Stimulus question 73

Circles, Polygons and Area

Teaching notes

Resources:

- Mini-whiteboards or similar individual display resource;
- Paper, pencils, rulers, pairs of compasses;
- Question 73 PowerPoint: 'Circles, Polygons and Area';
- **Question 73 Resource: Circumscribed and Inscribed Squares**;
- For the extension question for Higher tier, **Question 73 Resource: Inscribed Equilateral Triangle**;
- Enclosed teacher script, with supplementary and other probing questions.

Commentary/notes:

The main part of this question is suitable for all tiers. The extension questions are suitable only for Higher tier. It links with 72: Circles, Polygons, Circumference and Perimeters, (which will again be appropriate for all tiers), and question 74: Circles and Circumscribed Polygons, which is only appropriate for Higher tier.

The PowerPoint is constructed in four parts:

- The first section contains questions using the information that the radius of the circle is 1 unit;
- The second section generalises what they have found, with the radius having the value *r*;
- The third section has some extension questions suitable only for Higher tier;
- The fourth and final section of the presentation offers hints, and can be used at any time during the presentation. During planning, teachers may wish to identify which of the hints may be useful to insert into their own presentation to support learners as they tackle the questions.

Solutions

Radius 1 unit

- The circumscribed square has an area of 4 square units;
- The inscribed square has an area of 2 square units;
- The circle has an area of π square units. (Estimates of between 3 and 3½ are acceptable, if estimates are taken).

Radius *r*

- The circumscribed square has an area of $4r^2$ square units;
- The inscribed square has an area of $2r^2$ square units;
- The area of the circle is πr^2.

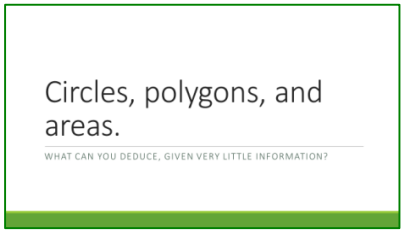
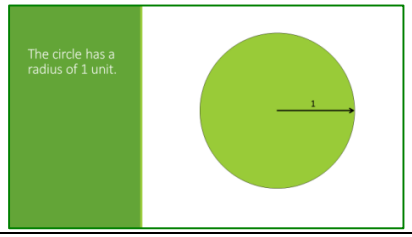
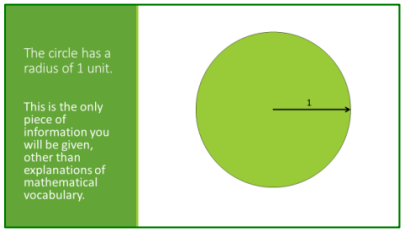
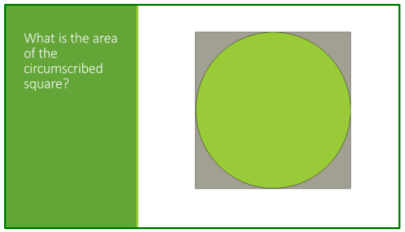
Extension (Higher tier only)

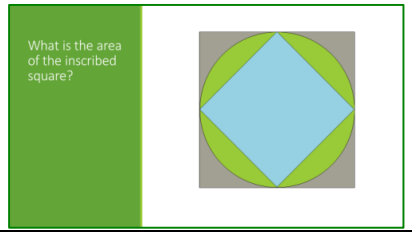
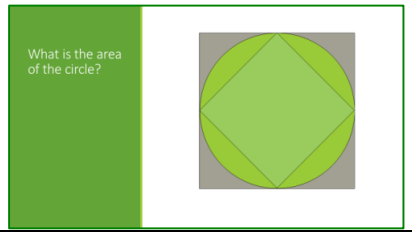
Slides 18-21 may help learners visualise how the equilateral triangle may be divide up into three isosceles triangles, which in turn may be divided into two congruent right-angled triangles. These right-angled triangles have interior angles of 30°, 60° and 90°, and whose sides are in the ratio (hypotenuse: short: other) of 1: ½ and ½√3. Each side of the equilateral triangle has a side length of √3. The perimeter is therefore 3√3. These slides may also help learners develop a method for solving the hexagon problem.

- **Triangle:** side length √3, height 1½. Area = (3√3)/4;
- **Hexagon** is made up of six equilateral triangles: Triangle side length 1, height (√3)/2. Area of triangle = (√3)/4.
  Area of hexagon = 6(√3)/4 = 3(√3)/2

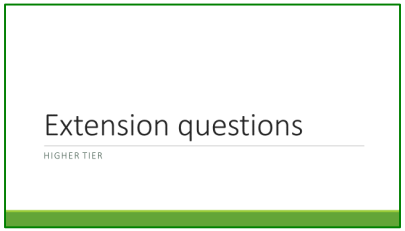
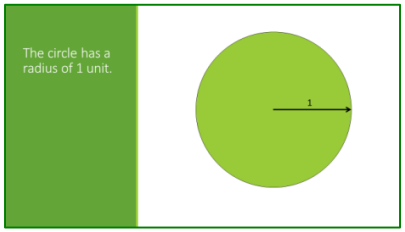
Teacher script (sections 1 and 3)

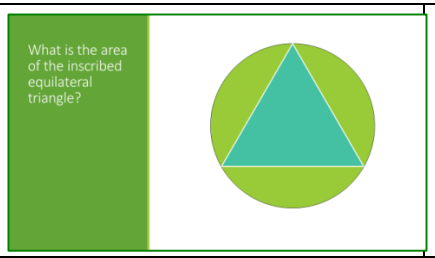
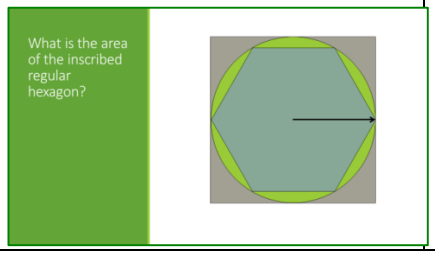
This script is focused only on sections 1 and 3. As the main questions and dialogue are written on the slides, this script only contains further supplementary and reasoning questions (in italics), which are designed to elicit deeper understanding and give explanations of vocabulary. These can be used at the teacher's discretion.

Slide 1		*Keep this slide on the screen until you are ready to start the presentation*
Slide 2		• *Give your own definition of a radius.*
Slide 3		Although you will only have this information to start with I hope you can generate your own information from the diagrams.
Slide 4		• *The square is circumscribed around the circle. Describe what this means.* • *Should the square touch the circle, just miss the circle, or cut through the circle?*

Slide 5		• *Can anyone describe the meaning of 'inscribed'?* • *Are there any lines we could add to the diagram that may help us?* • *Do we need to find the side of the square? What other ways could we use to find the area of this square?* • *What reasoning can we use to explain why the area of the inscribed square is half the area of the circumscribed square?*
Slide 6		• *Can you estimate the area of the circle, given what you know about the areas of the two squares?*

Section 3 script

Slide 13		*Keep this slide on the screen until you are ready to start the presentation*
Slide 14		*(Foundation Tier)* Notice that one is called circumference, the other is called perimeter. We only use circumference for the full circle. We are still using the radius of 1. • *Does the semicircle have half the area of the circle?* • *Can we find the exact measurement for the perimeter of this semicircle?*

Slide 15		*(Higher tier only)* **Resource sheet: Inscribed Equilateral Triangle** *is available for this question.* *Slides 18-21 are offered to support if learners are stuck, or to summarise a possible method.*
Slide 16		• *What can we take from the way we solved the problem for the triangle to help us with solving this problem?* • *What is the relationship between the area of the hexagon and the area of the triangle on the previous slide?* • *What can we say about the solution to this problem and pi?*

GCSE Subject Content		
Foundation	**Intermediate**	**Higher**
Vocabulary of triangles, quadrilaterals and circles: isosceles, equilateral, scalene, square, rectangle, parallelogram, rhombus, kite, trapezium, polygon, pentagon, hexagon, radius, diameter, tangent, circumference, chord, arc, sector, segment. Calculating: area of a square, triangle, circle, semicircle and composite shapes.		
	Using Pythagoras' theorem in 2-D.	Using Pythagoras' theorem in 2-D. Manipulating surds; using surds and π in exact calculations. Simplifying numerical expressions involving surds.

Learner Outcomes and Assessment	
Reasoning strand – **Learners are able to:**	**Assessment Guidance –** **Can learners:**
• Identify, measure or obtain required information to complete the task; • Identify what further information might be required and select what information is most appropriate; • Select appropriate mathematics and techniques to use; • **Develop and evaluate mathematical strategies and ideas creatively;** • Explain results and procedures precisely using appropriate mathematical language; • Use appropriate notation, symbols and units of measurement, including compound measures; • **Generalise in words, and use algebra, to describe patterns that arise in numerical, spatial or practical situations;** • Interpret mathematical information; draw inferences from diagrams.	• Identify that the side lengths and heights of the polygons is needed to find the area? • Find an alternative way to find the area of the inscribed square? • Make links between the radius of the circle and the sides of polygons? • Extend diagrams by drawing appropriate lines, radii, diameters to generate useful information? • Explain their solutions through using conventional mathematical notation and vocabulary? • Derive formulae for the areas of the circle and the polygons?