Stimulus question 86

The Dartboard

Teaching notes

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Key question / task
Which is the better half of the dartboard to aim for to give you a higher score?
What is your likely score for three darts?

Supplementary questions / discussion
▪ What mathematical calculations can we make?
▪ How can we show which half is better?
▪ Are we looking for totals? Averages?
▪ When totalling / averaging for both halves, do we have to include the 20 and the 3 in both calculations, to see which is better?
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Resources for learners:
▪ Pencils / pens and paper
▪ Calculators (optional).
Resources for teachers:
▪ PowerPoint.
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Reasoning questions
▪ Do we have to allow for the trebles and doubles to find which half of the dartboard is better?
▪ If we did account for the doubles and trebles, what further information would we need, and what calculations would we have to do?
▪ If you could only be sure of hitting ¼ of the dartboard, which quarter would you aim for?
▪ If we know the total of the scores on the dartboard (not including doubles, trebles and bull's eyes), how can we use that to make our work more efficient?

Extension questions
• Is there a way to organise the numbers on a dartboard, so that two halves have equal sums?
• Is it possible to organise the dartboard so that the numbers in four quarters have the same sum? How do we know if it is possible, without doing lots of examples?
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Commentary / notes:

This question is focused on identifying the information that is needed when comparing the scoring values of two halves of a dartboard. Learners need to identify this as a situation for comparing small distributions, and calculate the statistics accordingly. This work also helps develop strategies for calculating: e.g. knowing that the total of the dartboard scores can be found by adding $1 + 20$, $2 + 19$, then $3 + 18$ and so on will give us 10 lots of $21 = 210$. The two numbers 20 and 3 do not have to be counted (because they are in both halves), so there is an available total of 187. This technique will allow learners to quickly calculate the values of two halves of a dartboard however it is split.

The question is likely to arise about whether learners have to allow for the doubles, trebles and bulls. The multiple scores only make a difference in predicting a likely score for three darts, and learners at this level could argue that it is not likely to score a treble or double if you are not very good at darts, and so could suggest a score that ignores these extras. The doubles and trebles make no difference to which half is better.

At higher levels, learners may wish to have '**The Dartboard: additional information sheet**' if they wish to consider the probabilities of scoring doubles and trebles. This will make the task longer, by including more complex calculations about the fractions of the board the doubles and trebles make. It is also quite difficult, given the presentation of the dimensions in fractions of inches, but perhaps a more accessible problem is in the Task: A Narrow Chance, which deals with probabilities on an archery target.

Solutions

As the working above suggests, the total of the dartboard is 210, less 23 for the 20 and the 3, giving 187. The left hand side totals 101, the right must total 86. Learners should show how they check their calculations. An average dart will score 11.22 on the left and 9.56 on the right. A score for three darts on the left will therefore be 33.7 (rounded to 34) and on the right will be 28.7, rounded to 29.

At intermediate level learners should also show the range of the two distributions: on the left hand side the range is 14, lowest value 5 and greatest is 19; on the right hand side the range is 17, lowest value 1 and highest 18.

A better split is to draw a dividing line between 12 and 2. The numbers below this line sum to 104, giving an average dart score of about 11.6. The range at 16 is slightly greater however, lowest value now 3 and highest 19.

A strategy for looking at alternative halves could be to identify opposite pairs of numbers that have the lowest sums, leaving greater sums within the two halves; or areas of the board that house the higher numbers. Learners may look at all the possible halves but this will take too much time.

For the extension questions, there will be many ways to organise the numbers so that each half sums to 105, but it would be impossible to organise the numbers so that quarters of the board have the same sum. This is because there is not a whole number solution to 210 ÷ 4.

Mark scheme

Full credit

- Calculate means for both halves accurately, giving the answers for 3 darts as in the solution above.
- Calculates range for both halves.
- Gives answers to a suitable degree of accuracy: i.e. to the nearest whole number, when suggesting a score for three darts.

Partial Credit

Accurate calculations, but does not round answers for three darts.

No credit

Any other response.

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<tr><th colspan="3">GCSE Subject Content</th></tr>
<tr><th>Foundation</th><th>Intermediate</th><th>Higher</th></tr>
<tr><td colspan="2">Giving solutions in the context of a problem, interpreting the display on a calculator.
Knowing whether to round up or down as appropriate.
Mean median and mode for a discrete (ungrouped) frequency distribution.
Comparison of two distributions using one measure of central tendency (i.e. the mean or the median) <u>and/or one measure of spread.</u></td><td></td></tr>
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Learner Outcomes and Assessment	
Reasoning strand - Learners are able to:	**Assessment Guidance: Can learners:**
▪ Transfer mathematical skills across the curriculum in a variety of contexts and everyday situations ▪ Prioritise and organise the relevant steps needed to complete the task or reach a solution ▪ Choose an appropriate mental or written strategy and know when it is appropriate to use a calculator ▪ Identify, measure or obtain required information to complete the task ▪ Identify what further information might be required and select what information is most appropriate ▪ Select appropriate mathematics and techniques to use ▪ Refine methods of recording calculations ▪ Use appropriate notation, symbols and units of measurement, including compound measures ▪ Select and apply appropriate checking strategies ▪ Interpret answers within the context of the problem and consider whether answers, including calculator, analogue and digital displays, are sensible ▪ Interpret mathematical information; draw inferences from graphs, diagrams and data, including discussion on limitations of data	▪ Recognise that this is a situation involving two distributions? ▪ Identify a suitable strategy for comparing two distributions? ▪ Add totals mentally, or use a calculator correctly to find the mean? ▪ Find the range mentally? ▪ Identify a strategy for simplifying the problem: e.g. look for pairs of opposite numbers that have the lowest sum, enabling the possibility of a larger sum of the remaining numbers? ▪ Use the 'Gauss' method for summing all the numbers from 1 to 20 (i.e. 1 + 20, 2 + 19 etc) ▪ Present solutions to an appropriate degree of accuracy? ▪ Show checking e.g. adding all the numbers within a half, but check against the three sums: opposite pairs, left half total and right half total, compared with the total of all numbers (210)? ▪ Suggest that their estimate does not include a consideration of the doubles and trebles, and that they do not have the information necessary to calculate the probabilities of scoring doubles or trebles accidently.