Contextualised task 36

A Narrow Chance

Teaching notes

This task is suitable for higher tier. Many mathematical techniques and ideas are used in this task and similarly in *Question 86: The Dartboard* which provides further opportunities. Two ways of looking at probability are included here: one is to identify the probability of hitting a specific part of an archery target – assuming that an archer can always hit any part of the target at random – and the other is to identify probability through relative frequency. It is structured around three questions:

a) finding the areas of parts of the target,
b) finding the probability of hitting the centre of the target at random, and then any part of the target at random,
c) finding the probability that an Olympic archer will score a maximum in three consecutive arrows.

Learners will have to identify the mathematics to use in the third question. To find the probability of the archer scoring three consecutive tens requires the learner to consider the relative frequency of him scoring a ten from his previous record and then recognise they must multiply the probabilities in this situation. However, this will lead to a discussion about what data we should use, and what degree of accuracy we should use to present the answer.

Part 1: The Area of the Target

Learners are asked to find the area of the central circle of the Archery target. This is an optional part of the task, as they will have to find its area for part 2 anyway.

Part 2: The probability of a club archer scoring 10.

Learners calculate (or use ratio to identify) the area of the bull's eye as a proportion of the whole target to find the probability of scoring a 10. This is then extended to find the areas of all parts of the target and therefore the probabilities of hitting each ring. This is not such an arduous task as it sounds: it can be shortened by calculating with pi and by representing the areas of the rings symbolically.

Part 3: The probability of Im Dong-hyun scoring three tens in a row.

On a particular day in London 2012 the South Korean archer Im Dong-hyun needed to score three tens in a row to win his match. Up until that point he had scored 669 points from 69 arrows, scoring only tens and nines. Learners are asked to estimate the probability that he will succeed.

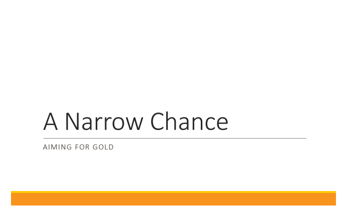
Resources you will need:

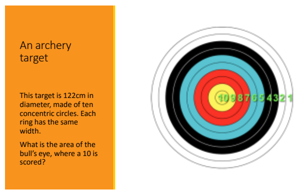
- PowerPoint slides.
- Learning Resource sheet: Archery Target (optional)

- Paper, pencils, rulers
- Calculators.

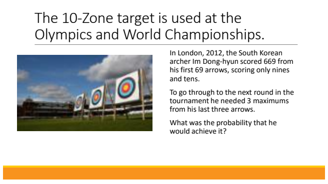
A Narrow Chance: Teachers' script for PowerPoint presentation

The text in the right-hand boxes provides a possible script to be read to students. However, it is probably preferable to use your own words and elaboration. When questions are asked, time for discussion in pairs / groups should be provided. Ensure that students are given the opportunity to explain their reasoning in response to these questions.

<table>
<tr>
<td>Slide 1</td>
<td>
</td>
<td>Keep this slide on the screen until you are ready to start the presentation.

Hand out the Learning Resource sheet: 'Archery Target', (a black and white sheet) which shows a target and the information on the next slide.</td>
</tr>
<tr>
<td>Slide 2</td>
<td>
</td>
<td>You may notice that on the screen there is a smaller central circle. This area scores 10 as well, but is sometimes marked as an 'X' on score sheets. This is used to separate archers' scores if there is a tie. Some tens are better than others! This smaller circle is not marked on your sheets.
Give a suitable time for the class to tackle the question on the slide.

There are at least two different mathematical techniques we could use to calculate this.
What methods have you used?
Does anyone have an alternative?
Did anyone notice anything once they calculated the areas of the whole target and the central circle?
What ratio is the radius of the central circle to the radius of the whole target?
What ratio is the area of the central circle to the area of the whole target?</td>
</tr>
<tr>
<td>Slide 3</td>
<td>
</td>
<td>Have a go at the question. Give a suitable time for the class to tackle it.
What information do you need to solve it?
What can you calculate from the information you have?
Why is the word 'random' important here?</td>
</tr>
<tr>
<td>Slide 4</td>
<td>
</td>
<td>When you tackle this question, I want you to look for the most efficient way you can solve it. This is likely to involve looking for links or relationships, and not making the full set of calculations.
Give about 3 or 4 minutes to start, and then compare methods.
We are looking for learners to use π in exact</td>
</tr>
</table>

		calculations here. Learners could also consider not using the exact measurements of the target, but a symbol to stand for the radii of the circles involved.
Slide 5		Have a go at this question, but you must explain your method, stating why you chose the figures you use. *Give a suitable time for the class to tackle the question.* • Is the probability of an Olympic archer hitting the ten the same as the probability of the club archer? • What information do we have to enable us to estimate the probability? • How does the equation $10A + 9(69 – A) = 669$ represent this situation? What will it tell us if we solved it? What does A stand for? **(A stands for the number of arrows that score 10)** • How should we present our answer? To what degree of accuracy? *Take ideas about how to solve this problem from the class.* • Why can we *calculate* a probability for the club archer, but only *estimate* a probability for this situation?
Slide 6		• If Im Dong-hyun had scored 69 bull's eyes up to that point, would the probability of scoring 3 more bull's eyes be 100%? Why / why not? • Does the probability change as he scores each maximum in his last three arrows? E.g. at the first of his last three arrows, is the probability 48 / 69, and then if he is successful, 49 / 70 for the next, and for his final arrow 50 / 71?

Solutions and Mark Scheme

Full credit

- Establishes the probability of a club archer randomly hitting the bull's eye is 1%.
- Finds the probabilities of randomly hitting each of the ten rings: 10 - 1%, 9 - 3%, 8 - 5%, 7 - 7%, 6 - 9%, 5 – 11%, 4 – 13%, 3- 15%, 2 – 17%, 1 – 19%.
- Suggests an estimated probability of Im Dong-hyun hitting the maximum three times in a row is about 34%.

Part 1:

The radius of the central circle is 6.1cm.
Its area is π x 6.1cm^2, which equals 116.9cm^2.

Part 2:

The area of the whole target is π x 61cm^2, which equals 11689.9cm^2.
The probability of hitting the bull's eye randomly is 1% or one hundredth.

An alternative way to consider this is that the radius of the central circle is one tenth of the whole target: therefore its area is one hundredth of the whole target.

If, instead of using exact measurements, we keep the calculations using π and a symbol (e.g. r, given on the learning resource sheet): then we can see the relationship between the areas of the rings of the target, and therefore the probabilities of hitting those rings:

Area of bull's eye: πr^2
Area of '9' ring = $\pi(2r)^2 - \pi r^2 = 4\pi r^2 - \pi r^2 = 3\pi r^2$
Area of '8' ring = $\pi(3r)^2 - \pi(2r)^2 = 9\pi r^2 - 4\pi r^2 = 5\pi r^2$
And so on...

Learners may begin to notice that the area of each ring falls into a pattern, and will be able to identify the other areas quickly: $7\pi r^2$, $9\pi r^2$, $11\pi r^2$, $13\pi r^2$, $15\pi r^2$, $17\pi r^2$, and $19\pi r^2$.
The probabilities match these areas accordingly, (note that the coefficients of these expressions sum to 100).
10 - 1%, 9 - 3%, 8 - 5%, 7 - 7%, 6 - 9%, 5 – 11%, 4 – 13%, 3- 15%, 2 – 17%, 1 – 19%.

Part 3:

Arguably this is a situation of experimental probability, where the experimental evidence is Im Don-hyun's record up to that point. This is also a situation of multiplying the probabilities of three events – three arrows hitting ten, otherwise he does not win the round. From 69 arrows, and only scoring nines and tens, to score 669 he must have shot 21 nines and 48 tens. The maximum he could have scored is 690, and he scored 21 points less than that.

At this point learners may calculate the probability of the next arrow scoring the maximum 10 would be 48 / 69. They may continue using this figure to estimate the probability of all three arrows hitting the target: $(48/69)^3 = 0.3366$, giving about 34%. However, there is also a reasonable argument that the evidence of the experimental probability will change as Im fires each arrow, and thus if he is successful, the probabilities would be 48/69 x 49/70 x 50/71. This works out to be 0.3429, still about 34%. Others may argue that an estimate of probability should be based on Im Dong-hyun's career average (for which we do not have the data), rather than the day he broke the world record.

The correct answer here is that we are dealing with limited data, and we can only produce an estimate from it. It may be preferable to maximise this data by adding the 70th and 71st arrows as we go along, but whichever way we consider it, we still come up with 34%.

Partial credit
Correct calculations, but probabilities are not presented to a suitable degree of accuracy – e.g. to whole percentages.

Methods of finding the areas of the rings and the probabilities of hitting them are accurate but not efficient – e.g. they make full calculations of each of the areas.

Insufficient explanation of otherwise correct working.

No credit
Any other response

Progression in reasoning			
Identify processes and connections • **consider connections between mathematical skills and contextualise these**	*e.g. consider finding the areas of both the whole target and the central circle, to calculate the probability of hitting the maximum at random.*	*e.g. consider the possibility of using the relationships between the ratios of the radii and the areas of the whole target and central circles to calculate the probability of scoring a maximum.*	*e.g. uses proportional reasoning to estimate the probability of scoring three successive maximums, or identifies how the equation* $10A + 9(69 - A) = 669$ *applies to this situation.*
Represent and communicate • explain results and procedures precisely using appropriate mathematical language	*e.g. explains how they calculate the probability of the club archer scoring a maximum.*	*e.g. explains why we calculate the probability in the first situation and estimate the probability on the second*	*e.g. correctly establishes that we are dealing with an estimated probability, and reasons that a whole-number percentage is adequate*
Review • draw conclusions from data and recognise that some conclusions may be misleading or uncertain	*e.g. may suggest that a club archer's performance can never be random, and is unlikely to be able to hit any part of the target with equal probability.*	*e.g. identifies that the archer's performance in one match may not be a sound indicator of usual performance, from which a meaningful estimate of probability may be made.*	*e.g. may suggest that it would be better to have a career average performance from which to base the probability of scoring ten, rather than the world record performance on this day.*

GCSE Content	
GCSE Mathematics – Numeracy and GCSE Mathematics	**GCSE Mathematics only**
Understanding number and place value • Rounding decimals to the nearest whole number or a given number of decimal places. <u>Rounding numbers to a given number of significant figures.</u> • Ordering and comparing whole numbers, decimals, fractions and percentages.	
Understanding number relationships and methods of calculation • Calculating using ratios in a variety of situations. • **Using surds and π in exact calculations.**	
Solving numerical problems • Giving solutions in the context of a problem, <u>selecting an appropriate degree of accuracy,</u> interpreting the display on a calculator, and <u>recognising limitations on the accuracy of data and measurements.</u> • <u>Rounding an answer to a reasonable degree of accuracy in the light of the context.</u>	
Understanding and using equations and formulae • Formation and manipulation of linear equations.	
Understanding and using properties of position, movement and transformation • **Using the relationships between the ratios of: lengths and areas of similar 2-D shapes**	
Understanding and using measures • Calculating area of circles.	
Estimating and calculating the probabilities of events • Understanding and using the vocabulary of probability, including notions of uncertainty and risk.	• Calculating theoretical probabilities based on equally likely outcomes. • Estimating the probability of an event as the proportion of times it has occurred. • **The multiplication law for (dependent) events.**

<u>Key</u>
Foundation tier content is in standard text.
Intermediate tier content which is in addition to foundation tier content is in <u>underlined text</u>.
Higher tier content which is in addition to intermediate tier content is in **bold text.**