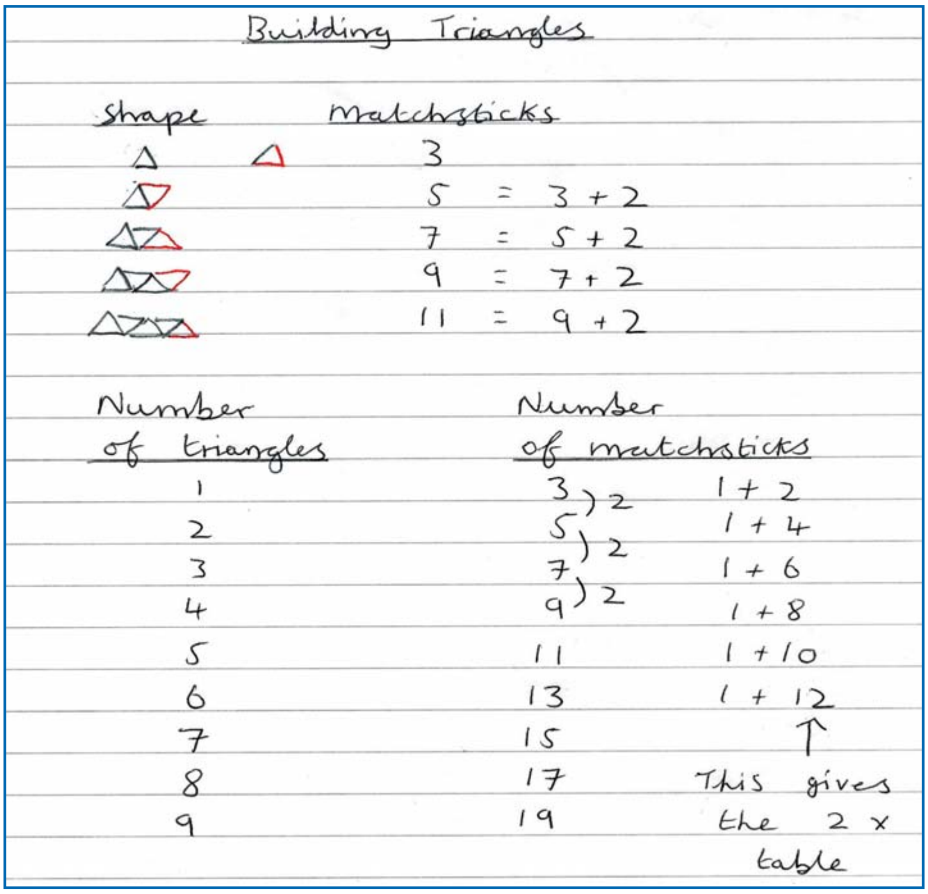
There are opportunities for pupils to:

- develop their own mathematical strategies and ideas
- devise and refine their own ways of recording
- recognise, and generalise in words, patterns that arise
- investigate and generalise repeating patterns and relationships; search for pattern in their own results
- use a variety of methods to represent data
- develop early ideas of algebra and mathematical structure by exploring number sequences and relationships; explain and predict subsequent terms.

Pupils are also:

- determining the process/method and strategy
- thinking logically and seeking patterns
- organising ideas and information
- recording and interpreting data and presenting findings.

The numbers for the triangles go up in twos, because I add two sides each time.

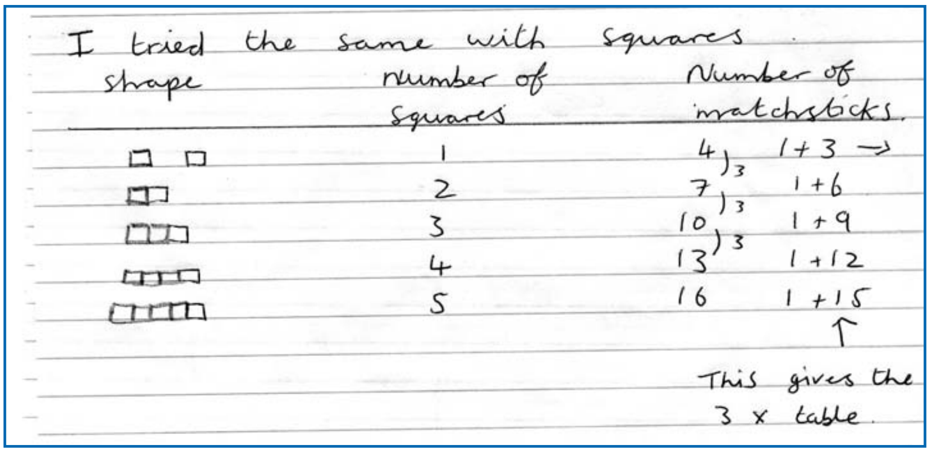


Building Triangles

Shape	Matchsticks
△ △	3
△▽	5 = 3 + 2
△▽△	7 = 5 + 2
△▽△▽	9 = 7 + 2
△▽△▽△	11 = 9 + 2

Number of triangles	Number of matchsticks	
1	3) 2	1 + 2
2	5) 2	1 + 4
3	7) 2	1 + 6
4	9	1 + 8
5	11	1 + 10
6	13	1 + 12
7	15	↑
8	17	This gives
9	19	the 2 x table

The numbers for the squares go up in threes because I add three sides each time.



I tried the same with squares.

shape	number of Squares	Number of matchsticks.	
□ □	1	4)3	1 + 3 →
□□	2	7)3	1 + 6
□□□	3	10)3	1 + 9
□□□□	4	13	1 + 12
□□□□□	5	16	1 + 15
			↑

This gives the 3 x table.

Commentary

Michelle used matchsticks to construct her triangles. She built up her patterns and went on to record and present her results systematically. She recognised the pattern of adding two that emerged for the triangles and used colour in her diagram to show this. She could then find the number of matchsticks needed for nine triangles by extending her table and repeatedly adding two. Her diagram also led her to think of the first triangle as one with two added sides; she then considered how many more than one there was each time, which led her to discover a pattern involving the two times table. When she went on to consider the pattern of squares, she used the same strategy of counting the additional matches, and using a similar strategy to see the three times table in the pattern. She did not use either of these results to predict the number needed for a particular case, though her comments to her teacher gave a simple justification of her results.

Activity | Our class

As part of a theme of 'Same and Different', pupils are given the opportunity to conduct a survey in their own class. They first discuss a variety of possibilities to survey, and Michelle chooses to gather data on environmental issues. She decides on the information she wants to gather, records her data using a tally chart, and chooses to represent it using a horizontal bar chart. She then draws her own conclusions from her data.

As a second step in this work, pupils are given the opportunity to interpret the representations of a partner. Michelle copies a pictogram constructed by her friend Siân about the different arrangements her classmates had for their midday meal, and draws her own conclusions from this.

There are opportunities for pupils to:

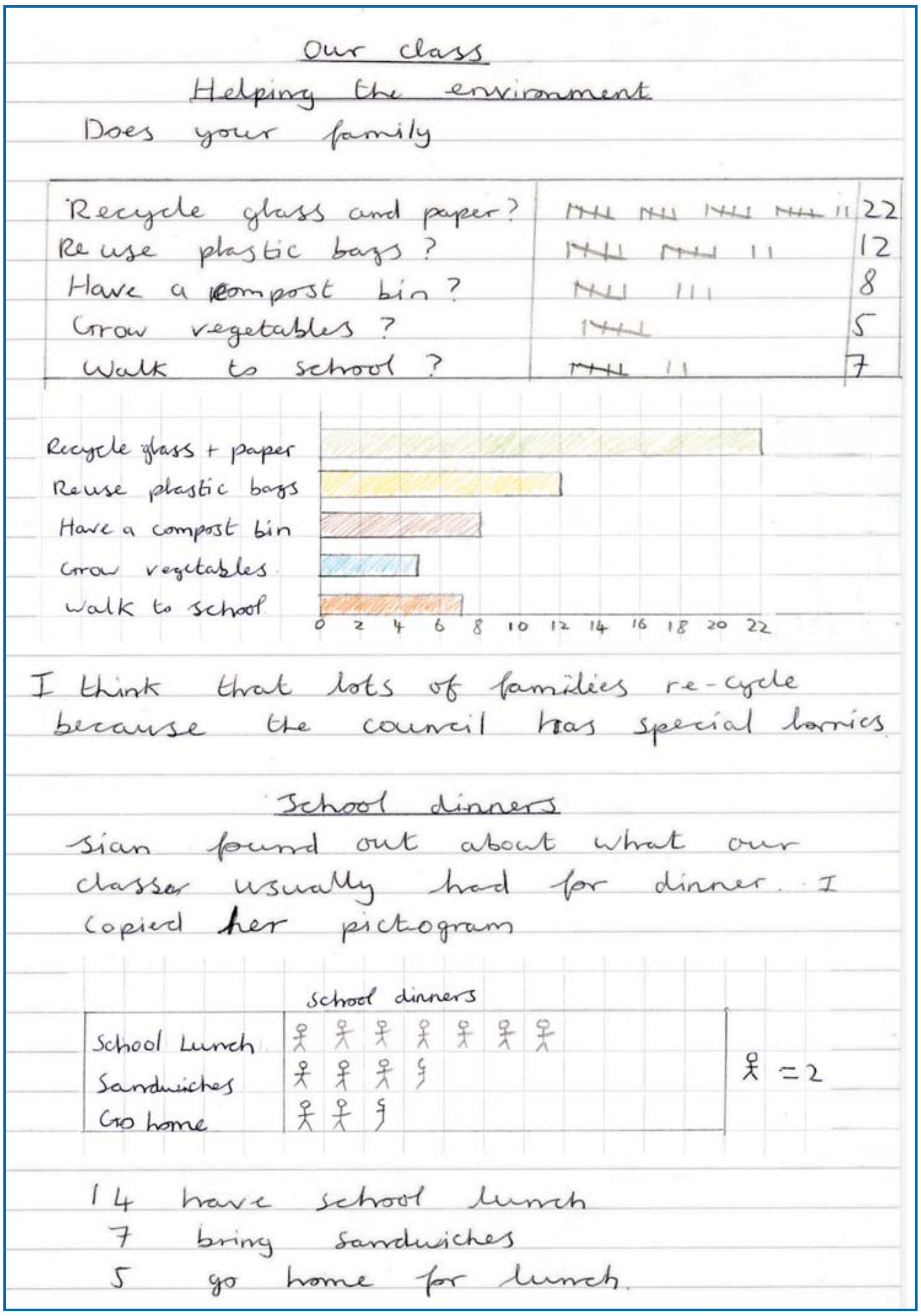
- identify, obtain and process information needed to carry out the work
- use a variety of methods to represent data
- read information from charts, diagrams, graphs and text
- devise and refine their own ways of recording
- present and interpret graphs and diagrams that represent data; draw conclusions from this data
- explain choices and conclusions in a variety of ways.

Pupils are also:

- determining the process/method
- gathering information
- using numbers
- recording and interpreting data and presenting findings
- communicating information
- talking about and explaining work.

I chose the environment because I am interested in it. We have a compost bin in the garden and I have planted some tomato seeds in a pot.

I chose a horizontal bar chart because I thought the headings would be too long for a vertical bar chart.



Our class

Helping the environment

Does your family

Recycle glass and paper?	𝍸 𝍸 𝍸 𝍸 ||	22
Re use plastic bags?	𝍸 𝍸 ||	12
Have a compost bin?	𝍸 |||	8
Grow vegetables?	𝍸	5
Walk to school?	𝍸 ||	7

I think that lots of families re-cycle because the council has special lorries

School dinners

Sian found out what our classes usually had for dinner. I copied her pictogram

14 have school lunch
7 bring sandwiches
5 go home for lunch.

Commentary

Michelle decided on her own topic for the survey, based on her own interests, and identified the data she would need. She justified her choice of the method she used to represent her data, and made a sensible conclusion about her findings. She interpreted Siân's pictogram correctly.

Summary and overall judgement

Levels 2, 3 and 4 were considered and Level 3 was judged to be the best fit.

In ‘Consecutive numbers’ and ‘Boxes’, Michelle *talked about her work using familiar mathematical language,* and *represented it using simple diagrams* when she used colour in ‘Building triangles’ to show how she built up the triangles. This is characteristic of Level 2.

In ‘Building triangles’, ‘Brecon Beacons’ and ‘Consecutive numbers’, Michelle has shown that she can *organise her work, check results and try different approaches.* In ‘Boxes’, ‘Building triangles’ and ‘Our class’, she *talked about and explained her work*. In ‘Boxes’, she *used and interpreted mathematical symbols.* In ‘Brecon Beacons’, she *used place value in numbers up to 1000* to order the heights of the mountains. Her work on ‘Consecutive numbers’ and ‘Boxes’ shows that she has *developed mental strategies for adding and subtracting numbers with at least two digits.* Her work on ‘Brecon Beacons’ also shows that she can *extract information presented in a simple list.* In ‘Our class’, she *constructed a bar chart and interpreted a pictogram*. These are all characteristics of Level 3.

In ‘Consecutive numbers’ and ‘Building triangles’, she *presented information and results systematically,* a characteristic of Level 4. In ‘Consecutive numbers’ and ‘Matchsticks’, she has begun to *recognise and describe number patterns,* a characteristic of Level 4, but she does not yet *use simple formulae expressed in words*, as shown by the way she needed to extend the table in ‘Building triangles’ to find the number of matches needed for nine triangles.

Key Stage 2

Emyr

Level 4

Emyr is a 10-year-old learner in Key Stage 2.

His teacher knows much more about Emyr's performance than can be included here. However, this profile has been selected to illustrate characteristic features of Emyr's work across a range of activities. Each example is accompanied by a brief commentary to provide a context and indicate particular qualities in the work.

Emyr's teacher judges that his performance in mathematics is best described as Level 4.

Activity | Boxes

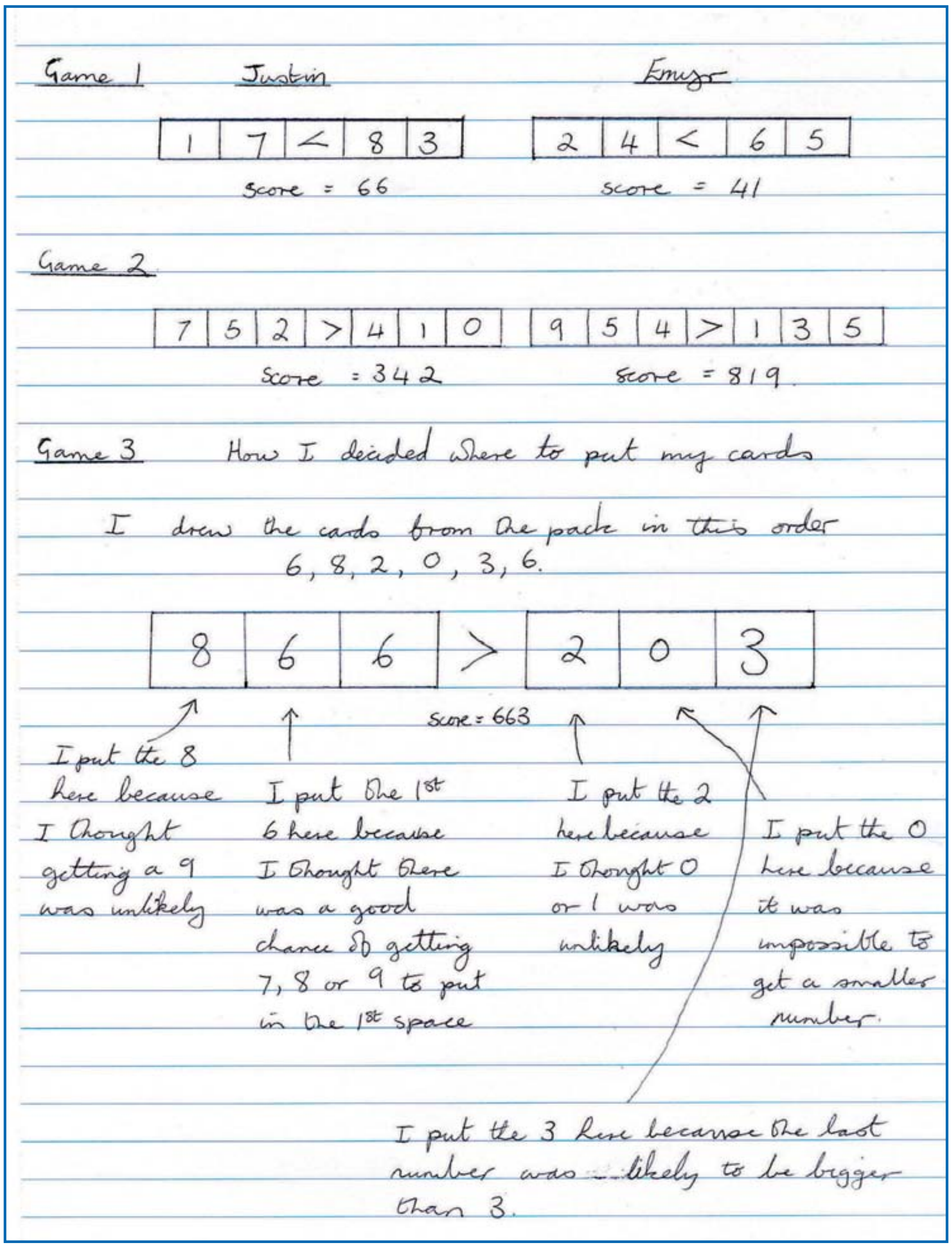
The focus of this activity is a game using place value and subtraction with numbers up to 1000. The game is played in pairs where pupils take turns to generate single digits using a set of cards. As each digit is generated, the player decides where to place it to form a valid inequality using either two-digit or three-digit numbers, and then replaces the card. The winner of the game is the player whose pair of numbers has the greater difference. In playing the game, pupils develop a strategy for where to place a digit by considering the likelihood of getting a higher or lower number, and explain their strategy.

There are opportunities for pupils to:

- develop their own mathematical strategies and ideas and consider those of others
- use flexible and effective methods of computation
- explain strategies, methods and choices
- use correct mathematical language, notation and symbols to talk about or represent their work.

Pupils are also:

- activating prior skills, knowledge and understanding
- thinking about cause and effect
- using numbers.



Game 1 Justin Emyr

1	7	<	8	3

Score = 66

2	4	<	6	5

score = 41

Game 2

7	5	2	>	4	1	0

Score = 342

9	5	4	>	1	3	5

score = 819

Game 3 How I decided where to put my cards

I drew the cards from the pack in this order 6, 8, 2, 0, 3, 6.

8	6	6	>	2	0	3

Score = 663

I put the 8 here because I thought getting a 9 was unlikely

I put the 1st 6 here because I thought there was a good chance of getting 7, 8 or 9 to put in the 1st space

I put the 2 here because I thought 0 or 1 was unlikely

I put the 0 here because it was impossible to get a smaller number.

I put the 3 here because the last number was likely to be bigger than 3.

Commentary

Emyr created appropriate inequalities using two-digit then three-digit numbers. He has started to use the language of probability, explaining how he could improve his chance of winning an inequality by considering the size of the digits and the likelihood of getting particular digits. He has demonstrated mental strategies in using place value of numbers up to 1000, and can add and subtract numbers up to three digits. He is beginning to think logically, and has checked that his results are sensible by considering the context and the size of the digits.

Activity | Using the library

As part of their work on using the library, pupils make and investigate a hypothesis about the relationship between word length and reader age in two fiction books. They choose two books appropriate for different ages and give a reason for which book they think will have the longer words. They decide how to collect and present data, which they then record and analyse by finding the mode and range of the data, and explain their findings.

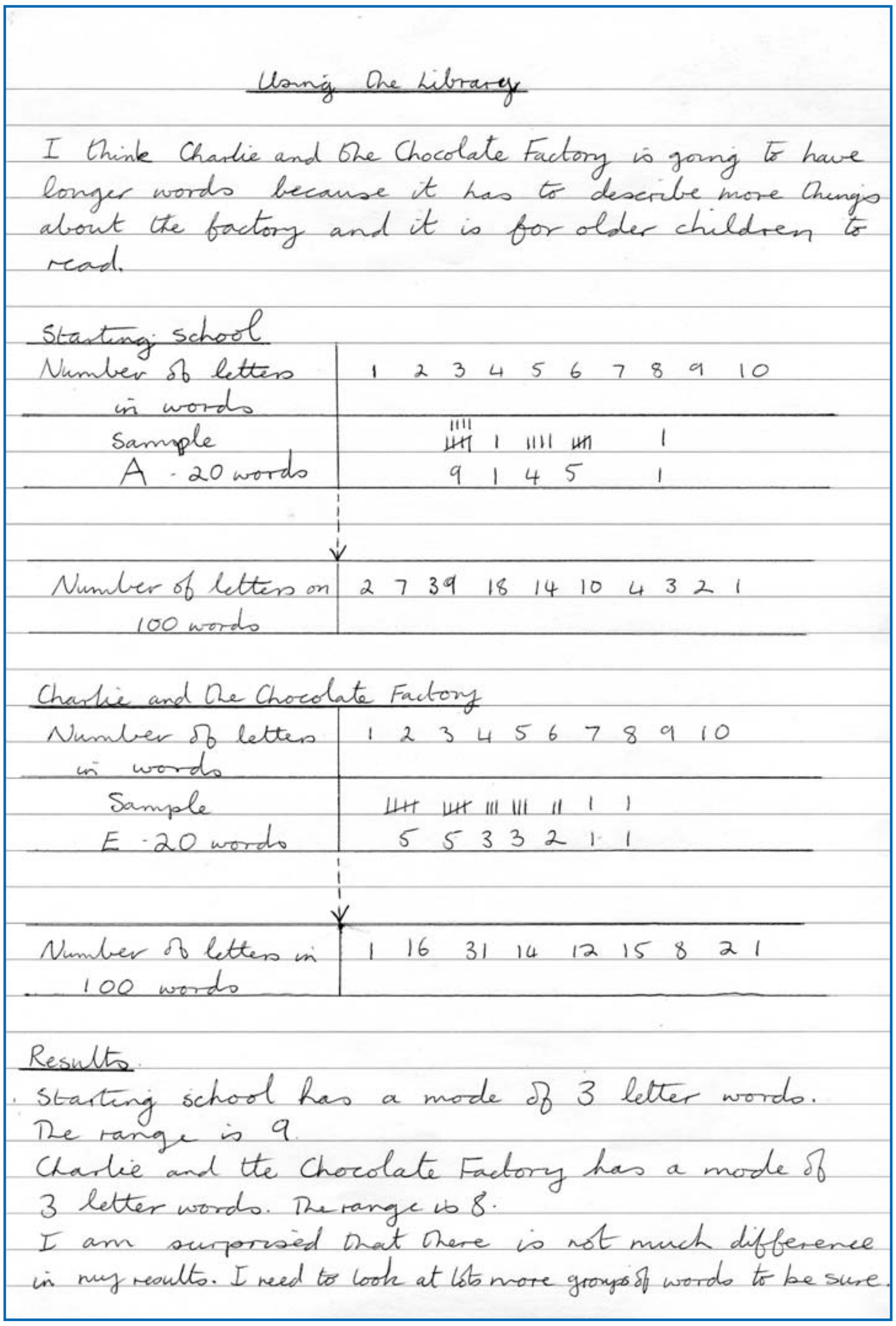
There are opportunities for pupils to:

- identify, obtain and process information
- use correct mathematical language, notation and symbols to represent their work
- make and investigate mathematical hypotheses
- devise and refine their own ways of recording
- explain strategies, methods, choices and conclusions
- develop a variety of mental strategies of computation.

Pupils are also:

- determining the process/method and strategy
- recording data and presenting findings
- reviewing the process/method.

I think *Charlie and the Chocolate Factory* will be harder to read than *Starting School*.



Using the Library

I think Charlie and the Chocolate Factory is going to have longer words because it has to describe more things about the factory and it is for older children to read.

Starting school

Number of letters in words	1	2	3	4	5	6	7	8	9	10
Sample A - 20 words			𝍸 |||| 9	| 1	|||| 4	𝍸 5		| 1		
Number of letters on 100 words	2	7	39	18	14	10	4	3	2	1

Charlie and the Chocolate Factory

Number of letters in words	1	2	3	4	5	6	7	8	9	10
Sample E - 20 words		𝍸 5	𝍸 5	||| 3	||| 3	|| 2	| 1	| 1		
Number of letters in 100 words	1	16	31	14	12	15	8	2	1	

Results.

Starting school has a mode of 3 letter words. The range is 9.

Charlie and the Chocolate Factory has a mode of 3 letter words. The range is 8.

I am surprised that there is not much difference in my results. I need to look at lots more groups of words to be sure.

Commentary

Emyr chose as his two books, *Starting School* by Janet and Alan Ahlberg and *Charlie and the Chocolate Factory* by Roald Dahl. He made an initial hypothesis that *Charlie and the Chocolate Factory* would be harder to read than *Starting School* because he thought that it would have longer words, and gave suitable reasons for his decision. He decided to count the numbers of letters in 100 words in each book by looking at five samples of twenty consecutive words for each book. He chose his samples by 'flipping through the pages'. He recorded his results systematically, drawing up tally charts for each individual sample of words and then finding the totals. He correctly identified the mode and range of the sets of data, and found that these were very similar so that he could not decide whether or not his hypothesis was correct. He suggested that he would need to consider lots more samples to decide one way or another.

Activity | Consecutive numbers

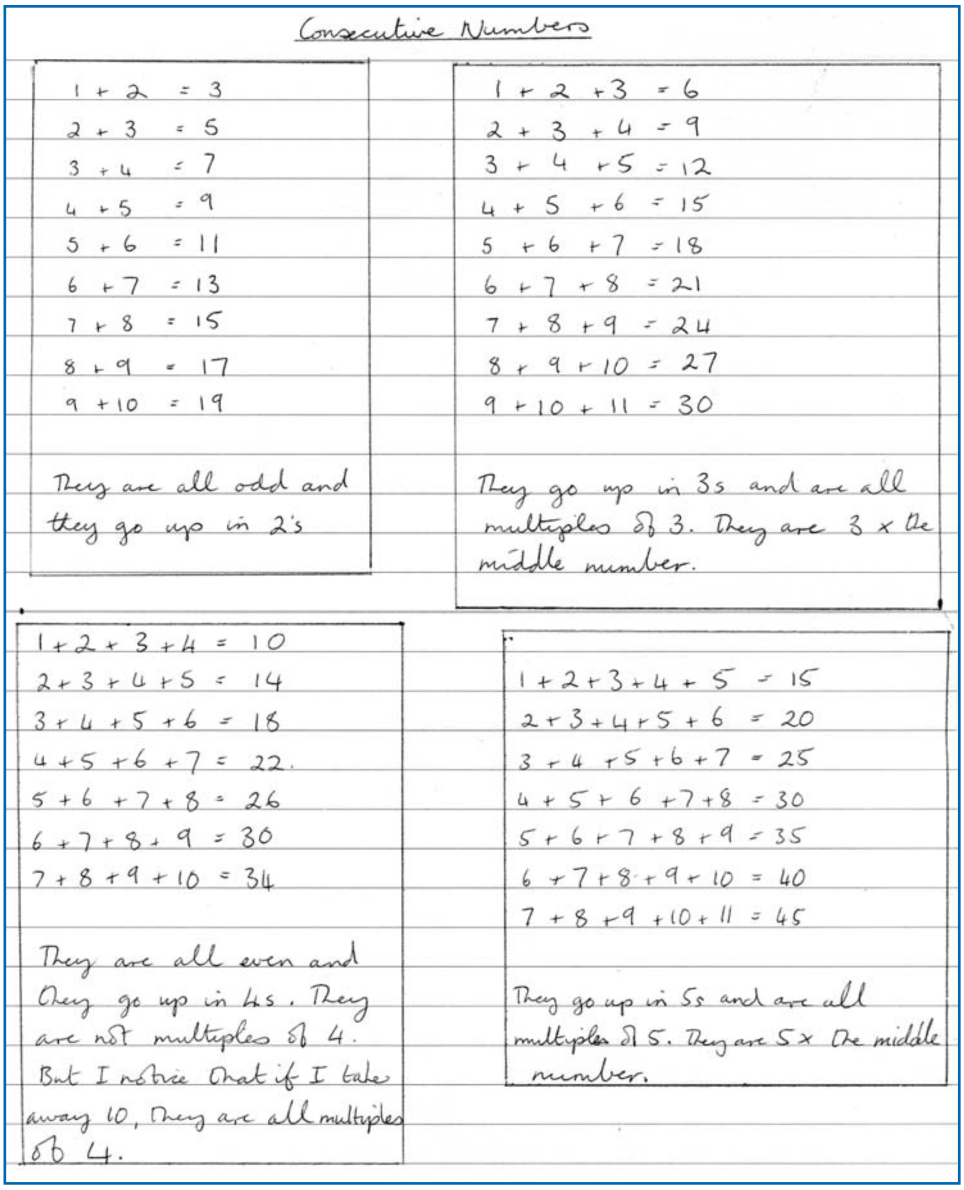
The focus of this activity is to identify and explain patterns that arise when adding together consecutive numbers.

There are opportunities for pupils to:

- develop their own mathematical strategies and ideas
- devise and refine their own ways of recording
- use flexible and effective methods of computation and recording
- estimate solutions to calculations; use alternative strategies to check the accuracy of answers
- recognise, and generalise in words, patterns that arise
- use their prior knowledge to find mathematical facts that they have not learned.

Pupils are also:

- determining the process/method and strategy
- thinking logically and seeking patterns
- communicating information
- talking about and explaining work
- recording data and presenting findings.



Consecutive Numbers

1 + 2 = 3
2 + 3 = 5
3 + 4 = 7
4 + 5 = 9
5 + 6 = 11
6 + 7 = 13
7 + 8 = 15
8 + 9 = 17
9 + 10 = 19

They are all odd and they go up in 2's

1 + 2 + 3 = 6
2 + 3 + 4 = 9
3 + 4 + 5 = 12
4 + 5 + 6 = 15
5 + 6 + 7 = 18
6 + 7 + 8 = 21
7 + 8 + 9 = 24
8 + 9 + 10 = 27
9 + 10 + 11 = 30

They go up in 3s and are all multiples of 3. They are 3 x the middle number.

1 + 2 + 3 + 4 = 10
2 + 3 + 4 + 5 = 14
3 + 4 + 5 + 6 = 18
4 + 5 + 6 + 7 = 22.
5 + 6 + 7 + 8 = 26
6 + 7 + 8 + 9 = 30
7 + 8 + 9 + 10 = 34

They are all even and they go up in 4s. They are not multiples of 4. But I notice that if I take away 10, they are all multiples of 4.

1 + 2 + 3 + 4 + 5 = 15
2 + 3 + 4 + 5 + 6 = 20
3 + 4 + 5 + 6 + 7 = 25
4 + 5 + 6 + 7 + 8 = 30
5 + 6 + 7 + 8 + 9 = 35
6 + 7 + 8 + 9 + 10 = 40
7 + 8 + 9 + 10 + 11 = 45

They go up in 5s and are all multiples of 5. They are 5 x the middle number.

Commentary

Emyr developed his own strategy for investigating the problem and used a variety of mental and written methods of computation. He approached the task systematically and identified patterns that arose. He presented information and results in a clear and organised way, described mathematical situations in words and formulated his own conclusions. He did not explain the patterns he found.

Activity | Building triangles

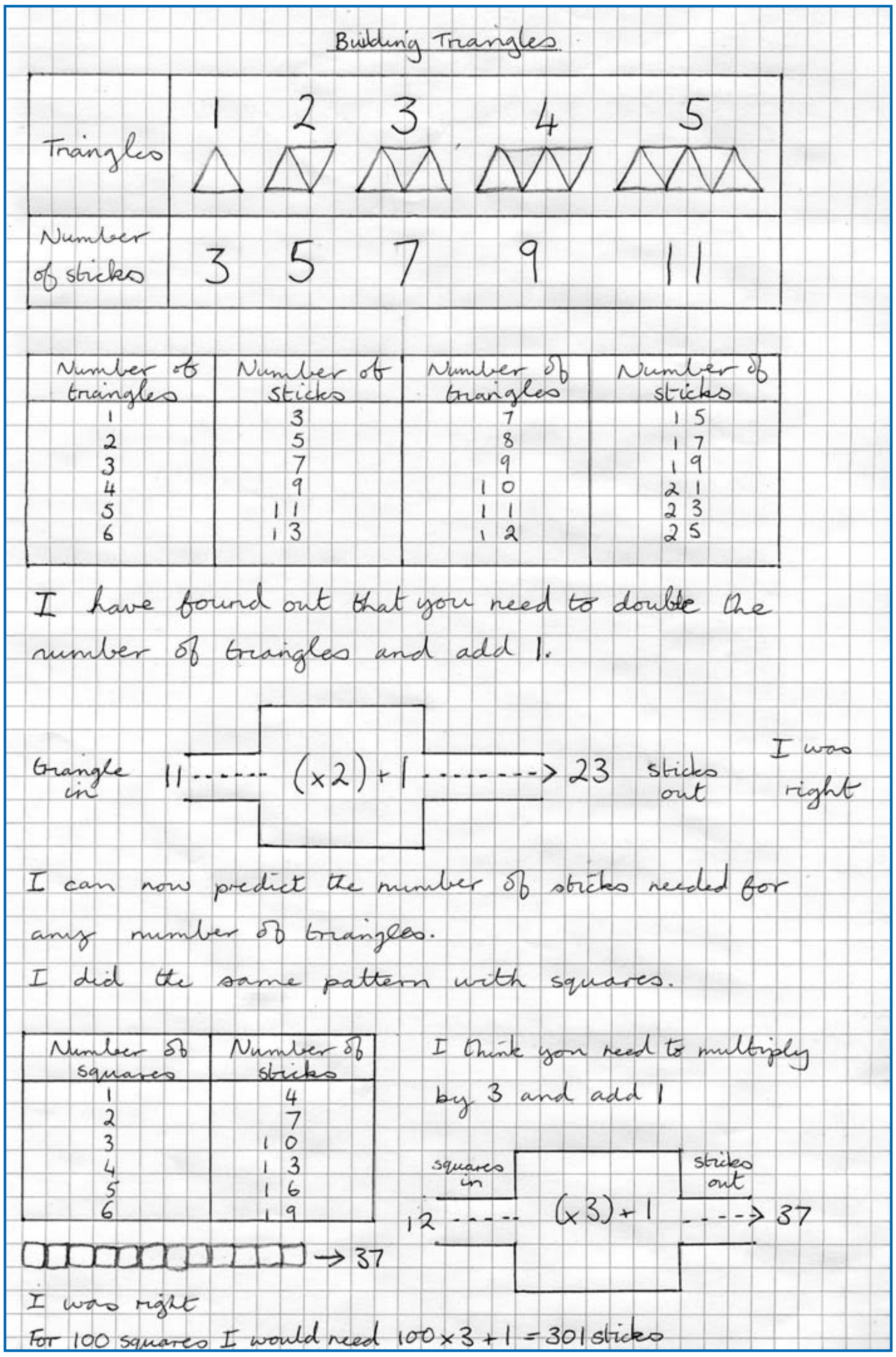
In this activity, pupils aim to identify patterns in a sequence, find a rule in words for the pattern, use this rule to predict later entries in the pattern, and prove their rule for a specific entry. They use matchsticks or straws to develop patterns of increasing numbers of equilateral triangles laid edge-to-edge in a straight line, and then attempt the same activity using squares.

There are opportunities for pupils to:

- develop their own mathematical strategies and ideas
- devise and refine their own ways of recording
- recognise, and generalise in words, patterns that arise
- investigate and generalise repeating patterns and relationships; search for pattern in their own results
- use a variety of methods to represent data
- develop early ideas of algebra and mathematical structure by exploring number sequences and relationships; explain and predict subsequent terms.

Pupils are also:

- determining the process/method and strategy
- thinking logically and seeking patterns
- organising ideas and information
- recording and interpreting data and presenting findings.



Building Triangles

Triangles	1	2	3	4	5
Number of sticks	3	5	7	9	11

Number of triangles	Number of sticks	Number of triangles	Number of sticks
1	3	7	15
2	5	8	17
3	7	9	19
4	9	10	21
5	11	11	23
6	13	12	25

I have found out that you need to double the number of triangles and add 1.

Triangle in 11 ------ (×2) + 1 --------> 23 sticks out

I was right

I can now predict the number of sticks needed for any number of triangles.

I did the same pattern with squares.

Number of squares	Number of sticks
1	4
2	7
3	10
4	13
5	16
6	19

I think you need to multiply by 3 and add 1

squares in 12 ---- (×3) + 1 ----> 37 sticks out

→ 37

I was right

For 100 squares I would need 100 × 3 + 1 = 301 sticks

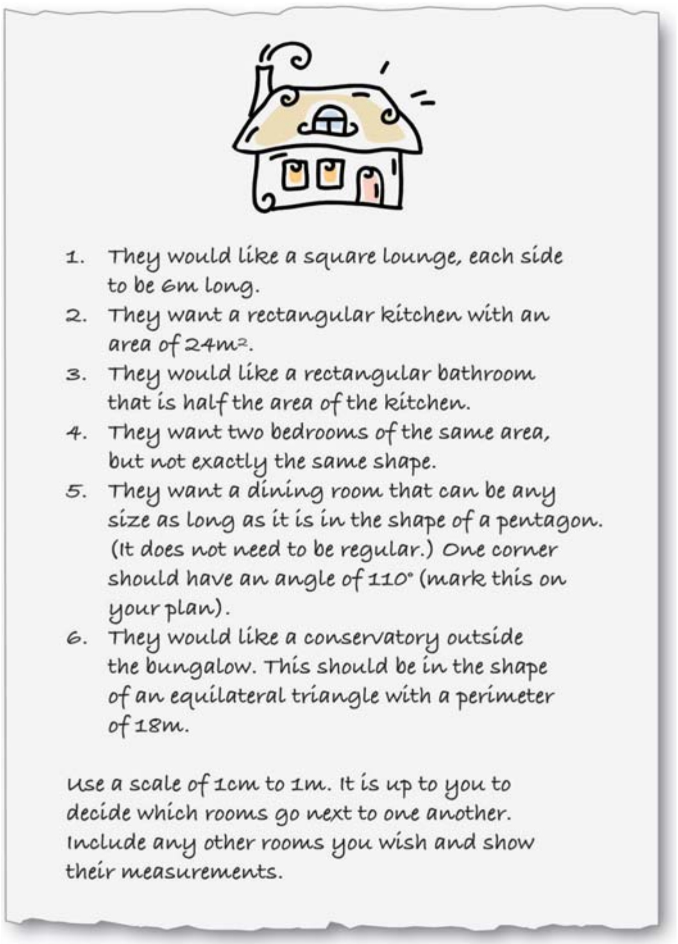
I can find out how many matches I need to make any number of triangles in a row and any number of squares in a row.

Commentary

Emyr worked consistently in building up the patterns of triangles, and recorded his findings systematically. He was able to identify and express in words the pattern he found using triangles, and constructed in symbols a simple formula involving two operations. He used his formula to predict the number of matchsticks required for a given number of triangles, and checked his formula using a specific case. He then used the same strategy to find the number of matches he would need to build up a row of squares.

Activity | Designing a bungalow

In this task pupils follow a sequence of instructions giving shapes and precise measurements in order to prepare a plan of a bungalow. They use their knowledge of the properties of 2-D shapes, including perimeter and area, and draw and measure angles and lengths to complete the task.



1. They would like a square lounge, each side to be 6m long.
2. They want a rectangular kitchen with an area of $24m^2$.
3. They would like a rectangular bathroom that is half the area of the kitchen.
4. They want two bedrooms of the same area, but not exactly the same shape.
5. They want a dining room that can be any size as long as it is in the shape of a pentagon. (It does not need to be regular.) One corner should have an angle of 110° (mark this on your plan).
6. They would like a conservatory outside the bungalow. This should be in the shape of an equilateral triangle with a perimeter of 18m.

Use a scale of 1cm to 1m. It is up to you to decide which rooms go next to one another. Include any other rooms you wish and show their measurements.

There are opportunities for pupils to:

- select and use the appropriate mathematics, materials, units of measure and resources to solve the problem
- identify, obtain and process information needed to carry out the work
- develop their own mathematical strategies and ideas
- visualise and describe shapes
- read information from text
- measure to an appropriate degree of accuracy.

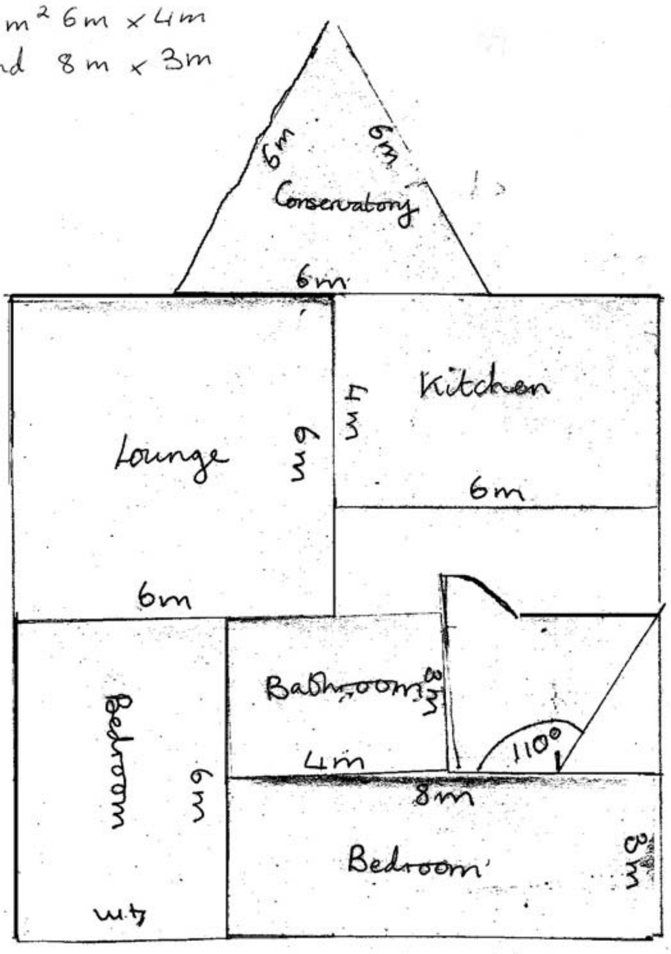
Pupils are also:

- activating prior skills, knowledge and understanding
- determining the process/method and strategy
- thinking about cause and effect
- measuring
- presenting findings.

Kitchen area = 24m²
Can be 6m x 4m

Bathroom area = 12m²
Can be 4m x 3m

2 bedrooms same area
Could be 24m² 6m x 4m
and 8m x 3m



Commentary

Emyr cut out shapes to scale, satisfying the conditions, in order to work out whether or not they would fit inside the given square (of area 12 cm x 12 cm). He followed the instructions accurately except for the angle of 110°. He was able to adjust his original sizes in order to fit inside the square.

Summary and overall judgement

Levels 4 and 5 were considered and Level 4 was judged to be the best fit.

In ‘Boxes’ and ‘Consecutive numbers’, Emyr has shown that he can *develop strategies for solving problems*. In ‘Building triangles’, ‘Consecutive numbers’ and ‘Using the library’, he has *presented information and results systematically*. In ‘Building triangles’ and ‘Consecutive numbers’, he has shown that he can *recognise and describe number patterns and relationships* and, in ‘Building triangles’, *use simple formulae expressed in words*. In ‘Designing a bungalow’, he *chose and used suitable instruments, reading, with appropriate accuracy, numbers on a range of measuring instruments*, and he *found perimeters of shapes and areas by counting squares*. In ‘Using the library’, he *collected discrete data and found the mode of a set of data*. In ‘Boxes’, he showed an understanding of *simple vocabulary associated with probability*. These are all characteristics of Level 4.

In ‘Building triangles’, he *described the situation mathematically using symbols* and *constructed a simple formula involving two operations,* both characteristic of Level 5. Emyr *drew his own conclusions* and *made general statements of his own based on available evidence* in ‘Consecutive numbers’, which is indicative of Level 5, but did not explain his reasoning which would have been further evidence of Level 5. He also showed some *appreciation that different outcomes may result from repeating an experiment*, when he recognised that he needed to take more samples in ‘Using the library’. This is also characteristic of Level 5.

Key Stage 2

Marya

Level 5

Marya is an 11-year-old learner in Key Stage 2.

Her teacher knows much more about Marya’s performance than can be included here. However, this profile has been selected to illustrate characteristic features of Marya’s work across a range of activities. Each example is accompanied by a brief commentary to provide a context and indicate particular qualities in the work.

Marya’s teacher judges that her performance in mathematics is best described as Level 5.

Activity | Consecutive numbers

The focus of this activity is to identify and explain patterns that arise when adding together consecutive numbers.

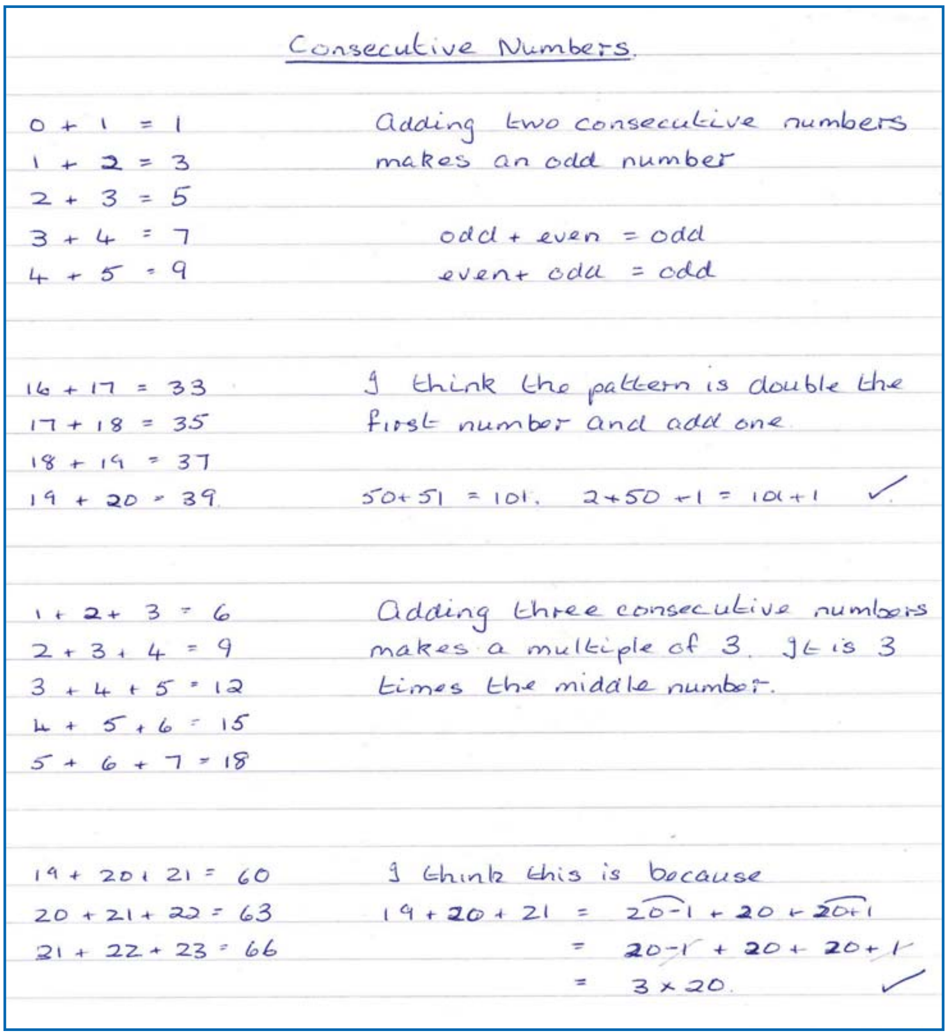
There are opportunities for pupils to:

- develop their own mathematical strategies and ideas
- devise and refine their own ways of recording
- use flexible and effective methods of computation and recording
- estimate solutions to calculations; use alternative strategies to check the accuracy of answers
- recognise, and generalise in words, patterns that arise
- use their prior knowledge to find mathematical facts that they have not learned.

Pupils are also:

- determining the process/method and strategy
- thinking logically and seeking patterns
- communicating information
- talking about and explaining work
- recording data and presenting findings.

Two consecutive numbers will always be even then odd or odd then even, so the sum will always be odd.



Consecutive Numbers.

0 + 1 = 1
1 + 2 = 3
2 + 3 = 5
3 + 4 = 7
4 + 5 = 9

Adding two consecutive numbers makes an odd number

odd + even = odd
even+ odd = odd

16 + 17 = 33
17 + 18 = 35
18 + 19 = 37
19 + 20 = 39

I think the pattern is double the first number and add one

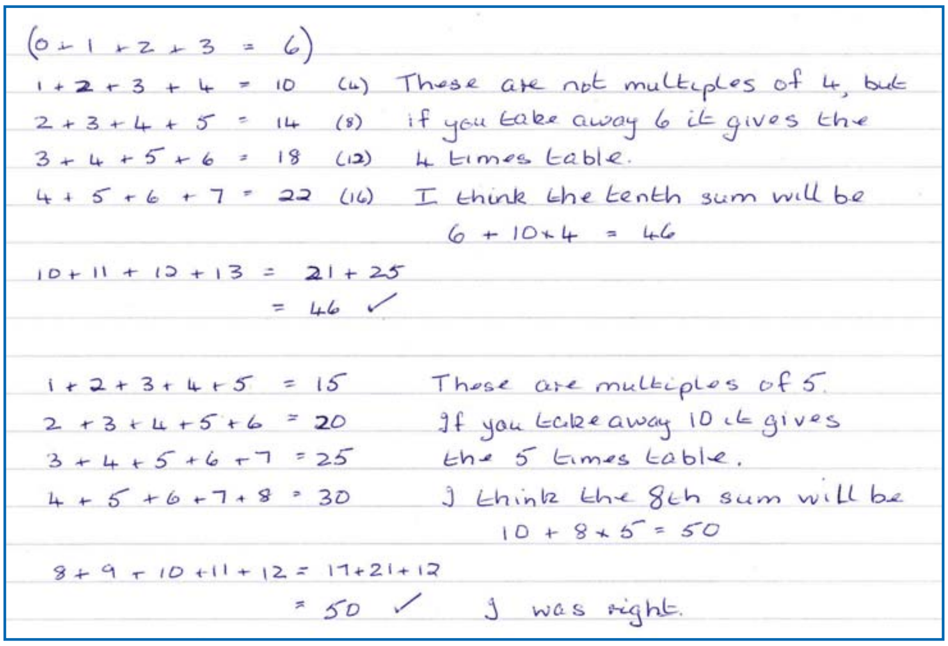
50+ 51 = 101. 2+50 +1 = 101+1 ✓

1 + 2+ 3 = 6
2 + 3 + 4 = 9
3 + 4 + 5 = 12
4 + 5 + 6 = 15
5 + 6 + 7 = 18

Adding three consecutive numbers makes a multiple of 3. It is 3 times the middle number.

19 + 20 + 21 = 60
20 + 21 + 22 = 63
21 + 22 + 23 = 66

I think this is because
19 + 20 + 21 = 20−1 + 20 + 20+1
= 20−1 + 20 + 20+1
= 3 × 20. ✓



(0 + 1 + 2 + 3 = 6)
1 + 2 + 3 + 4 = 10 (4)
2 + 3 + 4 + 5 = 14 (8)
3 + 4 + 5 + 6 = 18 (12)
4 + 5 + 6 + 7 = 22 (16)

These are not multiples of 4, but if you take away 6 it gives the 4 times table.
I think the tenth sum will be
6 + 10×4 = 46

10 + 11 + 12 + 13 = 21 + 25
= 46 ✓

1 + 2 + 3 + 4 + 5 = 15
2 + 3 + 4 + 5 + 6 = 20
3 + 4 + 5 + 6 + 7 = 25
4 + 5 + 6 + 7 + 8 = 30

These are multiples of 5.
If you take away 10 it gives the 5 times table.
I think the 8th sum will be
10 + 8×5 = 50

8 + 9 + 10 + 11 + 12 = 17+21+12
= 50 ✓ I was right.

I think that if you add an odd number of consecutive numbers you will get multiples of that number.

In further work, Marya noticed that adding five consecutive numbers gave multiples of five and adding seven consecutive numbers gave multiples of seven. She then made her own generalisation.

Commentary

Marya developed her own strategy for solving problems. She presented information and results in a clear and organised way. She initially investigated the totals when adding two and three consecutive numbers and discovered some patterns, then she moved on to four and more numbers. She did some of her rough calculations separately from the work presented on the previous page, and showed that she could use a variety of written and mental strategies for calculating the sums. She used her recall of multiplication tables in recognising patterns. She found rules in words, and explained her reasoning mathematically in some instances. She also gave a simple justification of her finding that adding two consecutive numbers always gave an odd number.

Activity | Frogs and toads

This investigation started as a whole class activity. Marya explains, "As a class we put seven chairs together. We had three toads (boys) and three frogs (girls). We put the frogs on one side and the toads on the other. We tried to move the frogs and toads to the opposite sides by hopping (two spaces) or sliding (one space) but you could not move backwards."

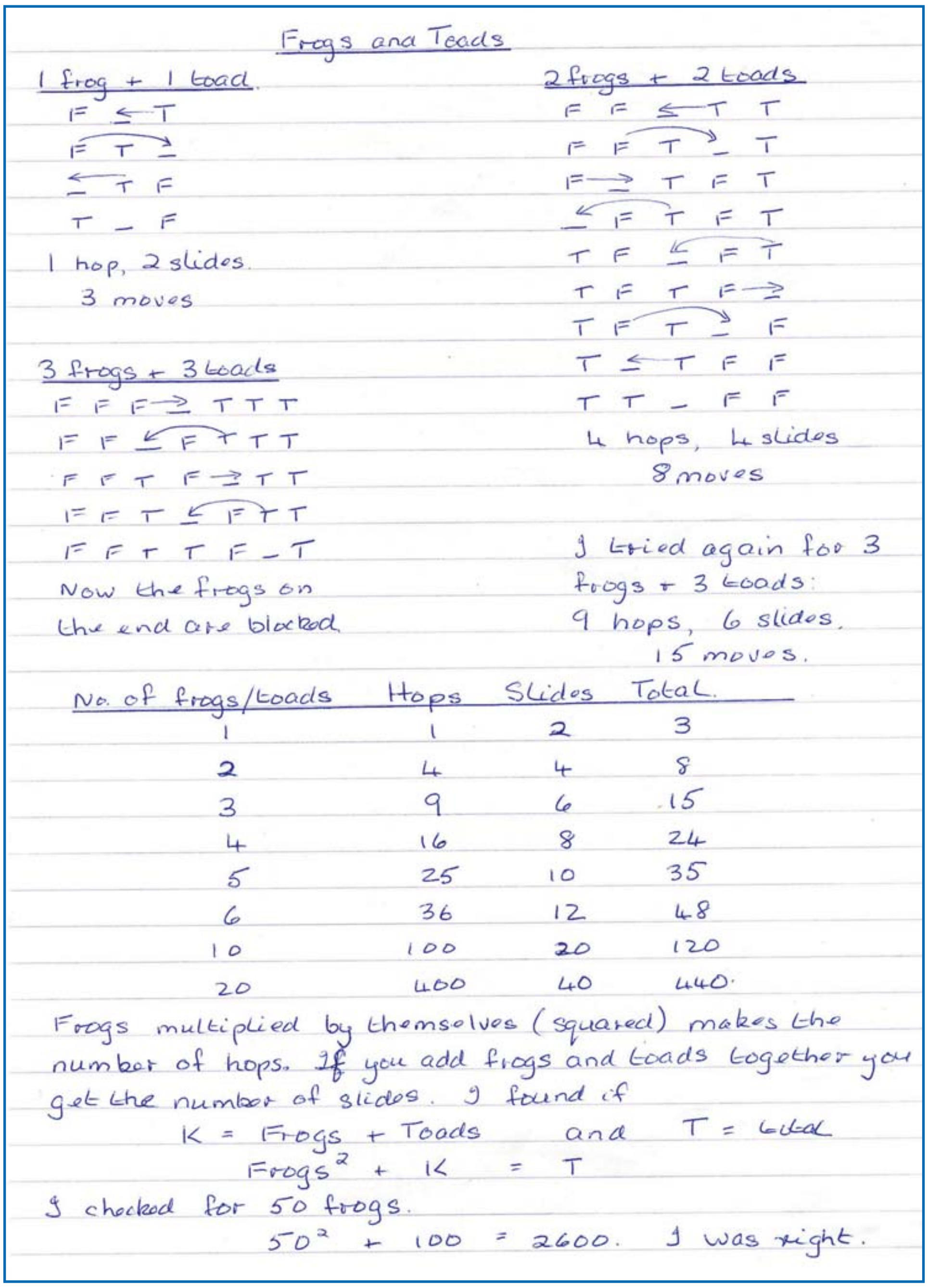
F	**F**	**F**		**T**	**T**	**T**

There are opportunities for pupils to:

- develop their own mathematical strategies and ideas
- devise and refine their own ways of recording
- search for pattern in their own results
- recognise, and generalise in words, patterns that arise in numerical, spatial or practical situations
- explain strategies, methods, choices and conclusions using informal written methods
- develop early ideas of algebra and mathematical structure by exploring number sequences and relationships.

Pupils are also:

- determining the process/method and strategy
- thinking logically and seeking patterns
- using the number system
- talking about and explaining work
- recording and interpreting data and presenting findings.



Frogs and Toads

1 frog + 1 toad

F _← T

F T ↷_

_← T F

T _ F

1 hop, 2 slides.

3 moves

3 frogs + 3 toads

F F F →_ T T T

F F _↶ F T T T

F F T F →_ T T

F F T _↶ F T T

F F T T F _ T

Now the frogs on the end are blocked.

2 frogs + 2 toads

F F _← T T

F F T ↷_ T

F →_ T F T

_↶ F T F T

T F _↶ F T

T F T F →_

T F T ↷_ F

T _← T F F

T T _ F F

4 hops, 4 slides

8 moves

I tried again for 3 frogs + 3 toads:

9 hops, 6 slides.

15 moves.

No. of frogs/toads	Hops	Slides	Total.
1	1	2	3
2	4	4	8
3	9	6	15
4	16	8	24
5	25	10	35
6	36	12	48
10	100	20	120
20	400	40	440.

Frogs multiplied by themselves (squared) makes the number of hops. If you add frogs and toads together you get the number of slides. I found if

K = Frogs + Toads and T = total

$\text{Frogs}^2 + K = T$

I checked for 50 frogs.

$50^2 + 100 = 2600$. I was right.

Commentary

After the whole class starter activity, Marya started by trying to describe with diagrams what happened with one frog and one toad and then with two frogs and two toads on either side. She decided that a frog could only be next to another frog in the starting or finishing positions, otherwise she could get blocked. She then used this strategy to find the number of hops and slides needed for three on each side. She continued the investigation using the ‘Investigating number patterns’ software programme (www.ngfl-cymru.org.uk) in order to extend the work to larger numbers, and drew up a table of results. She spotted patterns in the table for the numbers of hops and of slides, described her findings initially in words, and then derived an algebraic equation to describe her results. She used the computer programme to check her formula using the special case of 50 frogs and 50 toads.

Activity | Network Q Rally

Pupils are given a chart of prices for visits to the Network Q Rally. The task is to find various combinations of adult and child tickets that can be purchased with £200, using up as much of the money as possible.

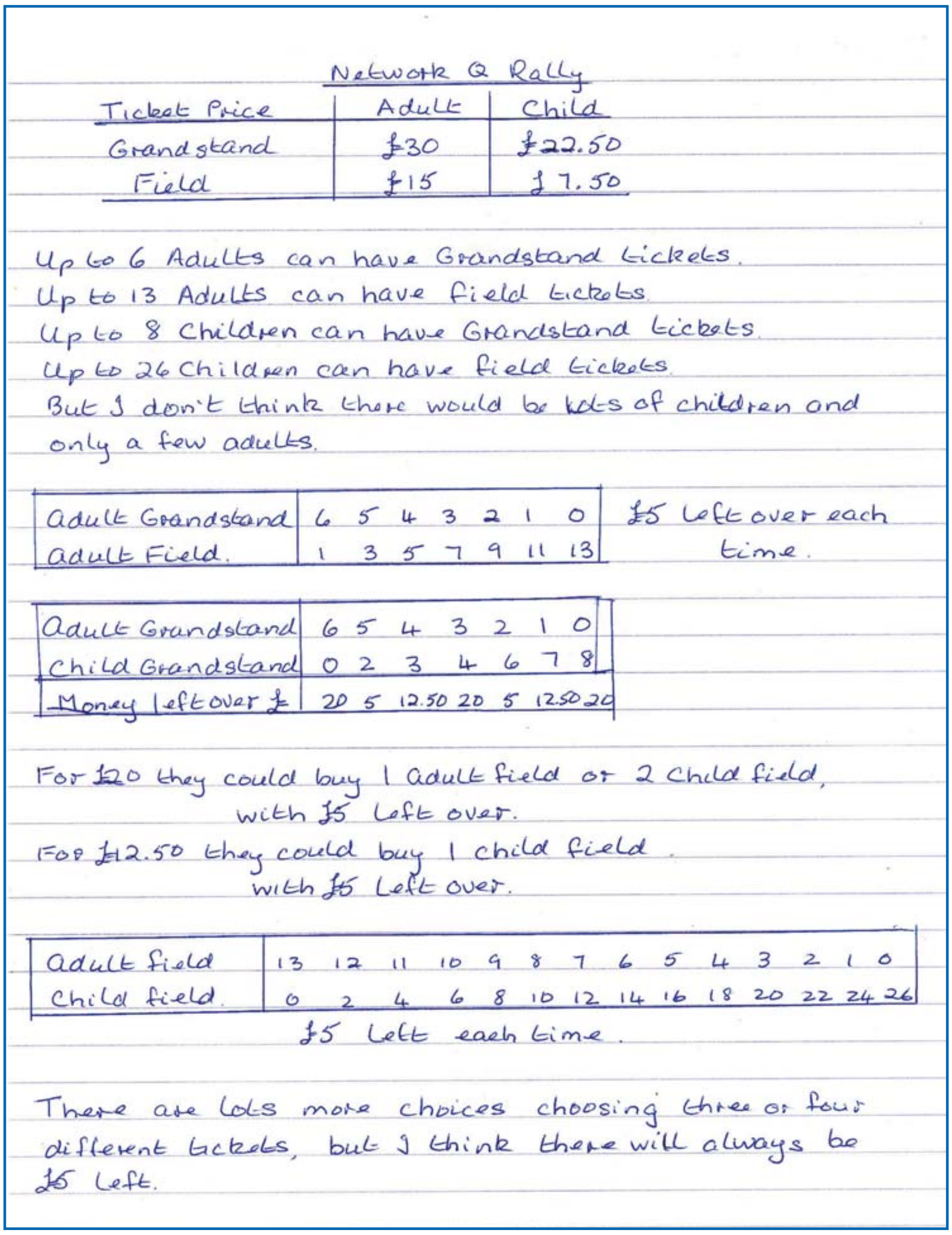
There are opportunities for pupils to:

- select and use the appropriate mathematics
- identify, obtain and process information needed to carry out the work
- devise and refine ways of recording
- explain methods and conclusions using informal methods
- use flexible and effective methods of computation and recording.

Pupils are also:

- determining the process/method
- thinking logically and seeking patterns
- gathering information
- talking about and explaining work
- recording and interpreting data and presenting findings.

All the ticket prices are multiples of £7.50. They are one times, two times, three times and four times. I think there are lots of choices for different prices together.



Network Q Rally

Ticket Price	Adult	Child
Grandstand	£30	£22.50
Field	£15	£7.50

Up to 6 Adults can have Grandstand tickets.
Up to 13 Adults can have field tickets.
Up to 8 Children can have Grandstand tickets.
Up to 26 Children can have field tickets.
But I don't think there would be lots of children and only a few adults.

Adult Grandstand	6	5	4	3	2	1	0
Adult Field.	1	3	5	7	9	11	13

£5 Left over each time.

Adult Grandstand	6	5	4	3	2	1	0
Child Grandstand	0	2	3	4	6	7	8
Money left over £	20	5	12.50	20	5	12.50	20

For £20 they could buy 1 Adult field or 2 Child field, with £5 Left over.

For £12.50 they could buy 1 child field. with £5 Left over.

Adult field	13	12	11	10	9	8	7	6	5	4	3	2	1	0
Child field.	0	2	4	6	8	10	12	14	16	18	20	22	24	26

£5 Left each time.

There are lots more choices choosing three or four different tickets, but I think there will always be £5 Left.

I think there is always at least £5 left because that is the remainder when you divide £200 by £7.50. Whenever there is more than £7.50 left you can buy a child field ticket.

Commentary

Marya noticed that there was a relationship between the ticket prices and used her finding that all the ticket prices were multiples of £7.50 to help find various combinations. She started by considering only adult tickets, then only stand tickets, considering the most expensive ticket price first and exploring the various ways in which to use the remaining money. She worked systematically to obtain the necessary information and found various combinations. She noticed that each combination gave a remainder of £5, and was able to give a valid mathematical reason for this. She also realised that some results were unlikely, for example, 26 children and no adults in the field.

Activity | Pentominoes

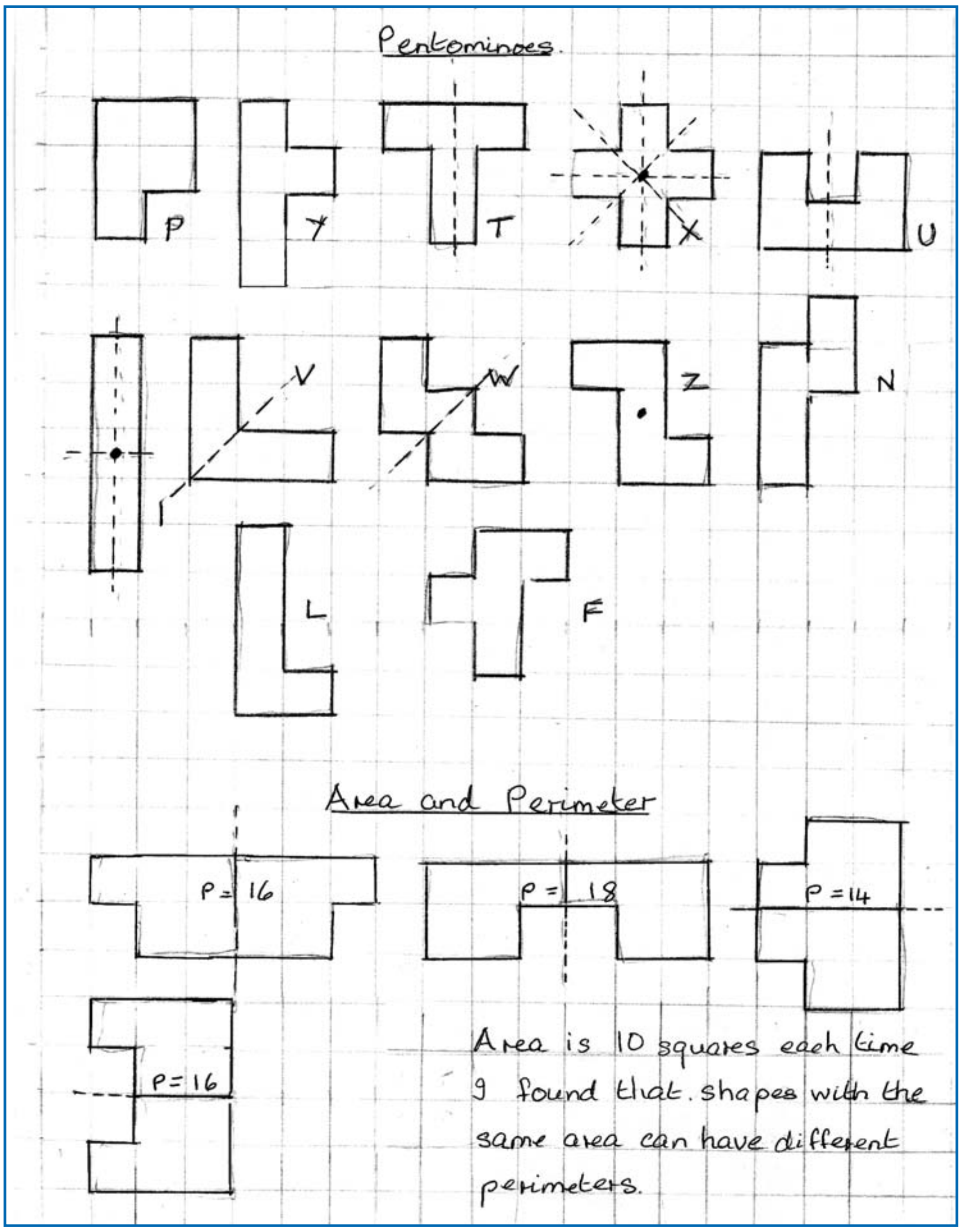
Pupils are asked to investigate how many different pentominoes they can create, and which of the pentominoes have line and/or rotational symmetry. Marya then continues her investigation by choosing one of the pentominoes, reflecting it in some of its sides, and finding the area and perimeter of the new shapes. She also investigates which pentominoes tessellate, and whether she can build new, larger pentominoes using some of the original pentominoes, though this work is not included here.

There are opportunities for pupils to:

- develop their own mathematical strategies and ideas
- try different approaches; use a variety of strategies
- use correct mathematical language to represent their work
- visualise and describe shapes and movements
- explain strategies, methods, choices and conclusions in a variety of ways.

Pupils are also:

- thinking logically and seeking patterns
- reviewing outcomes
- talking about and explaining work.



Area is 10 squares each time I found that. shapes with the same area can have different perimeters.

I think the perimeters are different because the side you reflect on ends up inside the shape, so it is not counted. If it is a long side the perimeter will be smaller.

Commentary

Marya succeeded in creating all 12 pentominoes. She correctly identified the line and rotational symmetries of some of the shapes. She noticed that different shapes, each comprising a pentomino and its reflection, could have different perimeters, and attempted to explain why this was.

Activity | Match up

Pupils are given a page of simple fractions, decimals and percentages that they are to organise into equivalent sets. They can choose to use a calculator to check their work. They then calculate fractional and percentage parts of quantities and measurements generated by two sets of cards, one containing the original numbers and the other the quantities and measurements.

There are opportunities for pupils to:

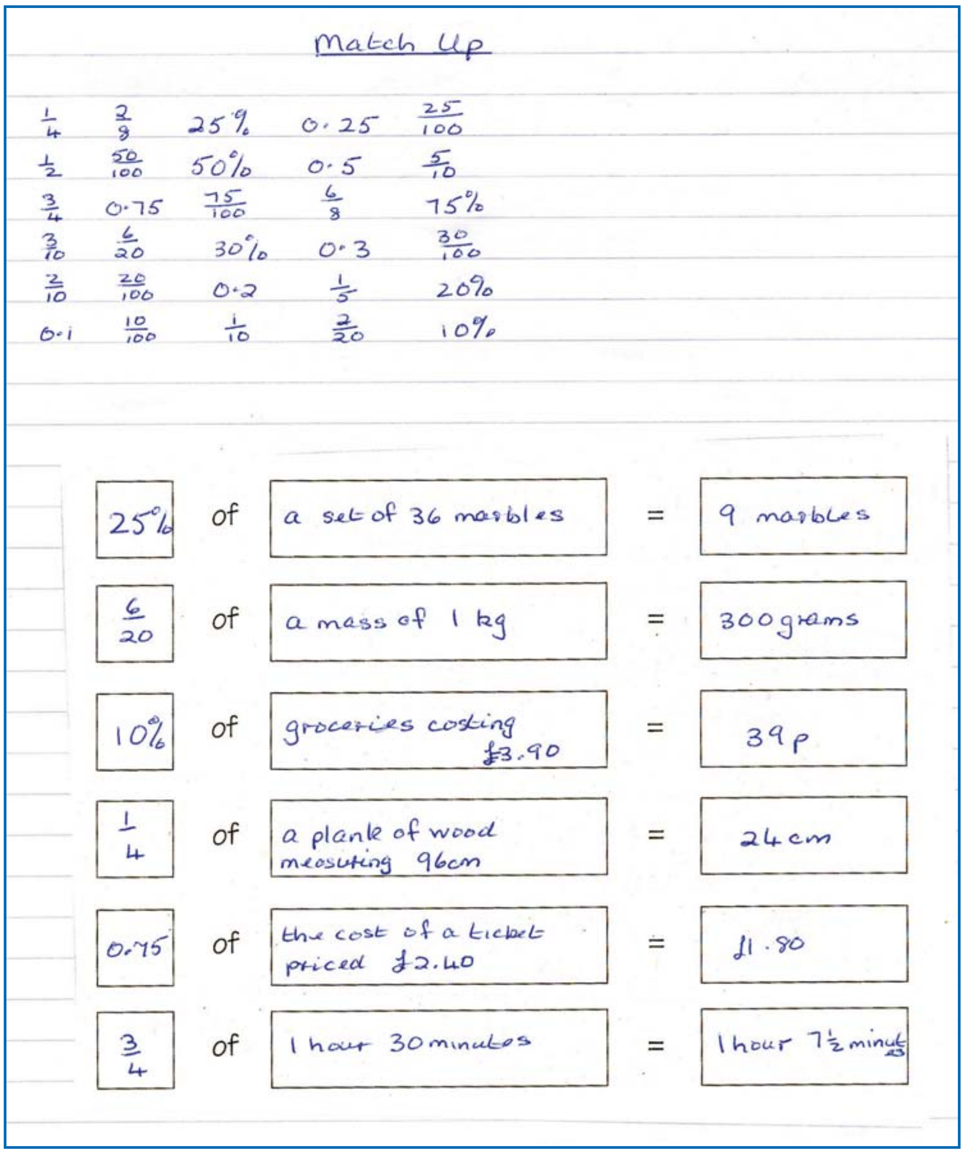
- select and use the appropriate mathematics
- develop a variety of mental and written strategies of computation
- use a variety of methods of calculating
- use alternative strategies to check the accuracy of answers.

Pupils are also:

- using numbers
- using the number system
- using a variety of methods.

$\frac{20}{100}$	$\frac{1}{2}$	$\frac{3}{10}$	0.75	$\frac{6}{20}$
30%	$\frac{75}{100}$	$\frac{3}{4}$	0.2	$\frac{2}{8}$
$\frac{50}{100}$	$\frac{6}{8}$	$\frac{2}{10}$	25%	$\frac{10}{100}$
$\frac{2}{20}$	0.3	50%	0.1	$\frac{30}{100}$
0.25	$\frac{1}{5}$	$\frac{1}{10}$	75%	$\frac{5}{10}$
20%	0.5	$\frac{1}{4}$	$\frac{25}{100}$	10%

A mass of 2000g	The cost of a ticket priced £2.40
A set of 36 marbles	A plank of wood measuring 96cm
A carpet of area $20m^2$	1 hour 30 minutes
A rope measuring 3.6m	A mass of 1kg
4 minutes 30 seconds	Groceries costing £3.90
A mass of 3.5kg	5¾ hours



Match Up

$\frac{1}{4}$	$\frac{2}{8}$	25%	0·25	$\frac{25}{100}$
$\frac{1}{2}$	$\frac{50}{100}$	50%	0·5	$\frac{5}{10}$
$\frac{3}{4}$	0·75	$\frac{75}{100}$	$\frac{6}{8}$	75%
$\frac{3}{10}$	$\frac{6}{20}$	30%	0·3	$\frac{30}{100}$
$\frac{2}{10}$	$\frac{20}{100}$	0·2	$\frac{1}{5}$	20%
0·1	$\frac{10}{100}$	$\frac{1}{10}$	$\frac{2}{20}$	10%

25%	of	a set of 36 marbles	=	9 marbles
$\frac{6}{20}$	of	a mass of 1 kg	=	300 grams
10%	of	groceries costing £3.90	=	39p
$\frac{1}{4}$	of	a plank of wood measuring 96cm	=	24cm
0.75	of	the cost of a ticket priced £2.40	=	£1.80
$\frac{3}{4}$	of	1 hour 30 minutes	=	1 hour $7\frac{1}{2}$ minutes

I changed the 1 kilogram to 1000 grams, and $^{6}/_{20}$ to $^{3}/_{10}$. Then I worked out $^{3}/_{10}$ of 1000 grams. I did $^{1}/_{10}$ first then multiplied by 3.

The hardest one was ¾ of 1 hour 30 minutes. I changed it to minutes and used a calculator for ¾. It gave me 67.5 minutes and I changed that back to hours and minutes.

Commentary

Marya correctly listed the equivalent fractions, decimals and percentages and calculated the proportions of the different measurements. In some cases, she used a calculator effectively to check that her fractions and decimals were equivalent, and to find the solutions.

Summary and overall judgement

Levels 4, 5 and 6 were considered and Level 5 was judged to be the best fit.

Marya has shown, in ‘Consecutive numbers’, ‘Frogs and toads’, and ‘Network Q Rally’, that she can *develop strategies for solving problems, and present information and results systematically*. In ‘Consecutive numbers’ and ‘Match up’, she *used a variety of mental and written methods for computation*, and in ‘Consecutive numbers’ she *recalled multiplication facts*. In ‘Pentominoes’, she *reflected simple shapes in a mirror line*, and *found perimeters of shapes and areas by counting squares*. These are all characteristic of Level 4.

In ‘Network Q Rally’, Marya *identified and obtained information to solve problems*, and *checked whether her results were sensible in the context of the problem*. In ‘Frogs and toads’ and ‘Consecutive numbers’, she *described the situation mathematically using symbols and words*; in ‘Frogs and toads’, she also *drew her own conclusions, explaining her reasoning.* She *made a general statement of her own, based on available evidence* in ‘Pentominoes’ and ‘Consecutive numbers’, and in ‘Network Q Rally’ and ‘Match up’, she *used her understanding of place value to multiply and divide whole numbers and decimals*. In ‘Match up’, she also *calculated fractional or percentage parts of quantities and measurements*. She *constructed and used a simple formula involving two operations* in ‘Frogs and toads’. In Pentominoes, *she recognised and identified symmetries of 2-D shapes*, and in ‘Match up’ she *converted one metric unit to another*. This work is all characteristic of Level 5.

In ‘Network Q Rally’, Marya started by considering only adult tickets, then only field tickets, before moving on to determine other possibilities, demonstrating that she has begun to *solve a complex problem by breaking it down into smaller tasks*, and she used the fact that all the ticket prices were multiples of £7.50 to *give a mathematical justification to support her conclusion* that there is always £5 left over. In ‘Match up’, she *used the equivalences between fractions, decimals and percentages*. These are characteristic of Level 6.

Key Stage 3

Oliver

Level 4

Oliver is a 14-year-old learner in Key Stage 3.

His teacher knows much more about Oliver’s performance than can be included here. However, this profile has been selected to illustrate characteristic features of Oliver’s work across a range of activities. Each example is accompanied by a brief commentary to provide a context and indicate particular qualities in the work.

Oliver’s teacher judges that his performance in mathematics in best described as Level 4.

Activity | Consecutive numbers

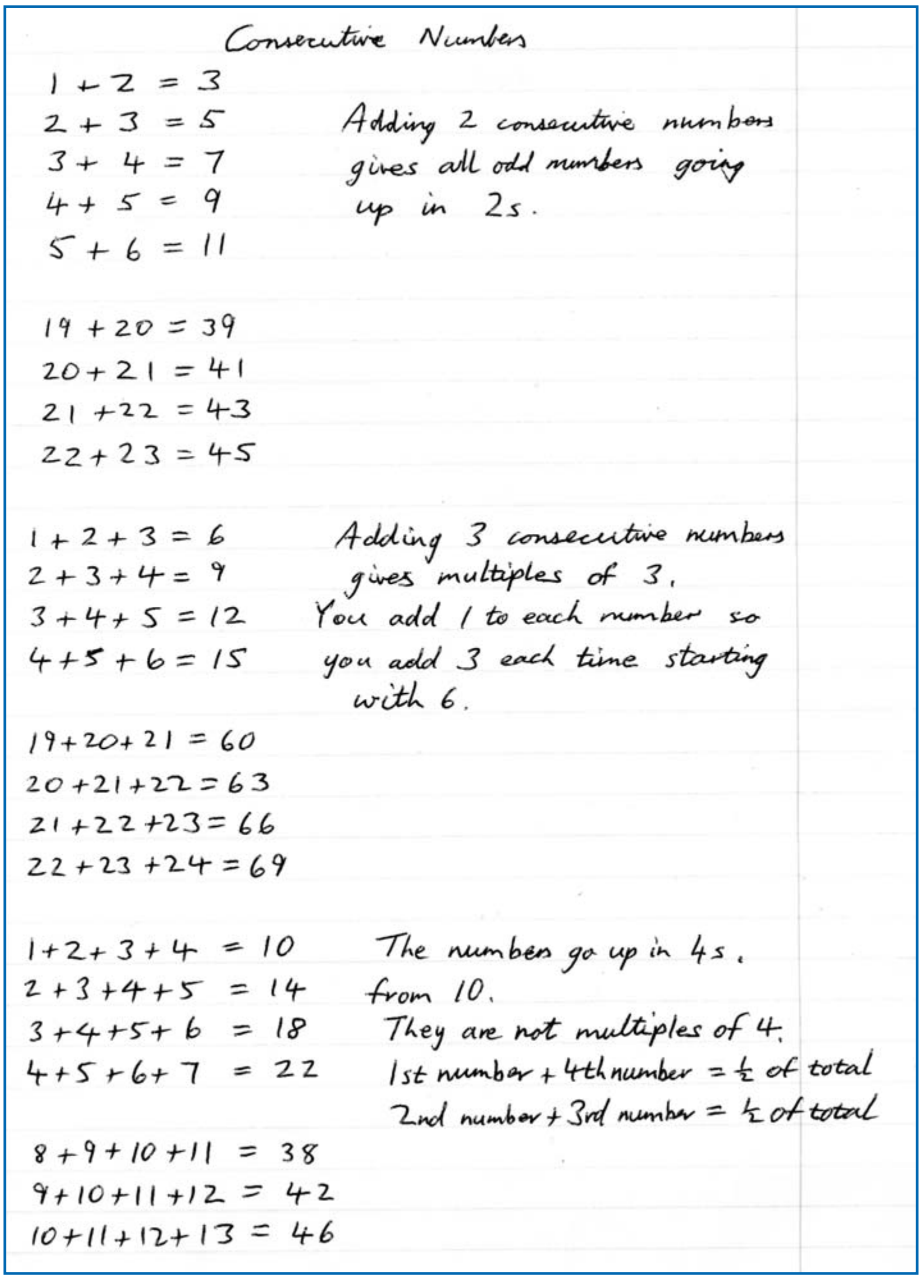
The focus of this activity is to identify and explain patterns that arise when adding together consecutive numbers.

There are opportunities for pupils to:

- develop and use their own mathematical strategies and ideas and consider those of others
- use a range of mental, written and calculator computational strategies
- use a variety of checking strategies, including mental estimation
- generalise and explain patterns and relationships in words and symbols.

Pupils are also:

- determining the process/method and strategy
- thinking logically and seeking patterns
- communicating information
- talking about and explaining work
- recording data and presenting findings.



Consecutive Numbers

1 + 2 = 3
2 + 3 = 5
3 + 4 = 7
4 + 5 = 9
5 + 6 = 11

Adding 2 consecutive numbers gives all odd numbers going up in 2s.

19 + 20 = 39
20 + 21 = 41
21 + 22 = 43
22 + 23 = 45

1 + 2 + 3 = 6
2 + 3 + 4 = 9
3 + 4 + 5 = 12
4 + 5 + 6 = 15

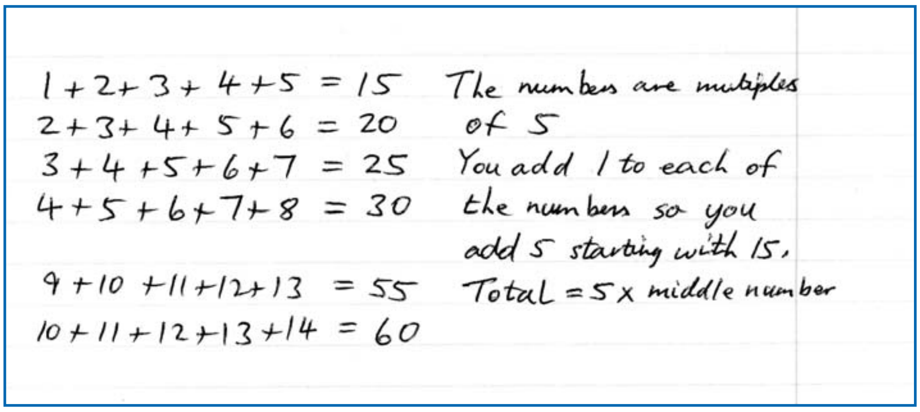
Adding 3 consecutive numbers gives multiples of 3. You add 1 to each number so you add 3 each time starting with 6.

19 + 20 + 21 = 60
20 + 21 + 22 = 63
21 + 22 + 23 = 66
22 + 23 + 24 = 69

1 + 2 + 3 + 4 = 10
2 + 3 + 4 + 5 = 14
3 + 4 + 5 + 6 = 18
4 + 5 + 6 + 7 = 22

The numbers go up in 4s. from 10.
They are not multiples of 4.
1st number + 4th number = $\frac{1}{2}$ of total
2nd number + 3rd number = $\frac{1}{2}$ of total

8 + 9 + 10 + 11 = 38
9 + 10 + 11 + 12 = 42
10 + 11 + 12 + 13 = 46



1 + 2 + 3 + 4 + 5 = 15
2 + 3 + 4 + 5 + 6 = 20
3 + 4 + 5 + 6 + 7 = 25
4 + 5 + 6 + 7 + 8 = 30

The numbers are multiples of 5. You add 1 to each of the numbers so you add 5 starting with 15.

9 + 10 + 11 + 12 + 13 = 55
10 + 11 + 12 + 13 + 14 = 60

Total = 5 x middle number

Commentary

Oliver worked systematically, starting with sums of two consecutive numbers and working his way up to sums of five consecutive numbers. He found several patterns in his results which he expressed in words, and gave valid reasons for his findings.

Activity | The chessboard

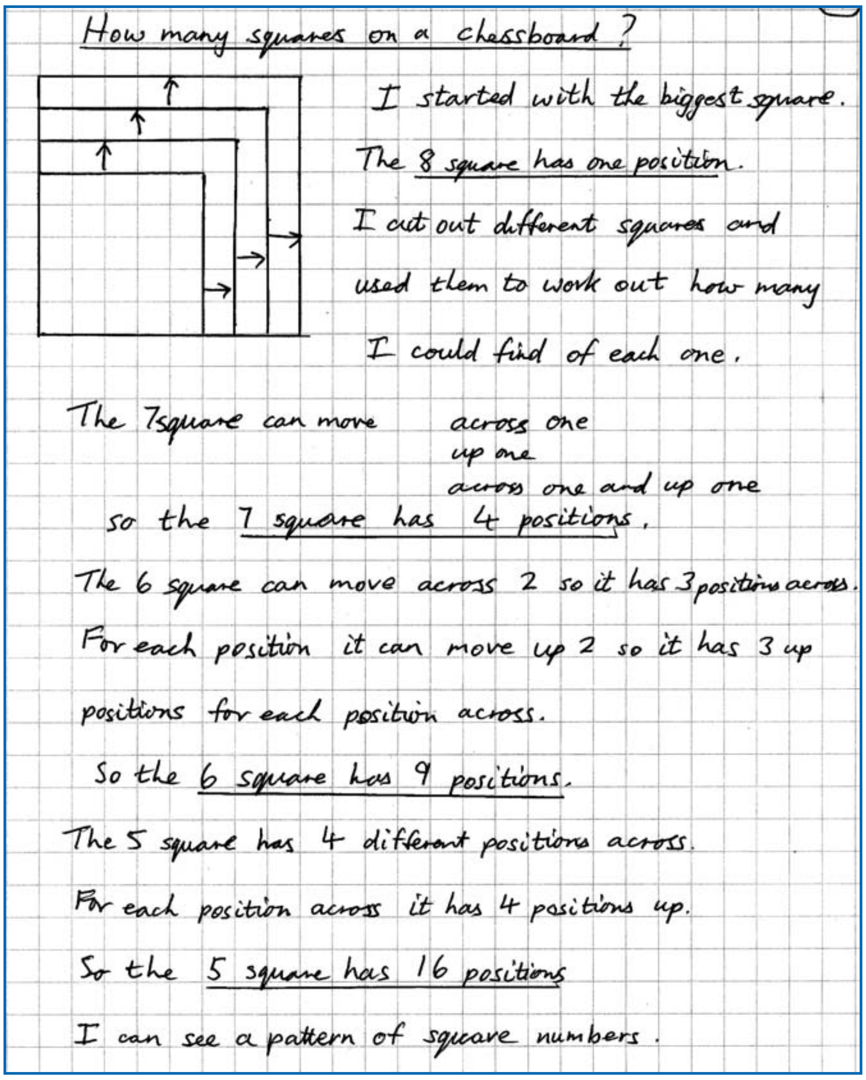
Pupils investigate the number of squares on a chessboard. Initially the teacher displays an image of the chessboard and discusses possible strategies with the class as a whole. Pupils are then challenged to develop a strategy for working out the number of each different size of square.

There are opportunities for pupils to:

- develop and use their own mathematical strategies and ideas and consider those of others
- select, trial and evaluate a variety of approaches; break complex problems into a series of tasks
- explain patterns in words and symbols; express simple functions in words and symbolically
- present work clearly using diagrams, labelled graphs and symbols
- predict subsequent terms in number sequences.

Pupils are also:

- determining the process/method and strategy
- thinking logically and seeking patterns
- presenting information and ideas
- recording and interpreting data and presenting findings.



How many squares on a chessboard?

I started with the biggest square.

The 8 square has one position.

I cut out different squares and used them to work out how many I could find of each one.

The 7square can move across one
up one
across one and up one

so the 7 square has 4 positions.

The 6 square can move across 2 so it has 3 positions across.

For each position it can move up 2 so it has 3 up positions for each position across.

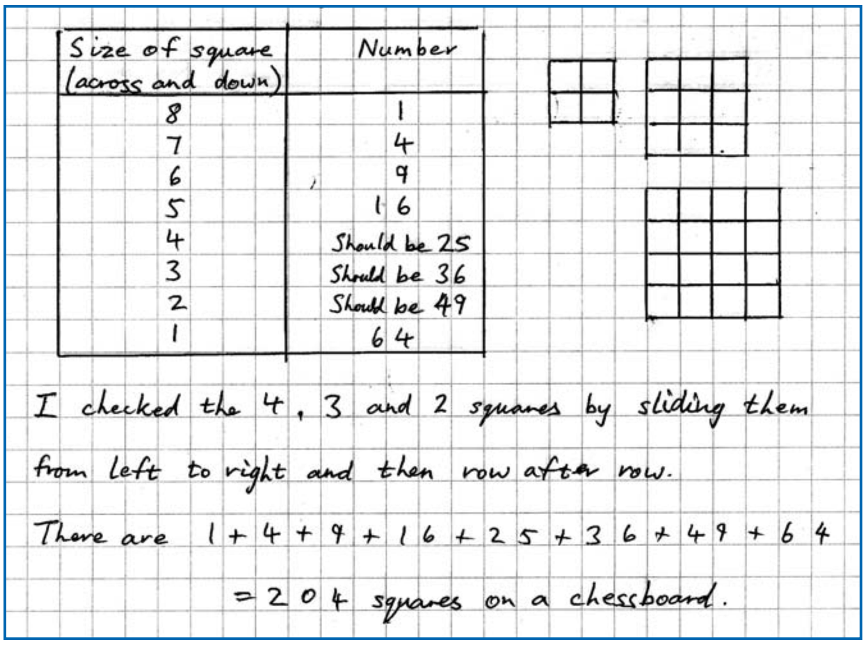
So the 6 square has 9 positions.

The 5 square has 4 different positions across.

For each position across it has 4 positions up.

So the 5 square has 16 positions

I can see a pattern of square numbers.



Size of square (across and down)	Number
8	1
7	4
6	9
5	16
4	Should be 25
3	Should be 36
2	Should be 49
1	64

I checked the 4, 3 and 2 squares by sliding them from left to right and then row after row.

There are 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64

= 204 squares on a chessboard.

Commentary

Oliver worked systematically and developed a strategy of sliding different sized cut-out squares along his chessboard in order to arrive at the totals for each different square. He recorded his results in a table and succeeded in spotting a pattern of square numbers. He was not able to give a reason for the pattern.

Activity | Healthy living

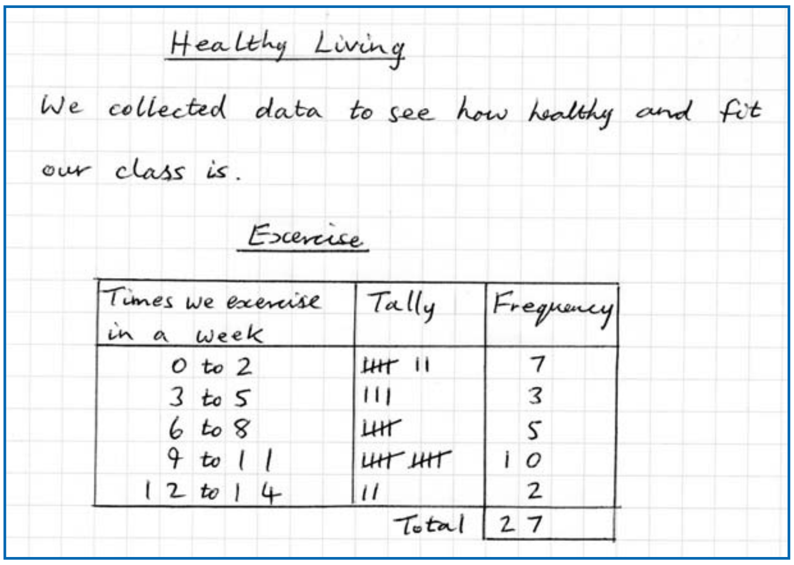
As part of a data enquiry into the importance of exercise and healthy eating, Oliver and his friend decide to find out how many times their classmates exercise during a week and whether they eat their ‘five a day’ fruits or vegetables. They collect and record data from all their classmates, and choose suitable data-handling techniques to represent and interpret their data.

There are opportunities for pupils to:

- select, organise and use the mathematics needed to solve problems
- identify data required in order to pursue a line of enquiry
- use a range of mathematical language to explain and communicate their work
- present work clearly, using diagrams and labelled graphs.

Pupils are also:

- considering evidence, information and ideas
- gathering information
- talking about and explaining work
- recording and interpreting data and presenting findings.

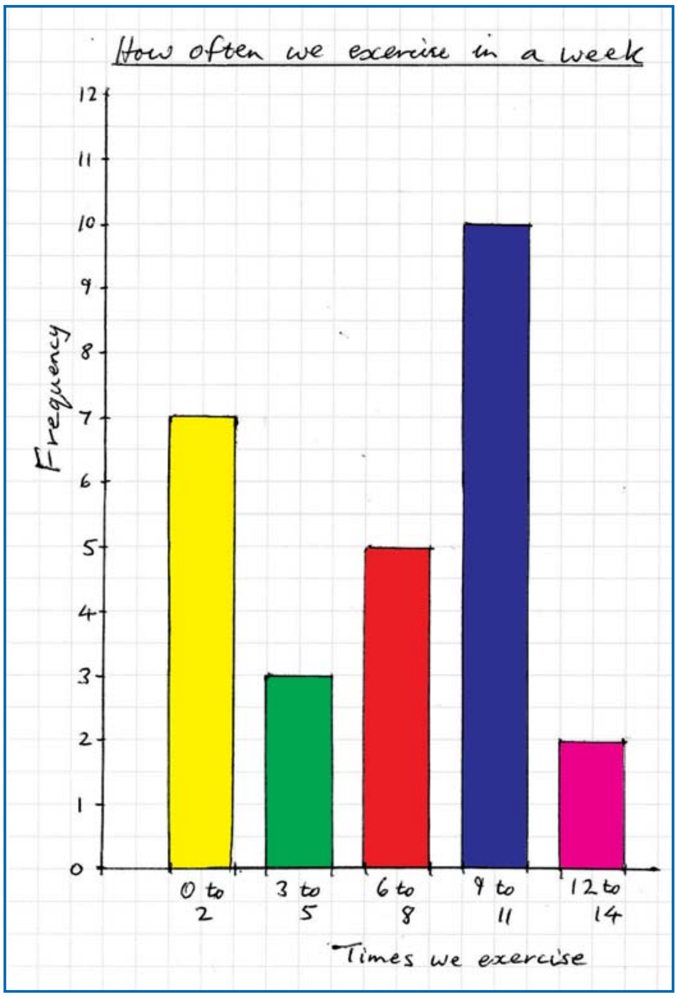


Healthy Living

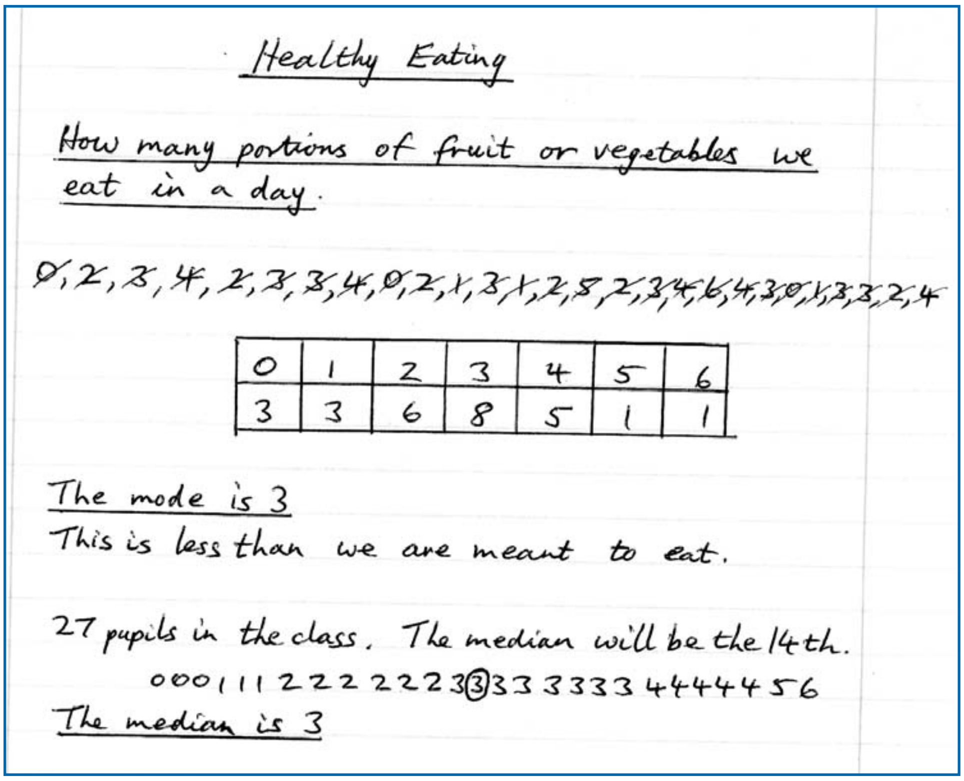
We collected data to see how healthy and fit our class is.

Exercise

Times we exercise in a week	Tally	Frequency
0 to 2	𝍸 ||	7
3 to 5	|||	3
6 to 8	𝍸	5
9 to 11	𝍸 𝍸	10
12 to 14	||	2
	Total	27



> I think that our class has quite a good record for the number of times we exercise in a week but we eat less than we are meant to of fruit and vegetables.



Healthy Eating

How many portions of fruit or vegetables we eat in a day.

~~0~~, ~~2~~, ~~3~~, ~~4~~, ~~2~~, ~~3~~, ~~3~~, ~~4~~, ~~0~~, ~~2~~, ~~1~~, ~~3~~, ~~1~~, ~~2~~, ~~5~~, ~~2~~, ~~3~~, ~~4~~, ~~6~~, ~~4~~, ~~3~~, ~~0~~, ~~1~~, ~~3~~, ~~3~~, 2, 4

0	1	2	3	4	5	6
3	3	6	8	5	1	1

The mode is 3
This is less than we are meant to eat.

27 pupils in the class. The median will be the 14th.
000111 222 2223③33 3333 44444 56
The median is 3

Commentary

Oliver grouped his data into sensible groups, and created a frequency table and diagram to display his data. He also offered some interpretation of his frequency diagram. His work with the 'number of portions of fruit and vegetables' shows a systematic way of arriving at the modal value for quite a large set of data.

Activity | Fraction triangles

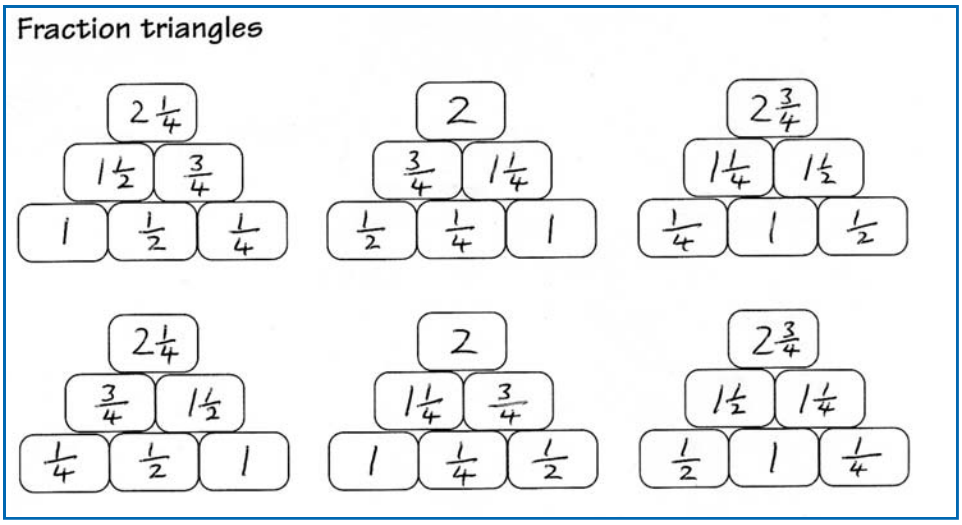
The focus of this activity is adding fractions (and/or decimals) in a triangle of numbers. The activity involves arranging simple fractions at the base of the triangle and adding adjacent pairs until the total is arrived at in the topmost block. Initially, only three fractions are used and pupils investigate which arrangements give the highest and lowest totals. The activity can then be extended to include four or more numbers at the base of the triangle and/or harder fractions.

There are opportunities for pupils to:

- select the mathematics needed to solve problems
- develop and use their own mathematical strategies
- use a range of mental and written computational strategies
- justify how they arrived at a conclusion to a problem.

Pupils are also:

- thinking logically and seeking patterns
- monitoring progress
- using numbers
- using a variety of methods
- talking about and explaining work.



Fraction triangles

$2\frac{1}{4}$
$1\frac{1}{2}$ $\frac{3}{4}$
1 $\frac{1}{2}$ $\frac{1}{4}$

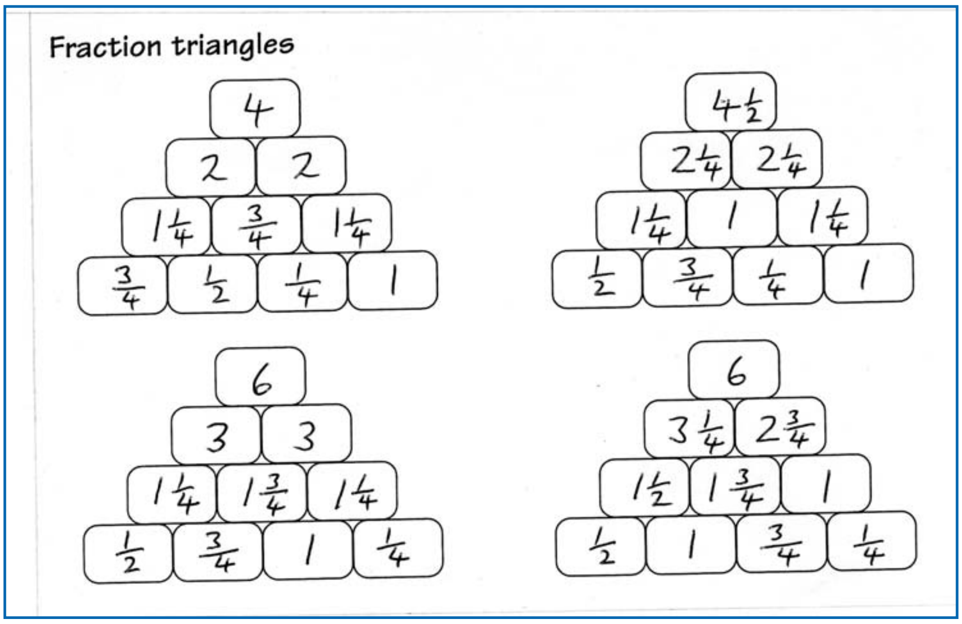
2
$\frac{3}{4}$ $1\frac{1}{4}$
$\frac{1}{2}$ $\frac{1}{4}$ 1

$2\frac{3}{4}$
$1\frac{1}{4}$ $1\frac{1}{2}$
$\frac{1}{4}$ 1 $\frac{1}{2}$

$2\frac{1}{4}$
$\frac{3}{4}$ $1\frac{1}{2}$
$\frac{1}{4}$ $\frac{1}{2}$ 1

2
$1\frac{1}{4}$ $\frac{3}{4}$
1 $\frac{1}{4}$ $\frac{1}{2}$

$2\frac{3}{4}$
$1\frac{1}{2}$ $1\frac{1}{4}$
$\frac{1}{2}$ 1 $\frac{1}{4}$



Fraction triangles

4
2 2
$1\frac{1}{4}$ $\frac{3}{4}$ $1\frac{1}{4}$
$\frac{3}{4}$ $\frac{1}{2}$ $\frac{1}{4}$ 1

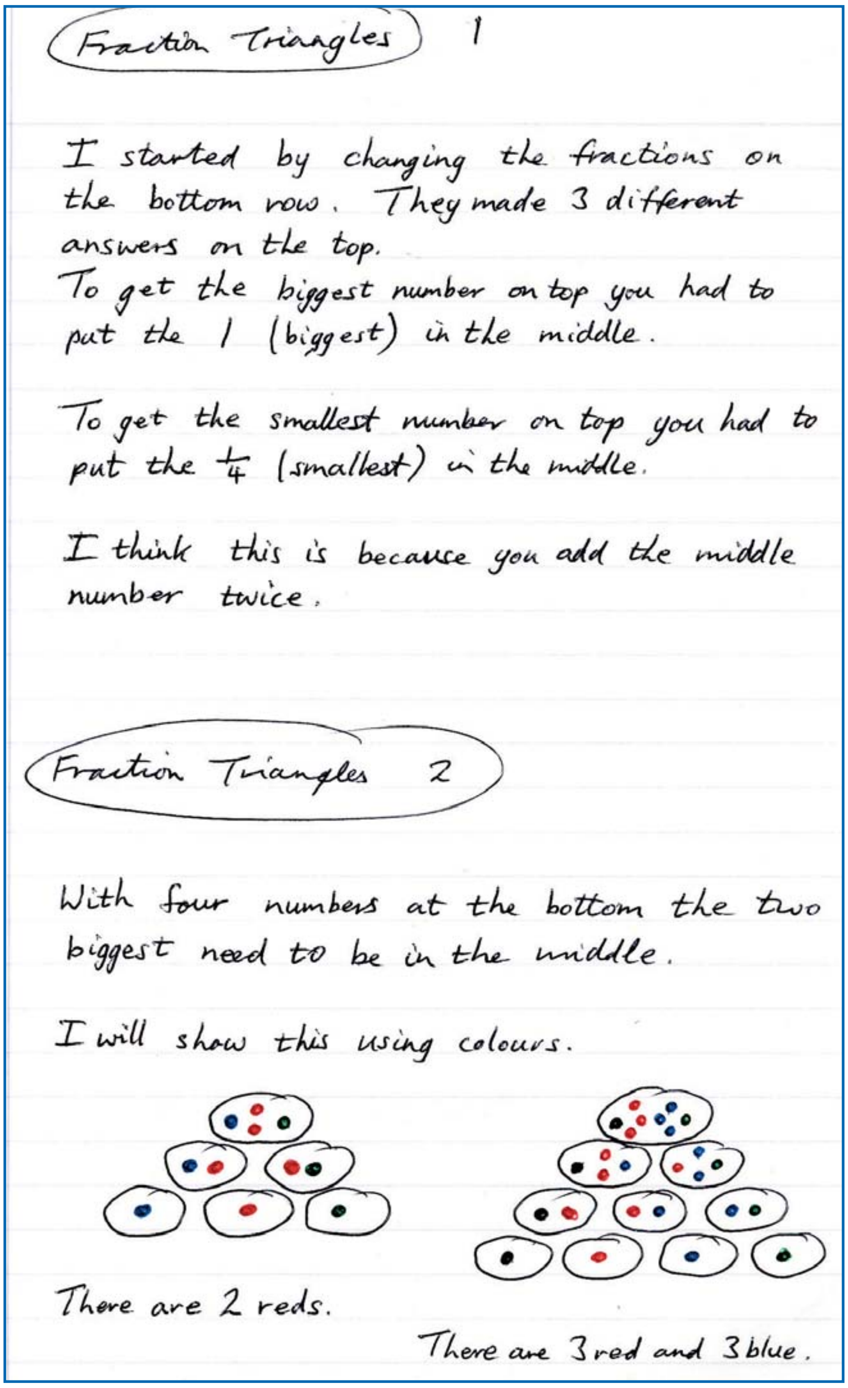
$4\frac{1}{2}$
$2\frac{1}{4}$ $2\frac{1}{4}$
$1\frac{1}{4}$ 1 $1\frac{1}{4}$
$\frac{1}{2}$ $\frac{3}{4}$ $\frac{1}{4}$ 1

6
3 3
$1\frac{1}{4}$ $1\frac{3}{4}$ $1\frac{1}{4}$
$\frac{1}{2}$ $\frac{3}{4}$ 1 $\frac{1}{4}$

6
$3\frac{1}{4}$ $2\frac{3}{4}$
$1\frac{1}{2}$ $1\frac{3}{4}$ 1
$\frac{1}{2}$ 1 $\frac{3}{4}$ $\frac{1}{4}$

Swapping the two outer numbers makes no difference to the total.

I think this is because you add the outer numbers once and the middle number twice.



Fraction Triangles 1

I started by changing the fractions on the bottom row. They made 3 different answers on the top.
To get the biggest number on top you had to put the 1 (biggest) in the middle.

To get the smallest number on top you had to put the $\frac{1}{4}$ (smallest) in the middle.

I think this is because you add the middle number twice.

Fraction Triangles 2

With four numbers at the bottom the two biggest need to be in the middle.

I will show this using colours.

There are 2 reds.

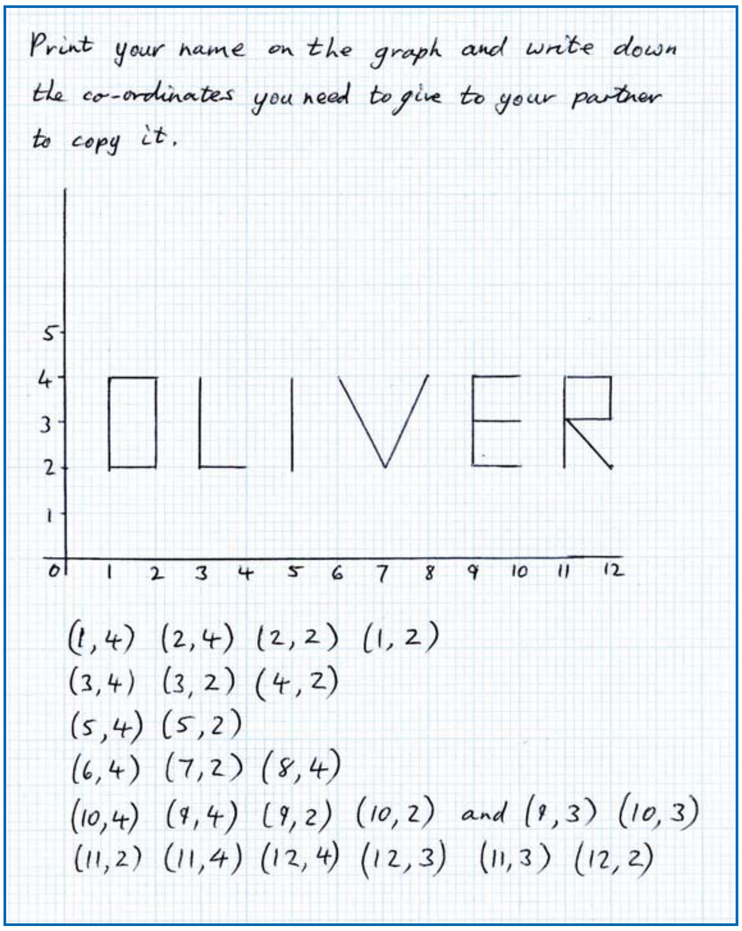
There are 3 red and 3 blue.

Commentary

Oliver used three simple values of 1, ½ and ¼ to begin his investigation and worked out all six arrangements accurately. He noticed that the arrangement with the biggest number in the middle of the base gave the highest total and the one with the smallest fraction in the middle gave the lowest total. He gave an explanation for this, then investigated a similar problem with four base numbers. He devised his own diagrams to explain why the position of the larger (or smaller) numbers was significant.

Bits and bobs

These are examples of short pieces of work completed by Oliver during the year, and include work with co-ordinates, reflections, reading scales and the use of a formula expressed in words.



Print your name on the graph and write down the co-ordinates you need to give to your partner to copy it.

(1,4) (2,4) (2,2) (1,2)

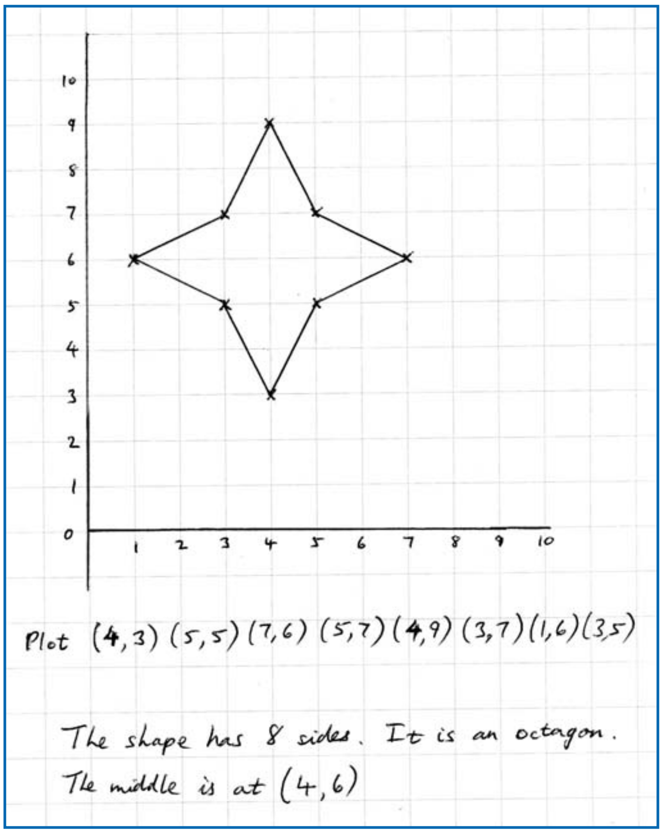
(3,4) (3,2) (4,2)

(5,4) (5,2)

(6,4) (7,2) (8,4)

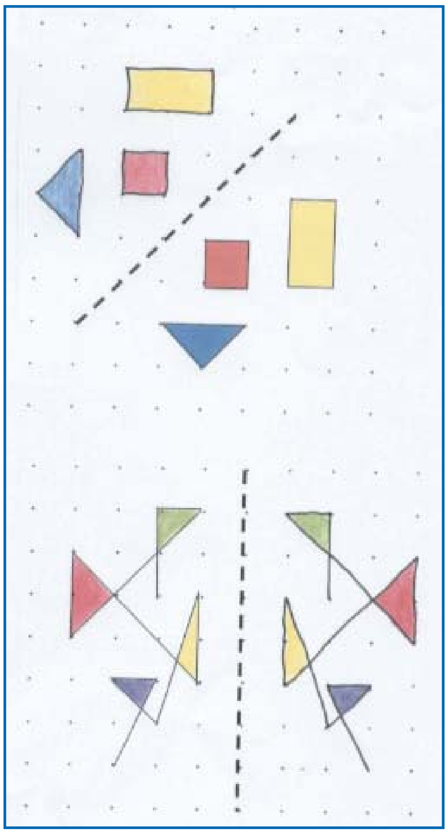
(10,4) (9,4) (9,2) (10,2) and (9,3) (10,3)

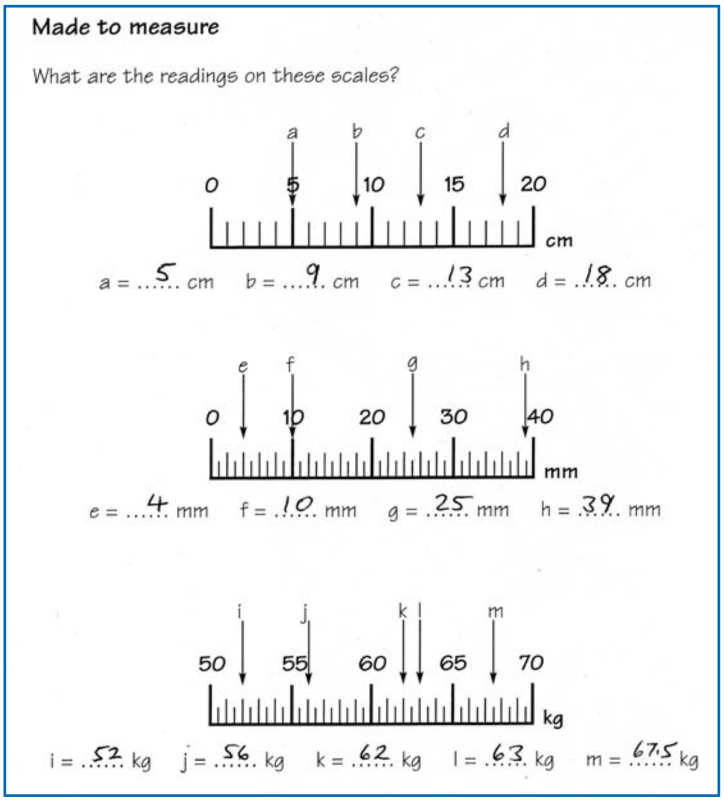
(11,2) (11,4) (12,4) (12,3) (11,3) (12,2)



Plot (4,3) (5,5) (7,6) (5,7) (4,9) (3,7)(1,6)(3,5)

The shape has 8 sides. It is an octagon.
The middle is at (4,6)





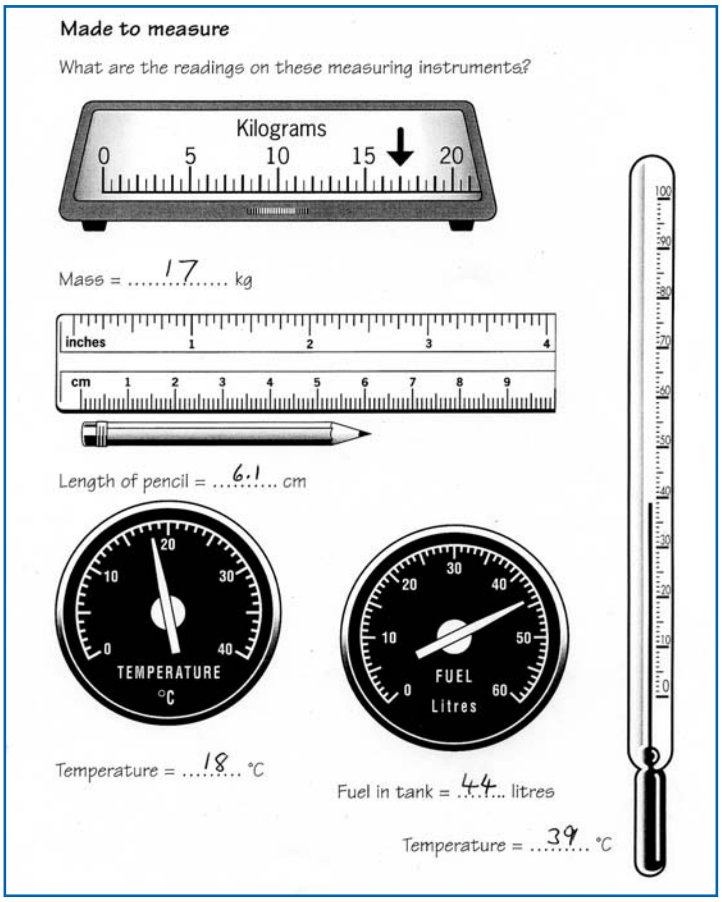
Made to measure

What are the readings on these scales?

a = ..5... cm b = ...9. cm c = ...13 cm d = .18. cm

e = ...4. mm f = .10. mm g = .25. mm h = .39. mm

i = .52. kg j = .56. kg k = .62. kg l = .63. kg m = 67.5 kg



Made to measure

What are the readings on these measuring instruments?

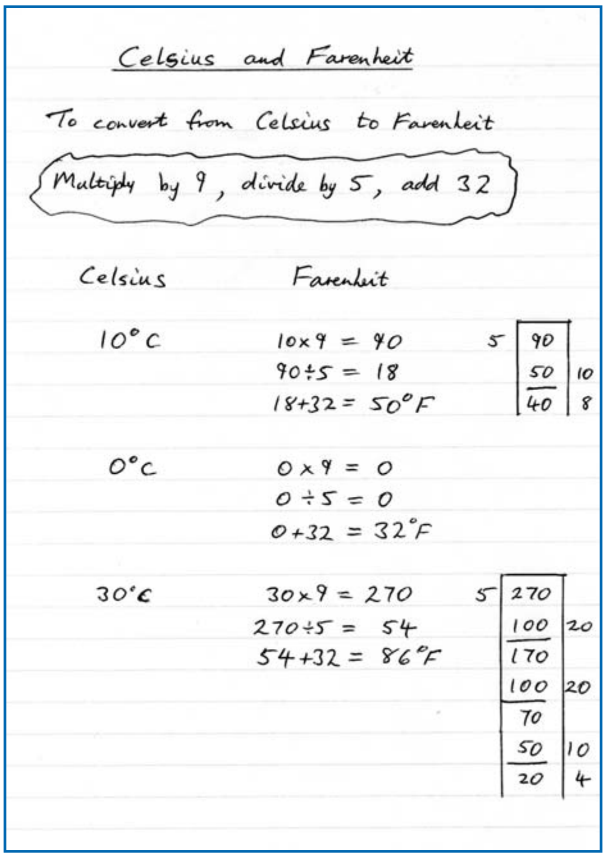
Mass = ...17...... kg

Length of pencil = ...6.1... cm

Temperature = ...18... °C

Fuel in tank = .44.. litres

Temperature = ...39.. °C



Celsius and Farenheit

To convert from Celsius to Farenheit

Multiply by 9, divide by 5, add 32

Celsius	Farenheit
10°C	10×9 = 90 90÷5 = 18 18+32 = 50°F
0°C	0×9 = 0 0÷5 = 0 0+32 = 32°F
30°C	30×9 = 270 270÷5 = 54 54+32 = 86°F

5 | 90
50 | 10
40 | 8

5 | 270
100 | 20
170
100 | 20
70
50 | 10
20 | 4

Commentary

Oliver correctly plotted and interpreted co-ordinates in the first quadrant and reflected patterns accurately in mirror lines with the help of a mirror. He interpreted a number of scales in the context of measurement and used a range of mental and written computational strategies whilst using a formula expressed in words.

Summary and overall judgement

Levels 4 and 5 were considered and Level 4 was judged to be the best fit.

Oliver has shown, in ‘The chessboard’ and ‘Consecutive numbers’, that he can *develop strategies for solving problems, present information and results systematically,* and *recognise and describe number patterns*. He has also shown, in ‘Bits and bobs’, that he can *draw 2-D shapes on grids, reflect simple shapes in a mirror line, use and interpret co-ordinates in the first quadrant*, and *read, with appropriate accuracy, numbers on a range of measuring instruments*. In ‘Healthy living’, he *collected discrete data, grouped the data, used the mode and median of a set of data*, and *drew and interpreted a frequency diagram*. All of these are characteristic of Level 4.

In ‘Consecutive numbers’ and ‘The chessboard’, Oliver *described situations mathematically using words, drew his own conclusions and explained his reasoning*. In ‘Fraction triangles’ and ‘Consecutive numbers’, he made *general statements of his own, based on available evidence*. These are characteristic of Level 5.

Key Stage 3

Adam

Level 5

Adam is a 14-year-old learner in Key Stage 3.

His teacher knows much more about Adam's performance than can be included here. However, this profile has been selected to illustrate characteristic features of Adam's work across a range of activities. Each example is accompanied by a brief commentary to provide a context and indicate particular qualities in the work.

Adam's teacher judges that his performance in mathematics is best described as Level 5.

Activity | Half-time scores

This activity is introduced to the pupils by looking at some of the local Saturday football scores and posing the questions 'What could the half-time score have been?' and 'How many different half-time scores were possible?' Strategies for developing the work are discussed as a class. Adam initially tries to find all half-time scores for up to four goals in a match, and then considers half-time draw scores for up to six goals in a match, leading to formulae for the number of draw scores.

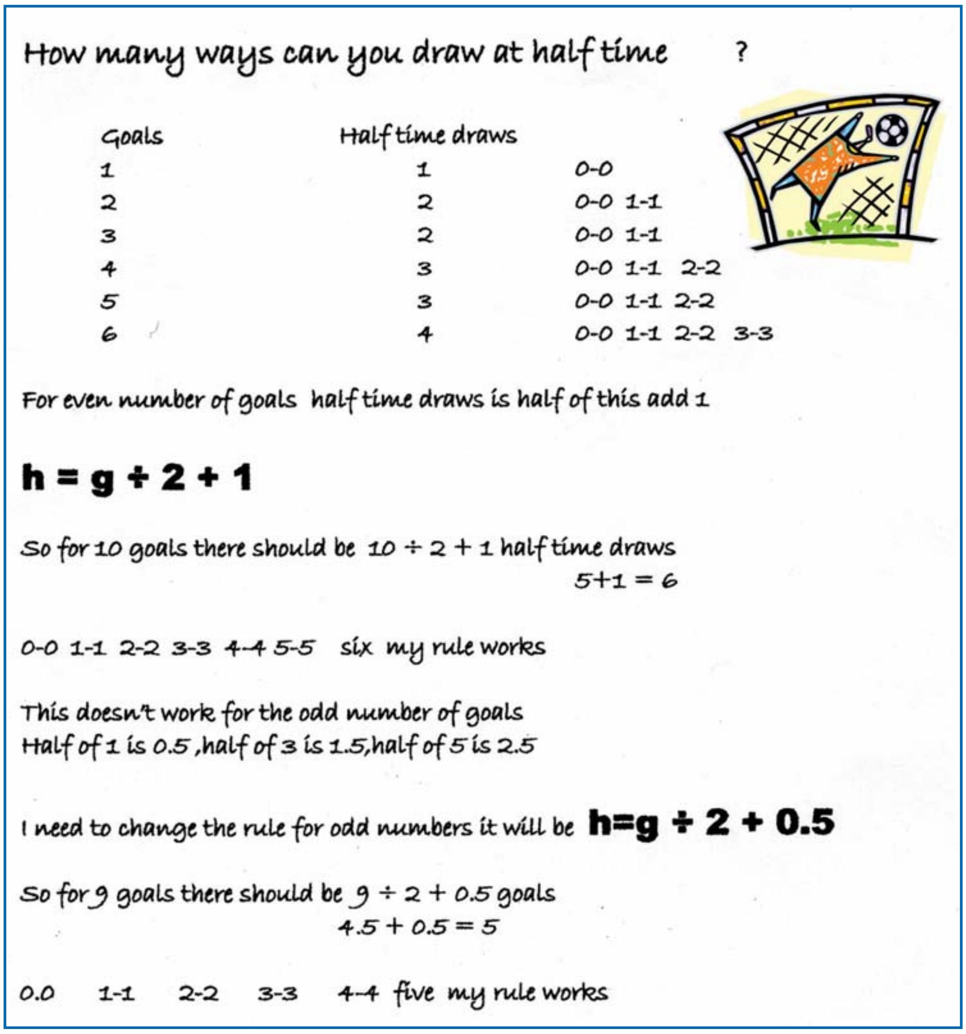
There are opportunities for pupils to:

- develop and use their own mathematical strategies and ideas and consider those of others
- break complex problems into a series of tasks
- trial and evaluate a variety of approaches
- explain patterns in words; express simple functions in words and symbolically
- predict subsequent terms in number sequences
- present work clearly using labelled graphs.

Pupils are also:

- determining the process/method and strategy
- thinking logically and seeking patterns
- using the number system
- recording and interpreting data and presenting findings (using ICT).

This is the second part of Adam's work, focusing on draw scores.



How many ways can you draw at half time ?

Goals	Half time draws	
1	1	0-0
2	2	0-0 1-1
3	2	0-0 1-1
4	3	0-0 1-1 2-2
5	3	0-0 1-1 2-2
6	4	0-0 1-1 2-2 3-3

For even number of goals half time draws is half of this add 1

$$h = g \div 2 + 1$$

So for 10 goals there should be $10 \div 2 + 1$ half time draws
$5+1 = 6$

0-0 1-1 2-2 3-3 4-4 5-5 six my rule works

This doesn't work for the odd number of goals
Half of 1 is 0.5 ,half of 3 is 1.5,half of 5 is 2.5

I need to change the rule for odd numbers it will be $h=g \div 2 + 0.5$

So for 9 goals there should be $9 \div 2 + 0.5$ goals
$4.5 + 0.5 = 5$

0.0 1-1 2-2 3-3 4-4 five my rule works

For an even number of total goals, there will be half that number of draw scores, plus nil-nil.

For an odd number of total goals, there will be half of one less than the number, plus nil-nil.

Commentary

Adam presented his results using ICT. He worked systematically and found patterns in his results. He discovered that the patterns for odd and even total goals were different. He expressed these in words and in simple algebra, and then checked his formulae by considering special cases.

Activity | Five a day

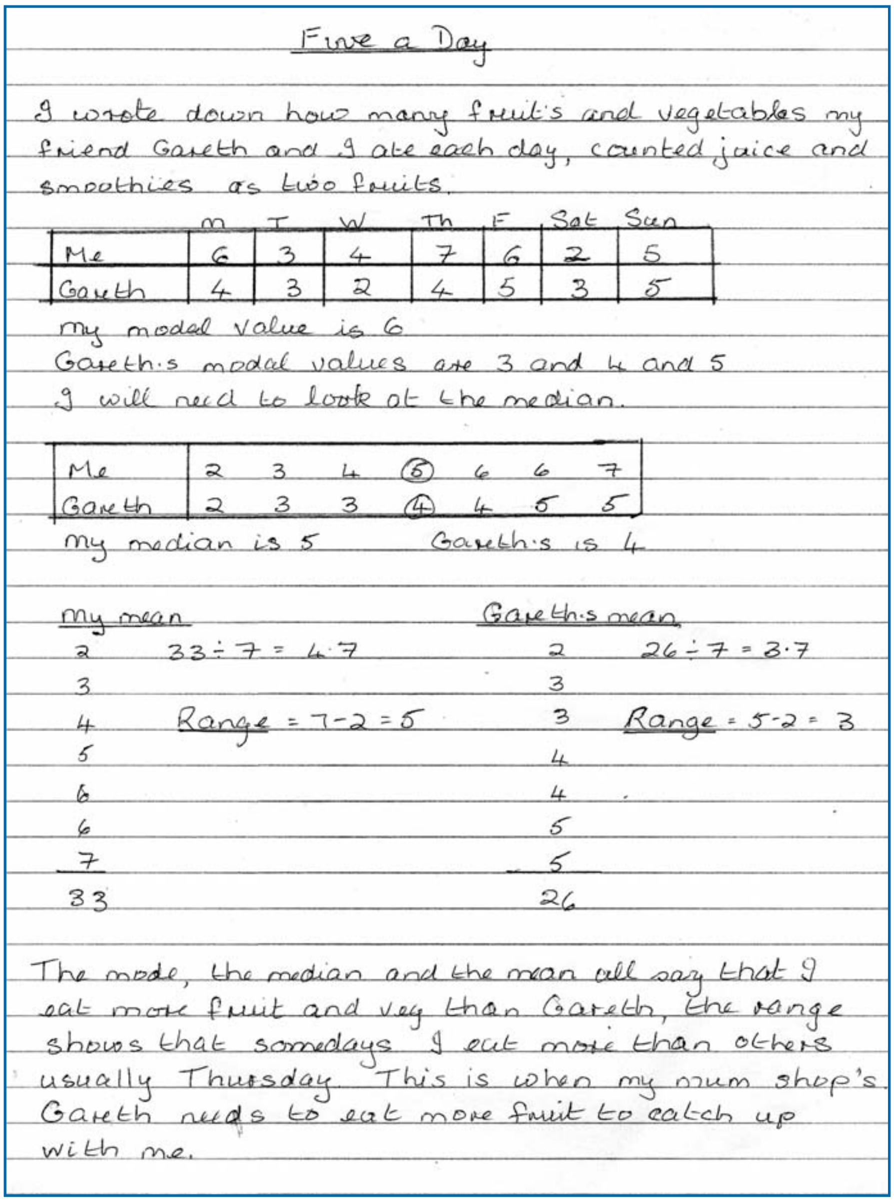
In this activity, pupils use averages and spread to compare two simple distributions. As part of a data enquiry into healthy eating habits in Year 9, Adam and his friend decide to find out which of them eats more fruit and vegetables and whether they are close to having their 'five a day'. They collect and record data over a period of two weeks, and use data-handling techniques in order to compare their results.

There are opportunities for pupils to:

- select and use the mathematics and methods of computation needed to solve problems
- identify the data required to pursue a line of enquiry
- make a hypothesis and design a method to test it
- use a range of mathematical language to explain and communicate their work
- use mental and calculator computational strategies
- use checking strategies
- present work clearly, using diagrams and labelled graphs.

Pupils are also:

- determining the process/method and strategy
- thinking logically
- monitoring progress
- communicating information
- gathering information
- recording and interpreting data and presenting findings.



Five a Day

I wrote down how many fruit's and vegetables my friend Gareth and I ate each day, counted juice and smoothies as two fruits.

	m	T	W	Th	F	Sat	Sun
Me	6	3	4	7	6	2	5
Gareth	4	3	2	4	5	3	5

my modal value is 6

Gareth.s modal values are 3 and 4 and 5

I will need to look at the median.

Me	2	3	4	⑤	6	6	7
Gareth	2	3	3	④	4	5	5

my median is 5 Gareth.s is 4

my mean		Gareth.s mean	
2	33 ÷ 7 = 4.7	2	26 ÷ 7 = 3.7
3		3	
4	Range = 7 − 2 = 5	3	Range = 5 − 2 = 3
5		4	
6		4	
6		5	
7		5	
33		26	

The mode, the median and the mean all say that I eat more fruit and veg than Gareth, the range shows that somedays I eat more than others usually Thursday. This is when my mum shop's. Gareth needs to eat more fruit to catch up with me.

Commentary

Adam decided to consider the mode, median and mean of the data, as well as the range, when attempting to compare the outcomes for his friend and himself. He approximated his answers for the means and checked that they were sensible. He also offered an explanation for the variations in the data over the two-week period and arrived at sensible conclusions.

Activity | Chances are

In this activity, pupils assign values to the probabilities of various events. Pupils are given a pack of 12 cards, each with an 'event' written on it, and assign numerical values where possible to each one. The activity is extended by asking pupils to generate statements of their own for specific probabilities.

There are opportunities for pupils to:

- develop and use their own mathematical ideas
- use checking strategies
- use a range of mathematical language
- explain their reasoning.

Pupils are also:

- activating prior skills, knowledge and understanding
- considering information
- using numbers
- talking about and explaining work.

A. Throw a 3 on one throw of a fair die numbered from 1 to 6.

B. Get a tail when spinning a fair coin.

C. Randomly draw a yellow counter from a bag containing only red and blue counters.

D. Win first prize when you buy one ticket in a raffle where 100 tickets are sold.

E. Randomly select a child with brown eyes from a class of thirty children, six of whom have blue eyes and the rest brown eyes.

F. Randomly select an apple from a bowl containing 10 apples and 5 oranges.

G. Throw an even number on one throw of a fair die numbered from 1 to 6.

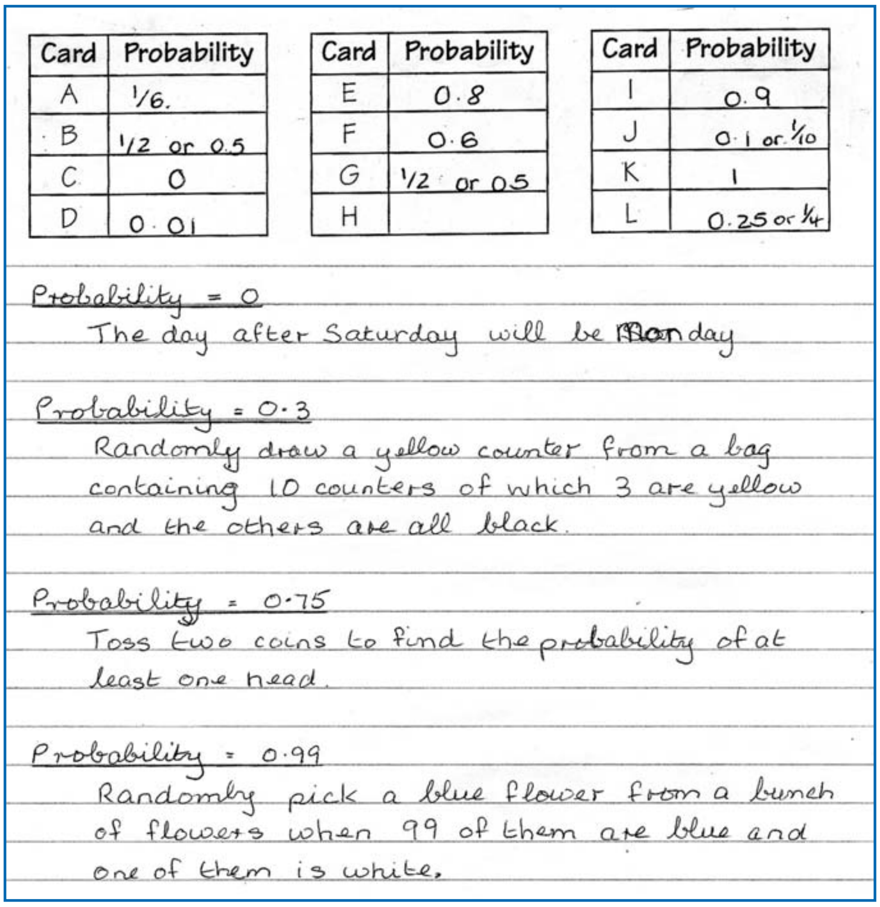
H. Tomorrow it will rain.

I. Randomly draw a black counter from a bag containing 50 counters, of which 45 are black and the other 5 are white.

J. Win first prize in an office draw when each of ten people in the office has one ticket in the draw.

K. The day after Wednesday will be Thursday.

L. When one card is dealt from a standard pack of 52 cards, it will be a diamond.



Card	Probability
A	1/6.
B	1/2 or 0.5
C	0
D	0. 01

Card	Probability
E	0.8
F	0.6
G	1/2 or 0.5
H	

Card	Probability
I	0.9
J	0.1 or 1/10
K	1
L	0.25 or 1/4

Probability = 0

The day after Saturday will be Monday

Probability = 0.3

Randomly draw a yellow counter from a bag containing 10 counters of which 3 are yellow and the others are all black.

Probability = 0.75

Toss two coins to find the probability of at least one head.

Probability = 0.99

Randomly pick a blue flower from a bunch of flowers when 99 of them are blue and one of them is white.

Commentary

Adam used the probability scale from 0 to 1 correctly when assigning most of the numerical values, and was able to suggest suitable events with the given probabilities.

Activity | Beat that

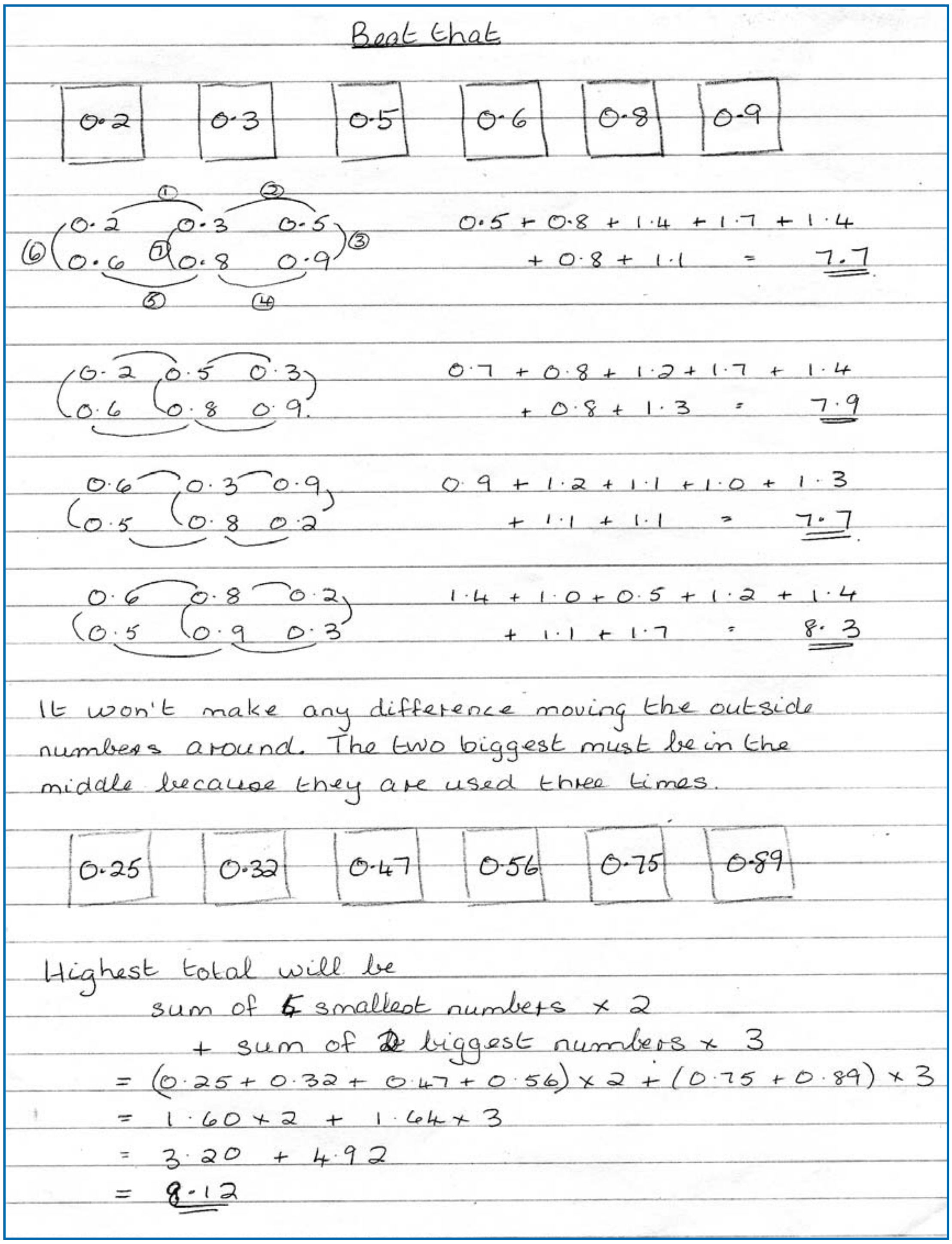
The focus of this activity is the addition of decimals. Pupils are given a set of six cards on each of which is a one-place decimal. They place the cards in a 2 x 3 grid and calculate all the pairs of totals of adjacent cards (seven pairs in all). The aim is to investigate the totals achieved by moving the cards and to produce the greatest total. The activity is repeated with numbers to two decimal places. Adam then makes an interesting extension to placing nine numbers on a square grid.

There are opportunities for pupils to:

- use a range of mental, written and calculator computational strategies
- use a variety of checking strategies, including mental estimation, approximation and inverse operations
- explain strategies and reasoning
- extend mental methods of computation to consolidate a range of non-calculator methods
- justify how they arrived at a conclusion to a problem.

Pupils are also:

- determining the process/method and strategy
- using the number system
- using a variety of methods
- talking about and explaining work.



Beat that

0·2	0·3	0·5	0·6	0·8	0·9

① ②
⑥ 0·2 0·3 0·5 ③
0·6 ⑦ 0·8 0·9
⑤ ④

0·5 + 0·8 + 1·4 + 1·7 + 1·4
+ 0·8 + 1·1 = 7.7

0·2 0·5 0·3
0·6 0·8 0·9.

0·7 + 0·8 + 1·2 + 1·7 + 1·4
+ 0·8 + 1·3 = 7·9

0·6 0·3 0·9
0·5 0·8 0·2

0·9 + 1·2 + 1·1 + 1·0 + 1·3
+ 1·1 + 1·1 = 7·7

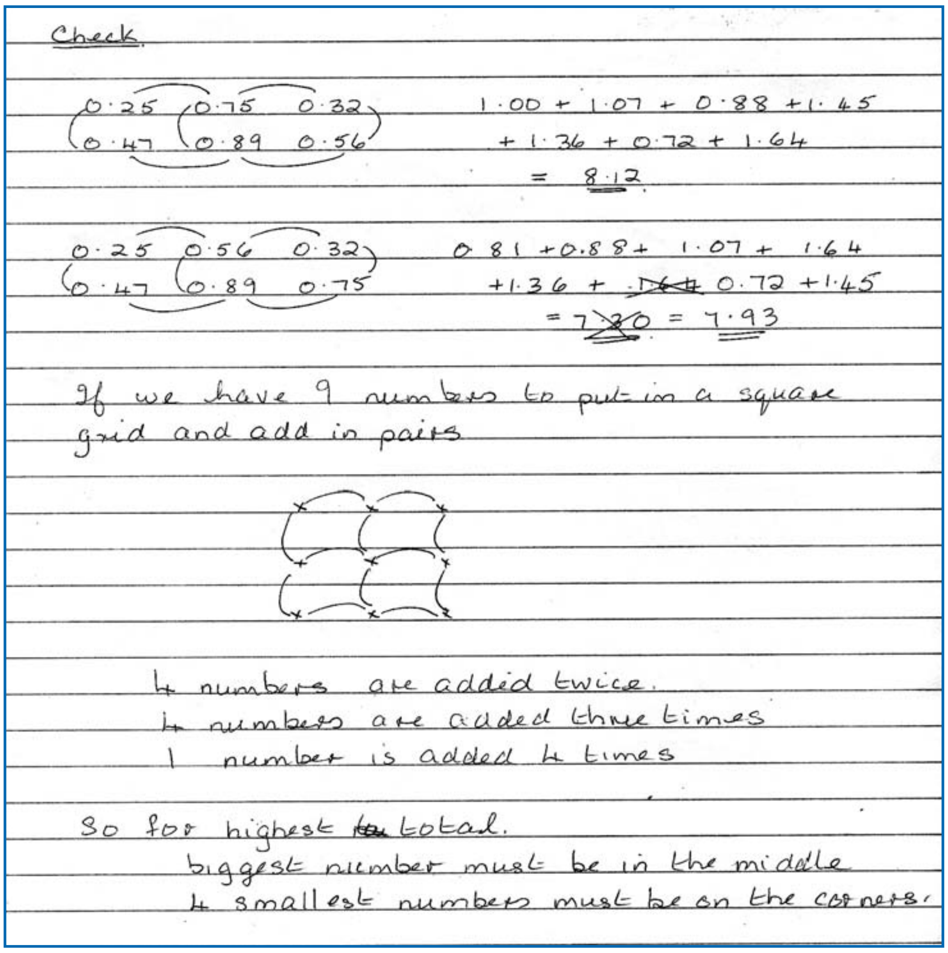
0·6 0·8 0·2
0·5 0·9 0·3

1·4 + 1·0 + 0·5 + 1·2 + 1·4
+ 1·1 + 1·7 = 8·3

It won't make any difference moving the outside numbers around. The two biggest must be in the middle because they are used three times.

0·25	0·32	0·47	0·56	0·75	0·89

Highest total will be
sum of 4 smallest numbers × 2
+ sum of 2 biggest numbers × 3
= (0·25 + 0·32 + 0·47 + 0·56) × 2 + (0·75 + 0·89) × 3
= 1·60 × 2 + 1·64 × 3
= 3·20 + 4·92
= 8·12



Check

0·25 0·75 0·32
0·47 0·89 0·56

1·00 + 1·07 + 0·88 + 1·45
+ 1·36 + 0·72 + 1·64
= 8·12

0·25 0·56 0·32
0·47 0·89 0·75

0 81 + 0.88 + 1·07 + 1·64
+1·36 + ~~1·64~~ 0·72 + 1·45
= ~~7·30~~ = 7·93

If we have 9 numbers to put in a square grid and add in pairs

4 numbers are added twice.
4 numbers are added three times
1 number is added 4 times

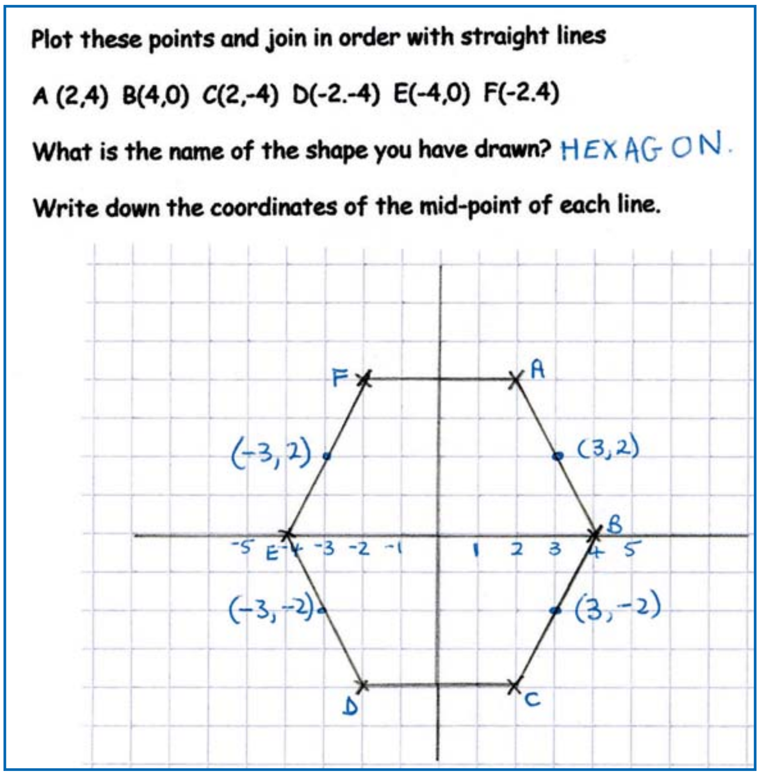
So for highest ~~to~~ total.
biggest number must be in the middle
4 smallest numbers must be on the corners.

Commentary

Adam added most of the pairs of one and two digit decimal sums correctly. He devised an effective way to show all the pairs of numbers that could be added. He noticed that the largest overall total occurred when the two biggest numbers were in the middle, and was able to explain why this was. He used his result when adding single digit decimals to predict the results for double digit decimals, and checked his prediction. His method of showing all the pairs helped him to identify an error when he noticed that he only had six pair-totals in one of his checks; he went on to correct his calculations. He then used a similar strategy and presentation to make a generalisation about placing nine numbers on a square grid.

Bits and bobs

These are examples of short pieces of work completed by Adam during the year, including work involving co-ordinates, negative numbers, the equivalences of fractions, decimals and percentages, and the outcomes of a class project to design a logo for the school mathematics club that involved creating enlargements of pupils' own designs.

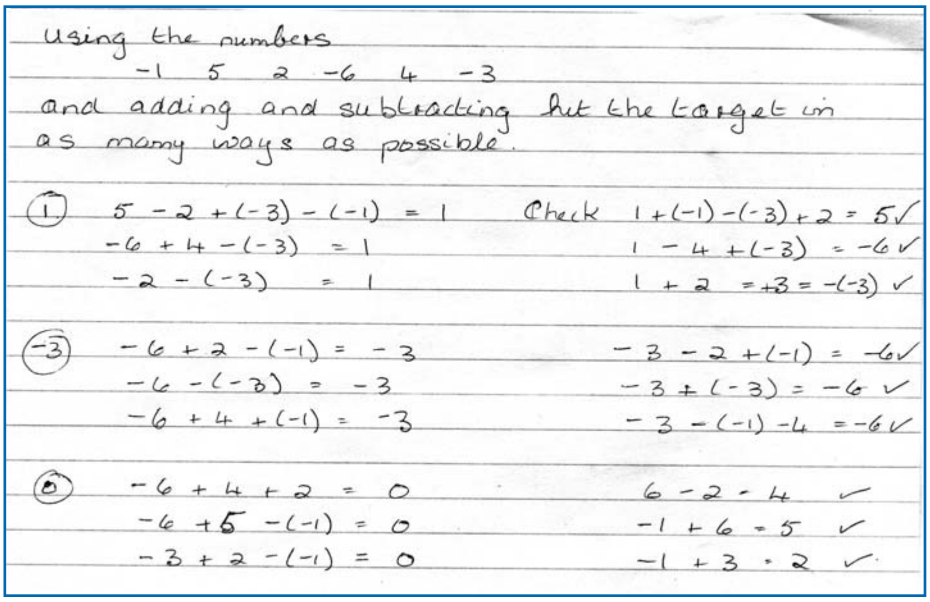


Plot these points and join in order with straight lines

A (2,4) B(4,0) C(2,-4) D(-2.-4) E(-4,0) F(-2.4)

What is the name of the shape you have drawn? HEXAGON.

Write down the coordinates of the mid-point of each line.



using the numbers

−1 5 2 −6 4 −3

and adding and subtracting hit the target in as many ways as possible.

① $5 - 2 + (-3) - (-1) = 1$ Check $1 + (-1) - (-3) + 2 = 5$ ✓

$-6 + 4 - (-3) = 1$ $1 - 4 + (-3) = -6$ ✓

$-2 - (-3) = 1$ $1 + 2 = +3 = -(-3)$ ✓

(−3) $-6 + 2 - (-1) = -3$ $-3 - 2 + (-1) = -6$ ✓

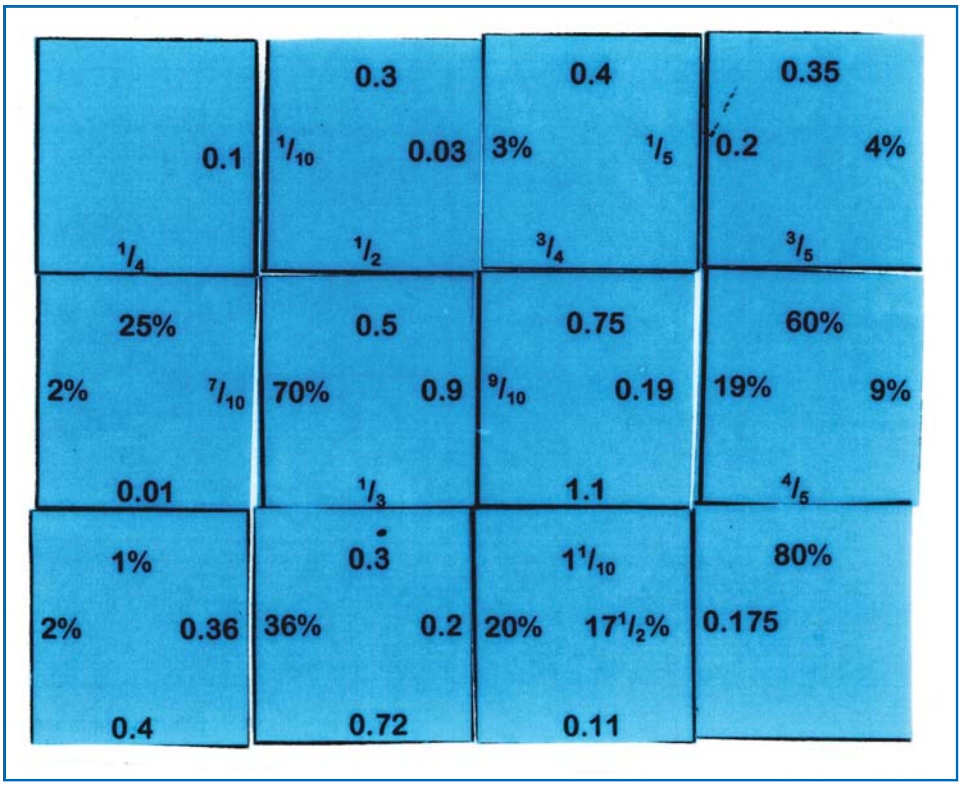
$-6 - (-3) = -3$ $-3 + (-3) = -6$ ✓

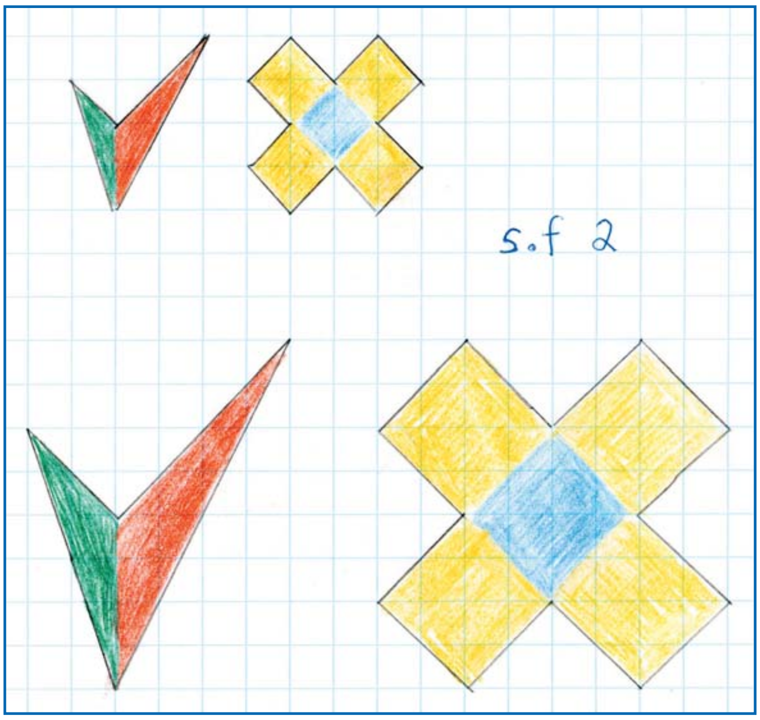
$-6 + 4 + (-1) = -3$ $-3 - (-1) - 4 = -6$ ✓

⓪ $-6 + 4 + 2 = 0$ $6 - 2 - 4$ ✓

$-6 + 5 - (-1) = 0$ $-1 + 6 - 5$ ✓

$-3 + 2 - (-1) = 0$ $-1 + 3 \cdot 2$ ✓





Commentary

Adam plotted and interpreted co-ordinates in all four quadrants. He added and subtracted negative numbers correctly, and used inverse operations to check this work. He accurately organised a set of cards to show the equivalences of fractions, decimals and percentages, and correctly enlarged his logo.

Summary and overall judgement

Levels 4, 5 and 6 were considered and Level 5 was judged to be the best fit.

In ‘Half-time scores’ and ‘Five a day’, Adam *presented information and results systematically*. In ‘Beat that’, he *added decimals to two places*, and in ‘Five a day’, he *used the mode and median as characteristics of a set of data*. These are all characteristic of Level 4.

In ‘Half-time scores’, Adam has shown that he can *identify and obtain information to solve a problem*. In ‘Half-time scores’ and ‘Beat that’, he *drew his own conclusions, explaining his reasoning* and *made a general statement, based on available evidence*. In ‘Bits and bobs’, he has shown that he can *add and subtract negative numbers*. He *constructed and used simple formulae involving two operations* in ‘Half-time scores’, and *used co-ordinates in all four quadrants* in ‘Bits and bobs’. In ‘Five a day’, he *used the mean of discrete data and compared two simple distributions*, and in ‘Chances are’ he *used the probability scale from 0 to 1*. These are all characteristic of Level 5.

In ‘Half-time scores’, Adam has shown that he can *break down a problem into smaller tasks* by considering odd and even numbers of goals separately. In ‘Bits and bobs’, he has also shown that he understands *the equivalences between fractions, decimals and percentages* and *can enlarge shapes by a positive whole-number scale factor*. These are all characteristic of Level 6.

Key Stage 3

Sarah

Level 7

Sarah is a 14-year-old learner in Key Stage 3.

Her teacher knows much more about Sarah's performance than can be included here. However, this profile has been selected to illustrate characteristic features of Sarah's work across a range of activities. Each example is accompanied by a brief commentary to provide a context and indicate particular qualities in the work.

Sarah's teacher judges that her performance in mathematics is best described as Level 7.

Activity | Hidden faces

In this activity, Sarah investigates the number of faces 'hidden' when a cuboid made up of unit cubes is placed on a surface. The pupils consider shapes that are more complex than the initial row of cubes used to introduce the task, and are expected to offer spatial justifications for their results.

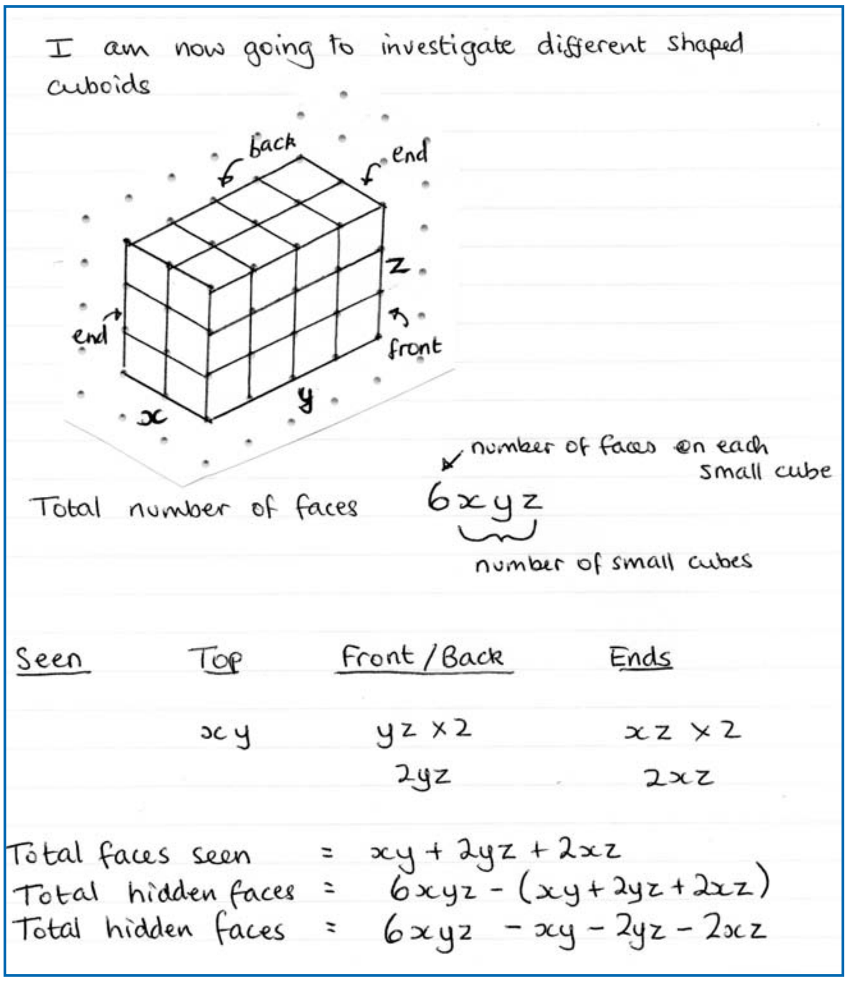
There are opportunities for pupils to:

- break complex problems into a series of tasks
- make and test generalisations
- explain strategies and reason orally and in writing
- justify how they arrived at a conclusion to a problem
- use a wide range of mathematical language and symbols to communicate their work
- visualise, describe and represent shapes
- present work clearly using diagrams and symbols.

Pupils are also:

- determining the process/method and strategy
- thinking logically and seeking patterns
- reviewing the process/method
- using numbers
- recording and interpreting data and presenting findings.

The following is an extract showing Sarah's work with a cuboid of any dimensions.



I am now going to investigate different shaped cuboids

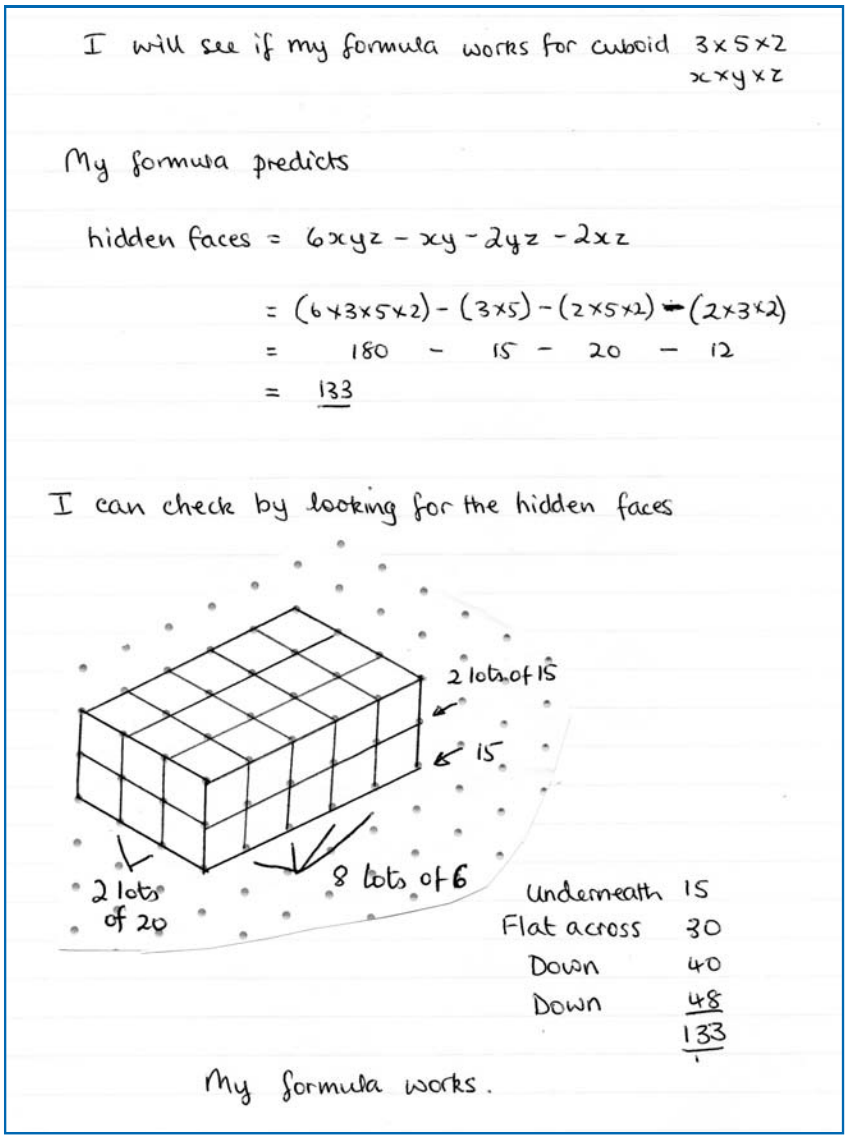
Total number of faces $\overset{\text{number of faces on each small cube}}{6}\underbrace{xyz}_{\text{number of small cubes}}$

Seen	Top	Front / Back	Ends
	xy	$yz \times 2$	$xz \times 2$
		$2yz$	$2xz$

Total faces seen = $xy + 2yz + 2xz$

Total hidden faces = $6xyz - (xy + 2yz + 2xz)$

Total hidden faces = $6xyz - xy - 2yz - 2xz$



I will see if my formula works for cuboid $3 \times 5 \times 2$
$x \times y \times z$

My formula predicts

$$\text{hidden faces} = 6xyz - xy - 2yz - 2xz$$
$$= (6 \times 3 \times 5 \times 2) - (3 \times 5) - (2 \times 5 \times 2) - (2 \times 3 \times 2)$$
$$= 180 - 15 - 20 - 12$$
$$= \underline{133}$$

I can check by looking for the hidden faces

2 lots of 15

15

2 lots of 20

8 lots of 6

Underneath	15
Flat across	30
Down	40
Down	48
	133

My formula works.

Commentary

Sarah worked systematically beginning with rows of cubes and eventually arriving at a general case for an *x* by *y* by *z* cuboid. She adopted a sensible strategy to arrive at her formula by considering the total number of faces for all the unit cubes and subtracting the faces that are seen. She explained how she arrived at her formula and checked that it worked for a special case, by directly counting the hidden faces.

Activity | Probability game

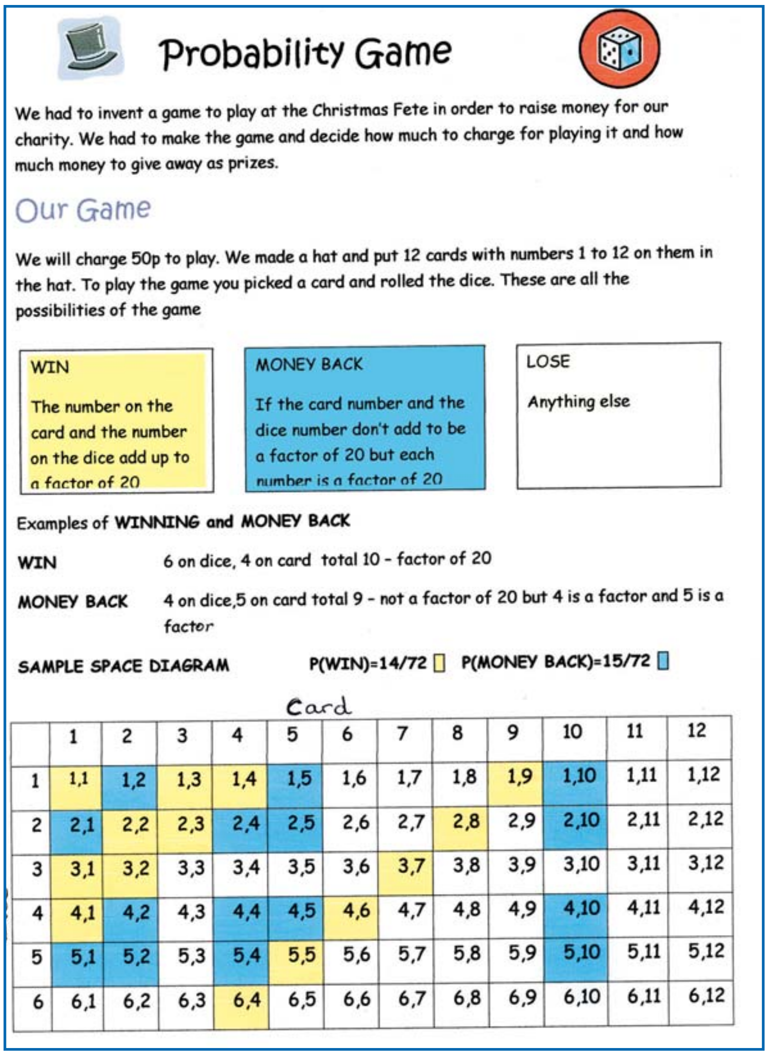
In this activity, pupils work in groups to devise their own games of chance to play during the school's Christmas Fête. They are told that the game should involve two events and that they would need a clear understanding of the probabilities of both events in order to ensure that their game generates a profit. Ideas for games and strategies for calculating profits are offered and discussed as part of an initial whole class activity. Sarah's group discuss a number of options and decide upon a game that involves picking one of twelve cards at random and throwing a die.

There are opportunities for pupils to:

- select, organise and use the mathematics needed to solve problems
- develop and use their own mathematical strategies and consider those of others
- explain choices and conclusions in writing
- justify how they arrived at a conclusion to a problem
- use mental and calculator strategies
- use mathematical language and notation
- present work clearly, using diagrams.

Pupils are also:

- determining the process/method and strategy
- generating and developing ideas
- thinking about cause and effect
- thinking logically
- monitoring progress
- reviewing outcomes
- creating and presenting information and ideas (using ICT)
- using numbers
- recording and interpreting data and presenting findings.



Probability Game

We had to invent a game to play at the Christmas Fete in order to raise money for our charity. We had to make the game and decide how much to charge for playing it and how much money to give away as prizes.

Our Game

We will charge 50p to play. We made a hat and put 12 cards with numbers 1 to 12 on them in the hat. To play the game you picked a card and rolled the dice. These are all the possibilities of the game

WIN	MONEY BACK	LOSE
The number on the card and the number on the dice add up to a factor of 20	If the card number and the dice number don't add to be a factor of 20 but each number is a factor of 20	Anything else

Examples of **WINNING** and **MONEY BACK**

WIN 6 on dice, 4 on card total 10 - factor of 20

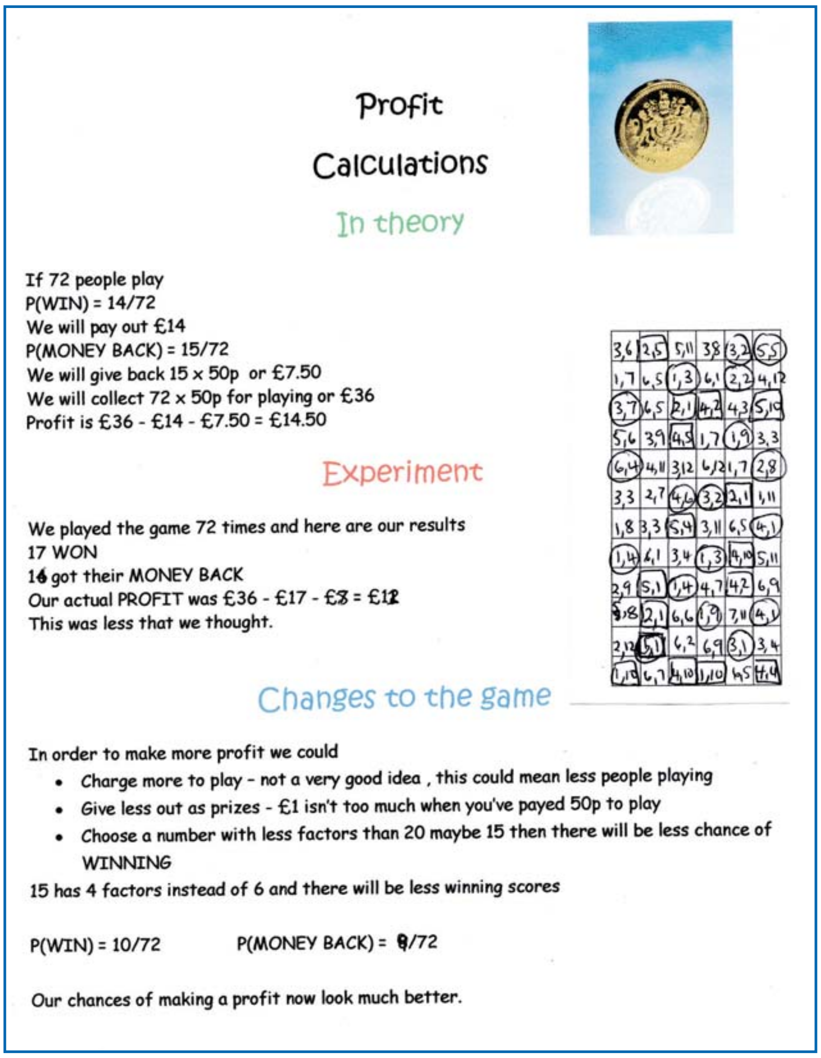
MONEY BACK 4 on dice,5 on card total 9 - not a factor of 20 but 4 is a factor and 5 is a factor

SAMPLE SPACE DIAGRAM P(WIN)=14/72 🟨 P(MONEY BACK)=15/72 🟦

Card

	1	2	3	4	5	6	7	8	9	10	11	12
1	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	1,10	1,11	1,12
2	2,1	2,2	2,3	2,4	2,5	2,6	2,7	2,8	2,9	2,10	2,11	2,12
3	3,1	3,2	3,3	3,4	3,5	3,6	3,7	3,8	3,9	3,10	3,11	3,12
4	4,1	4,2	4,3	4,4	4,5	4,6	4,7	4,8	4,9	4,10	4,11	4,12
5	5,1	5,2	5,3	5,4	5,5	5,6	5,7	5,8	5,9	5,10	5,11	5,12
6	6,1	6,2	6,3	6,4	6,5	6,6	6,7	6,8	6,9	6,10	6,11	6,12

In theory, we would expect to make about £14.50 profit, but when we tried it out as an experiment we made less than that so we need to change the rules.



Profit Calculations

In theory

If 72 people play
P(WIN) = 14/72
We will pay out £14
P(MONEY BACK) = 15/72
We will give back 15 x 50p or £7.50
We will collect 72 x 50p for playing or £36
Profit is £36 - £14 - £7.50 = £14.50

3,6	2,5	5,11	3,8	3,2	5,5
1,7	6,5	1,3	6,1	2,2	4,12
3,7	6,5	2,1	4,2	4,3	5,10
5,6	3,9	4,5	1,7	1,9	3,3
6,4	4,11	3,12	6,12	1,7	2,8
3,3	2,7	4,6	3,2	2,1	1,11
1,8	3,3	5,4	3,11	6,5	4,1
1,4	6,1	3,4	1,3	4,10	5,11
2,9	5,1	1,4	4,7	4,2	6,9
5,8	2,1	6,6	1,9	7,11	4,1
2,12	5,1	6,2	6,9	3,1	3,4
1,10	6,7	4,10	1,10	4,5	4,4

Experiment

We played the game 72 times and here are our results
17 WON
16 got their MONEY BACK
Our actual PROFIT was £36 - £17 - £8 = £12
This was less that we thought.

Changes to the game

In order to make more profit we could

- Charge more to play - not a very good idea , this could mean less people playing
- Give less out as prizes - £1 isn't too much when you've payed 50p to play
- Choose a number with less factors than 20 maybe 15 then there will be less chance of WINNING

15 has 4 factors instead of 6 and there will be less winning scores

P(WIN) = 10/72 P(MONEY BACK) = 8/72

Our chances of making a profit now look much better.

Commentary

Sarah chose to create a game that combined scores from two events (choosing a numbered card and rolling an ordinary die). She constructed a sample space diagram and correctly highlighted winning combinations and their probabilities. She calculated projected profits based on the theoretical probabilities of winning and losing, and compared these profits with the outcomes of her experiment. She used ICT to create a poster describing her game and explaining how the profit was calculated. She evaluated her game, suggested a sensible amendment to ensure greater profit and justified her final decision by using her understanding of probability.

Activity | Calls and texts

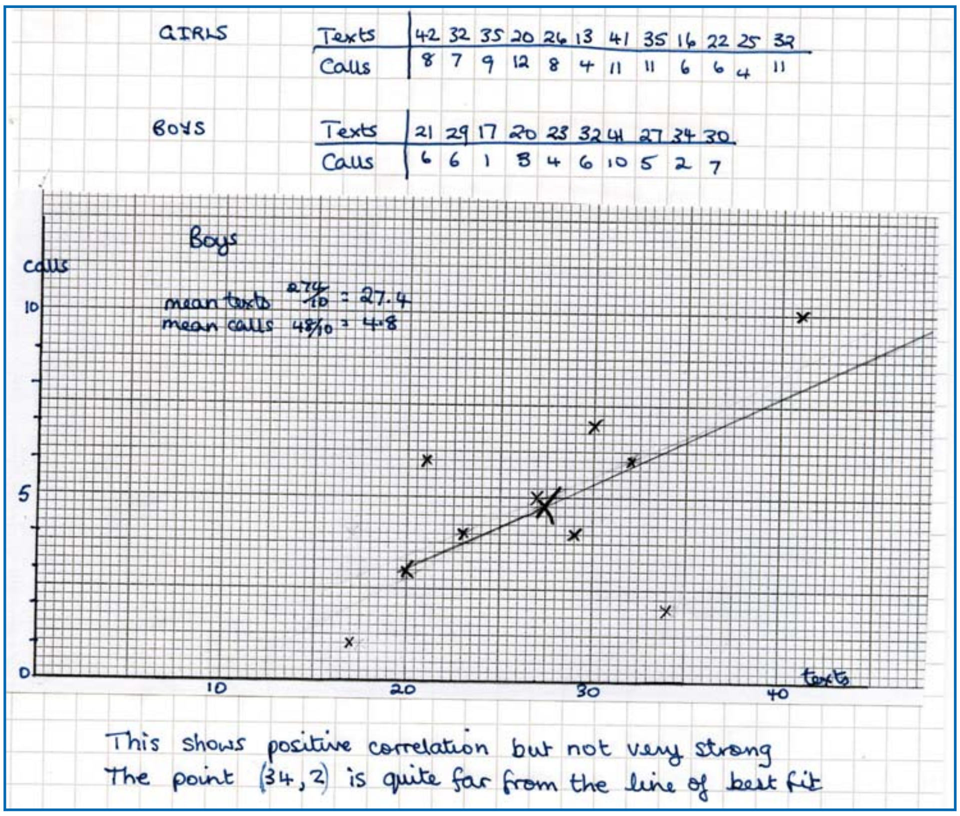
As part of a data study carried out by pupils to compare the use made of mobile phones by boys and by girls in Year 9, Sarah decides to test the hypothesis ‘Pupils who call the most text the least’, analysing the data for boys and girls separately.

There are opportunities for pupils to:

- develop their own strategies
- identify the data required to pursue a particular line of enquiry
- make hypotheses and design methods to test them
- draw inferences from graphs
- select the mathematics needed to solve problems
- use a range of mathematical language to explain and communicate their work
- present work clearly, using diagrams and labelled graphs.

Pupils are also:

- gathering information
- determining the process/method and strategy
- generating and developing ideas
- thinking logically
- considering evidence
- comparing data
- recording and interpreting data and presenting findings.



GIRLS

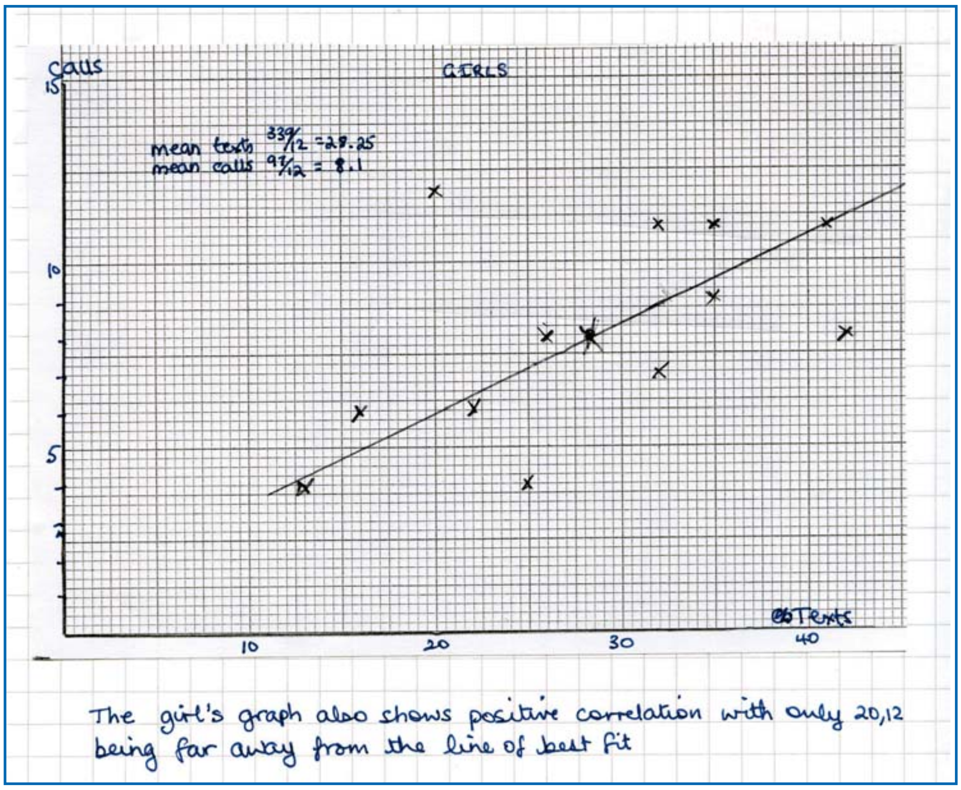
Texts	42	32	35	20	26	13	41	35	16	22	25	32
Calls	8	7	9	12	8	4	11	11	6	6	4	11

BOYS

Texts	21	29	17	20	23	32	41	27	34	30
Calls	6	6	1	8	4	6	10	5	2	7

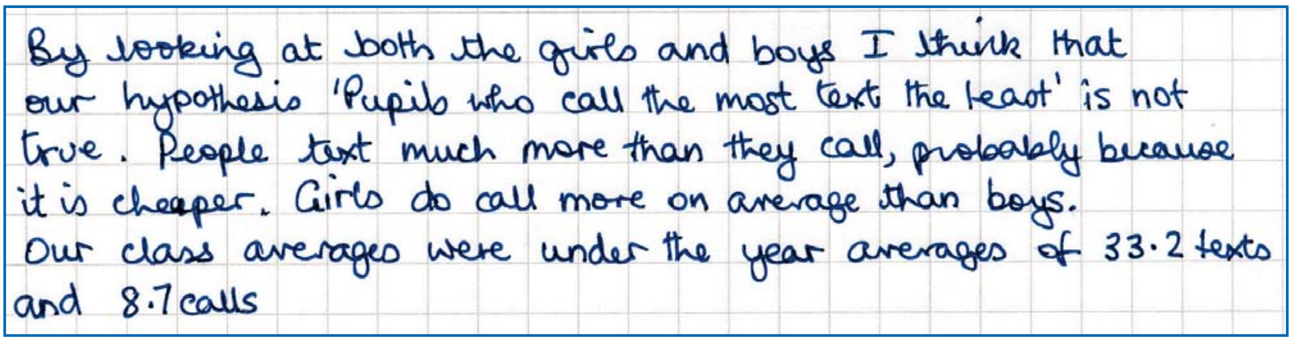
mean texts $^{274}/_{10}$ = 27.4
mean calls $48/10$ = 4·8

This shows positive correlation but not very strong
The point (34, 2) is quite far from the line of best fit



mean texts $^{339}/_{12}$ = 28.25
mean calls $^{97}/_{12}$ = 8.1

The girl's graph also shows positive correlation with only 20,12 being far away from the line of best fit



By looking at both the girls and boys I think that our hypothesis 'Pupils who call the most text the least' is not true. People text much more than they call, probably because it is cheaper. Girls do call more on average than boys.

Our class averages were under the year averages of 33.2 texts and 8.7 calls

Looking at the mean texts and calls for the boys and girls separately, there is not much difference in the numbers of texts but the girls call more than the boys.

Commentary

Sarah formed her own hypothesis to test and set about proving its validity. She drew lines of best fit on her separate graphs of the girls' and boys' data, and used her knowledge and understanding of correlation to arrive at some sensible conclusions. She judged that her hypothesis was not correct because the graphs demonstrated weak positive correlation rather than inverse correlation, which would be expected had her hypothesis been correct. She compared both graphs and came up with some sensible comparisons. She also calculated the mean numbers of calls and texts for boys and for girls separately, and used these to make comparisons.

Activity | Growing shapes

This activity is introduced to pupils by simply posing the question 'What happens to the areas of shapes as they grow?' In groups, pupils are asked to define what they understand by 'growing' and to form a plan of action in order to investigate the question further. In turn, the groups share their ideas for interesting lines of enquiry and are given the opportunity to modify their plans in the light of any good ideas picked up from other groups. They then set about investigating.

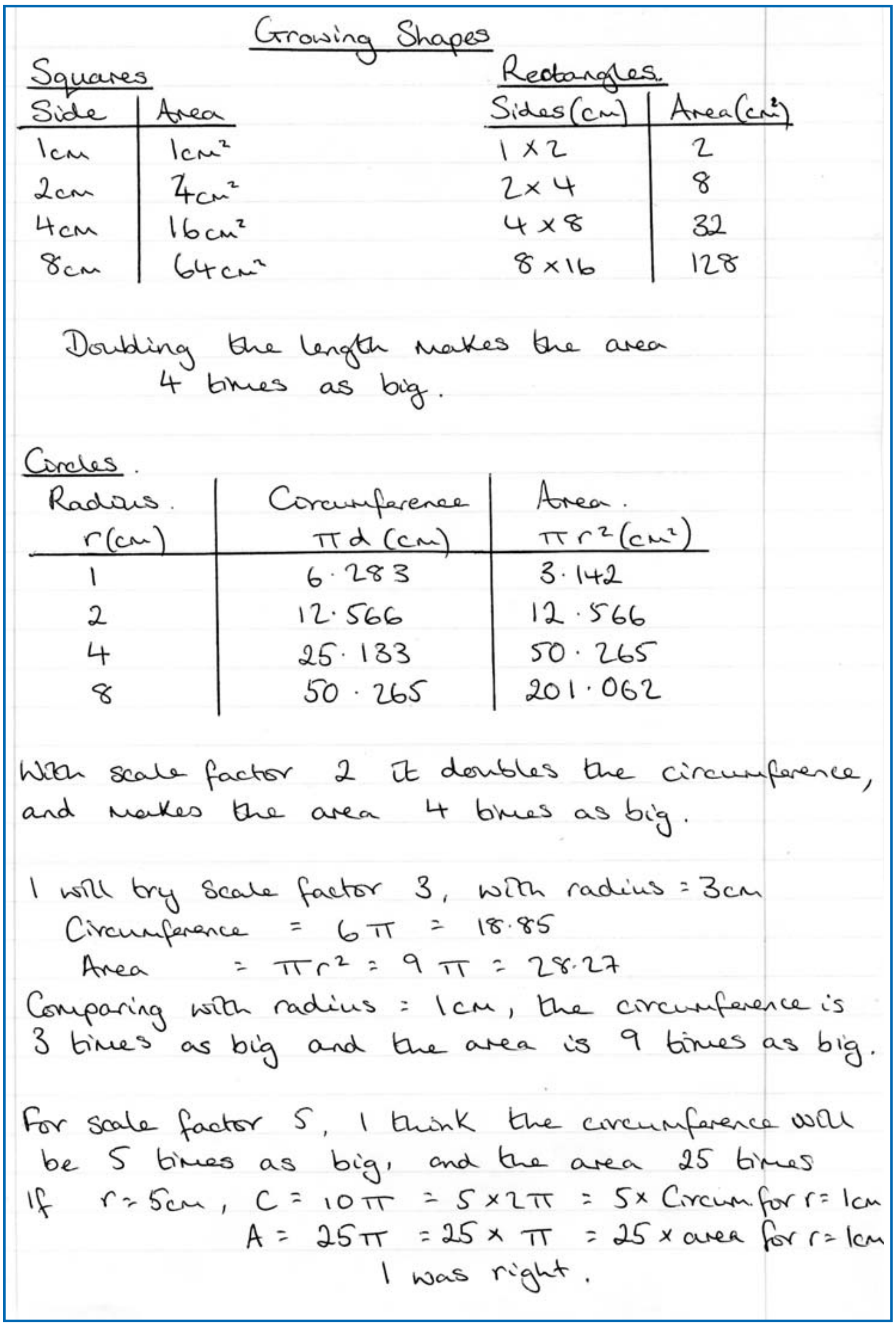
There are opportunities for pupils to:

- develop and use their own strategies and consider those of others
- break complex problems into a series of tasks
- generalise and explain patterns and relationships in words and symbols
- make and test generalisations
- use a range of mental, written and calculator strategies
- use mathematical language, notation and symbols to explain and communicate their work
- explain strategies and reasoning in writing.

Pupils are also:

- generating and developing ideas
- thinking about cause and effect
- thinking logically and seeking patterns
- comparing data
- recording and interpreting data and presenting findings.

I think ‘growing’ means enlarging a shape by a positive scale factor. I will use a positive whole number.



Growing Shapes

Squares

Side	Area
1cm	$1cm^2$
2cm	$4cm^2$
4cm	$16cm^2$
8cm	$64cm^2$

Rectangles.

Sides (cm)	Area (cm^2)
1×2	2
2×4	8
4×8	32
8×16	128

Doubling the length makes the area 4 times as big.

Circles.

Radius. r(cm)	Circumference πd (cm)	Area. πr^2 (cm^2)
1	6·283	3·142
2	12·566	12·566
4	25·133	50·265
8	50·265	201·062

With scale factor 2 it doubles the circumference, and makes the area 4 times as big.

I will try scale factor 3, with radius = 3cm

Circumference $= 6\pi = 18{\cdot}85$

Area $= \pi r^2 = 9\pi = 28{\cdot}27$

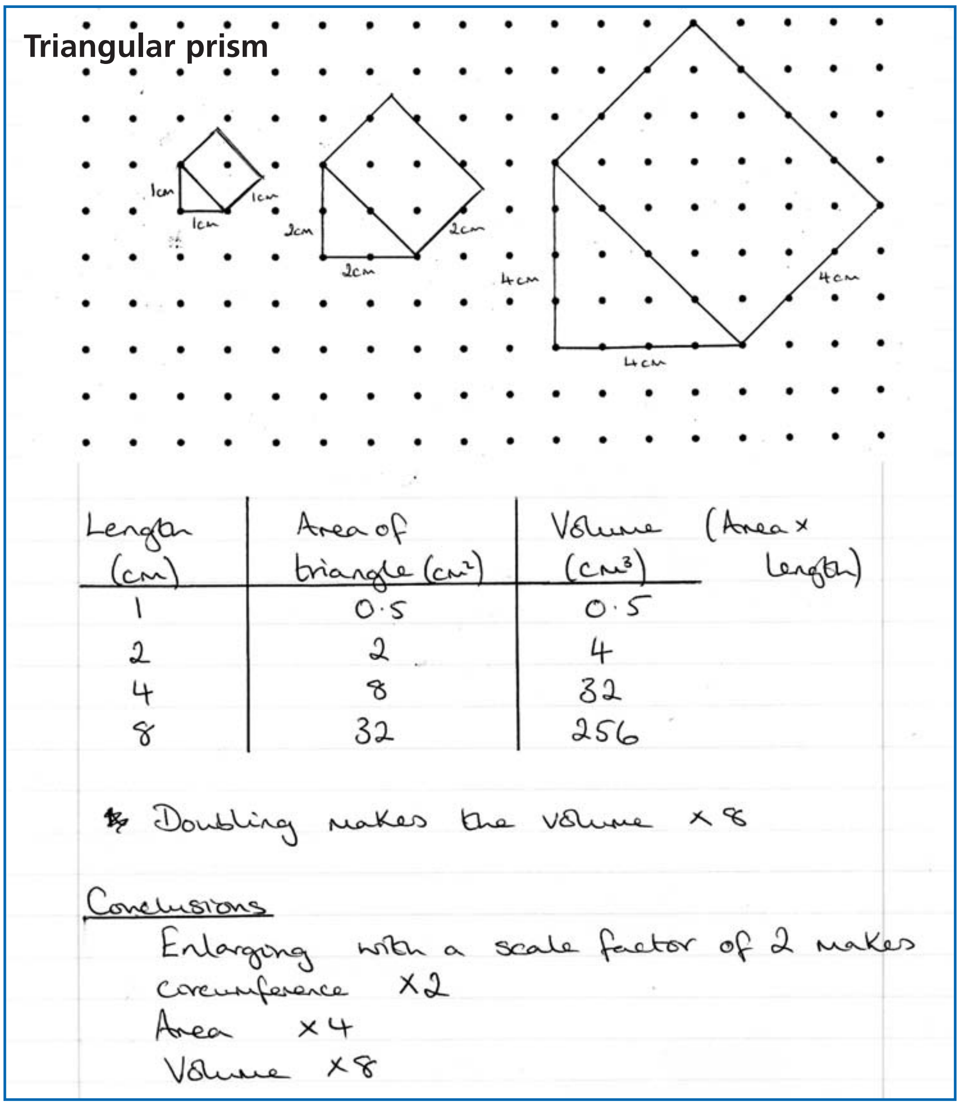
Comparing with radius = 1cm, the circumference is 3 times as big and the area is 9 times as big.

For scale factor 5, I think the circumference will be 5 times as big, and the area 25 times

If $r = 5$cm, $C = 10\pi = 5 \times 2\pi = 5 \times$ Circum. for $r = 1$cm

$A = 25\pi = 25 \times \pi = 25 \times$ area for $r = 1$cm

I was right.



Triangular prism

Length (cm)	Area of triangle (cm²)	Volume (cm³) (Area × length)
1	0·5	0·5
2	2	4
4	8	32
8	32	256

* Doubling makes the volume × 8

Conclusions

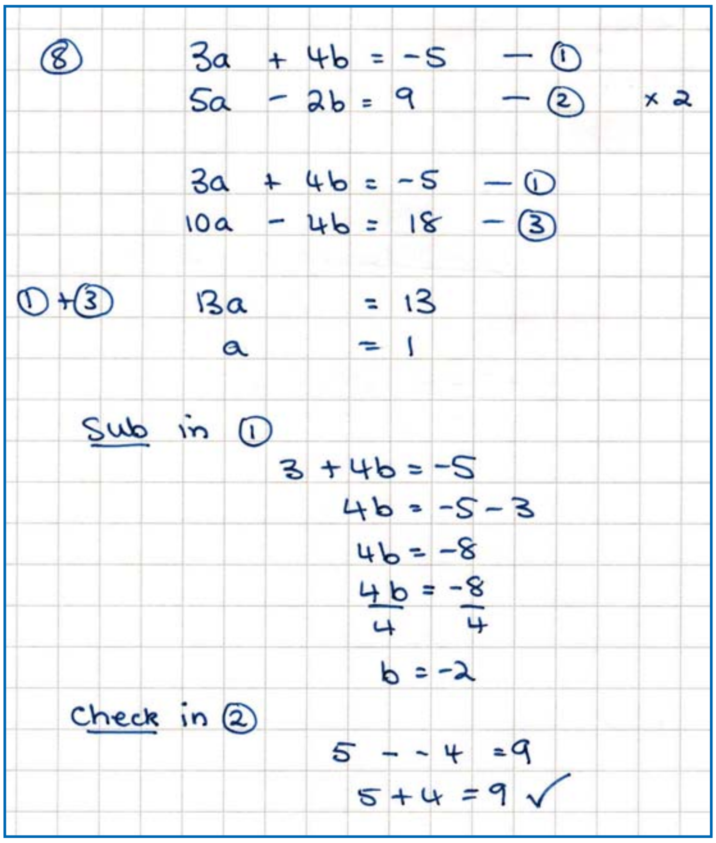
Enlarging with a scale factor of 2 makes
circumference ×2
Area ×4
Volume ×8

Commentary

Sarah began her investigation by enlarging squares and rectangles by a scale factor of two, recording her results for their areas, and looking for patterns and rules in her results. She progressed to the circle and decided to investigate circumference as well as area. She enlarged the circle systematically, used a calculator effectively to calculate the circumference and area, tabulated her results, and found a rule connecting the scale factor and both the circumference and area of each enlargement. She made a prediction for circles enlarged by a scale factor of five based on her findings for circles enlarged by scale factors of two and three. She extended her investigation to consider 3-D shapes, and discovered a rule connecting the scale factor and the volume of the enlarged shape.

Bits and bobs

These are examples of short pieces of work completed by Sarah during the year, including solving simultaneous equations and multiplying two linear expressions, work with measures and approximations, and a question where Sarah was asked to reflect a right-angled triangle in the line *y* = -*x*, before using Pythagoras' theorem to calculate the length of the hypotenuse.



⑧

$3a + 4b = -5$ — ①

$5a - 2b = 9$ — ② $\times 2$

$3a + 4b = -5$ — ①

$10a - 4b = 18$ — ③

① + ③ $13a = 13$

$a = 1$

Sub in ①

$3 + 4b = -5$

$4b = -5 - 3$

$4b = -8$

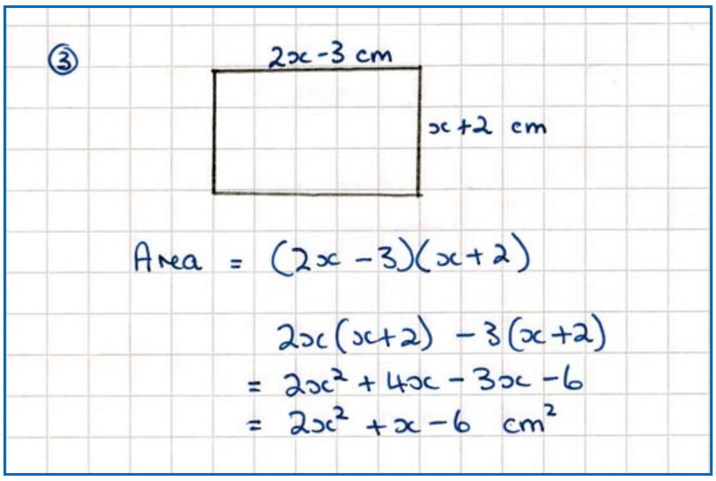
$\frac{4b}{4} = \frac{-8}{4}$

$b = -2$

Check in ②

$5 - -4 = 9$

$5 + 4 = 9$ ✓



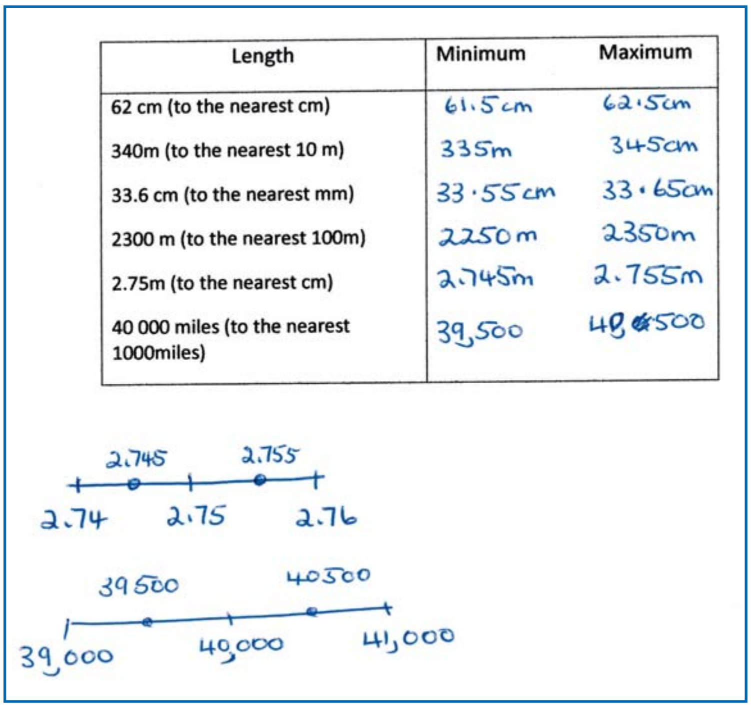
③

$\text{Area} = (2x - 3)(x + 2)$

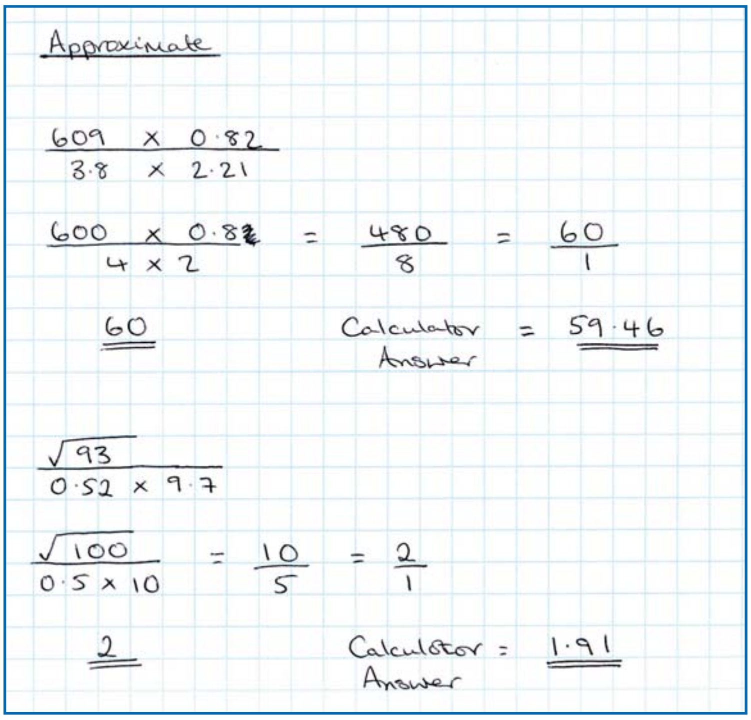
$2x(x+2) - 3(x+2)$

$= 2x^2 + 4x - 3x - 6$

$= 2x^2 + x - 6 \text{ cm}^2$



Length	Minimum	Maximum
62 cm (to the nearest cm)	61.5cm	62.5cm
340m (to the nearest 10 m)	335m	345cm
33.6 cm (to the nearest mm)	33.55cm	33.65cm
2300 m (to the nearest 100m)	2250m	2350m
2.75m (to the nearest cm)	2.745m	2.755m
40 000 miles (to the nearest 1000miles)	39,500	40,500



Approximate

$\frac{609 \times 0.82}{3.8 \times 2.21}$

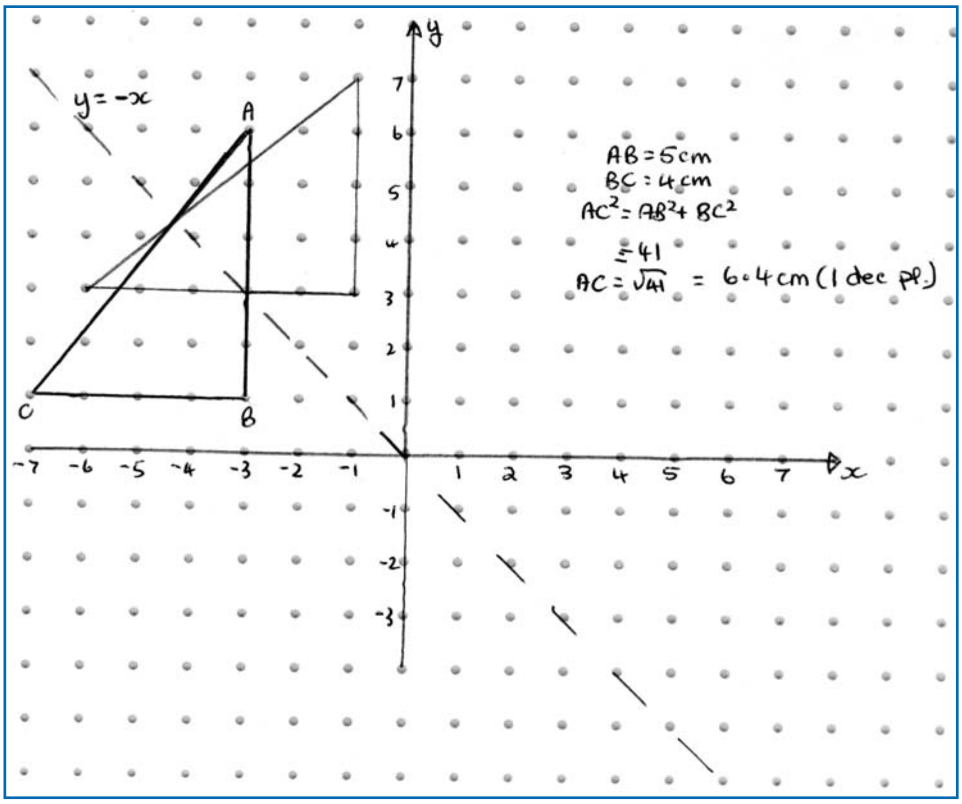
$\frac{600 \times 0.8}{4 \times 2} = \frac{480}{8} = \frac{60}{1}$

60 Calculator Answer = 59.46

$\frac{\sqrt{93}}{0.52 \times 9.7}$

$\frac{\sqrt{100}}{0.5 \times 10} = \frac{10}{5} = \frac{2}{1}$

2 Calculator Answer = 1.91



Commentary

Sarah has shown that she can work accurately in solving and checking simultaneous equations and expanding algebraic expressions involving brackets. She developed helpful strategies such as the use of number lines when rounding measurements, and approximated complex calculations sensibly. She correctly reflected a triangle in an oblique line, and was able to recall previous work on Pythagoras' theorem in order to calculate the length of the hypotenuse.

Summary and overall judgement

Levels 6 and 7 were considered and Level 7 was judged to be the best fit.

Sarah has shown, in ‘Hidden faces’, that she can *solve complex problems by breaking them down into smaller tasks*, *and give some mathematical justification to support her conclusions*. In ‘Growing shapes’, she *used formulae for finding the circumferences and areas of circles* and *enlarged shapes by a positive whole-number scale factor*. In ‘Probability game’, she *identified all the outcomes when dealing with a combination of two experiments*. These are characteristic of Level 6.

In ‘Hidden faces’, Sarah *justified her generalisations*; in ‘Probability game’, she *considered alternative approaches*, *and appreciated the difference between mathematical explanation and experimental evidence*. In ‘Bits and bobs’, she *used an algebraic method to solve simultaneous linear equations in two variables*; she also *used Pythagoras’ theorem in two dimensions*, and showed that she *appreciates the imprecision of measurement*. In ‘Growing Shapes’, she *calculated lengths, areas and volumes in right prisms.* In ‘Calls and texts’, she *specified and tested hypotheses, used measures of average to compare distributions*, and *drew lines of best fit on scatter diagrams by inspection*. This is all characteristic of Level 7.

In ‘Bits and Bobs’, she *manipulated algebraic expressions*, which is characteristic of Level 8.

Acknowledgements

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