

## Proving Number Situations

a) Three students, Seren, Ffion and Jac have been given the question:

*The sum of two consecutive numbers is always odd.  
The product of two consecutive numbers is always even.  
Are these statements true, and can you prove it?*

Look at the three solutions. Which one is the best and why?

*Seren*

$1 + 2 = 3$	<i>odd</i>	$65 + 66 = 131$	<i>odd</i>
$147 + 148 = 295$	<i>odd</i>	$1999 + 2000 = 3999$	<i>odd</i>

*Every combination I try comes out odd. I have tried pairs of numbers in units, tens, hundreds and thousands, so it is bound to be true.*

$1 \times 2 = 2$	<i>even</i>	$65 \times 66 = 4290$	<i>even</i>
$147 \times 148 = 21756$	<i>even</i>	$1999 \times 2000 = 3998000$	<i>even</i>

*Again, every combination I try comes out even. Because I have tried pairs of different sizes of numbers it is bound to be true.*

*Ffion*

*If a is the first number, then b is the next number along.  
 $a + b = c$ , and c is the third letter and is odd  
 $a \times b = b$ , which is the second number, and is even.*

**Jac**

If the first number is called  $n$ , then the next number (the consecutive number) must be one more than  $n$  so it is called  $n + 1$ .

**Adding two consecutive numbers**

$$n + n + 1 = 2n + 1$$

$2n$  must be even because it is a multiple of 2  
 $2n + 1$  must therefore be odd because it is one more than an even number.

**Multiplying two consecutive numbers**

$$n \times (n + 1) = n(n + 1) = n^2 + n$$

If  $n$  is even,  $n^2$  must also be even, because you are multiplying an even number by itself. Also when you add two even numbers, the result is even, so  $n^2 + n$  is even.

If  $n$  is odd,  $n^2$  must also be odd, because you are multiplying an odd number by itself. But when you add two odd numbers, the result is even, so  $n^2 + n$  is again even.

## More Number Situations

Use what you have learned about using algebra effectively to try the following questions.

Part b) is similar to the example above, but parts c) and d) both consider place value, and you will have to use symbols carefully to show this.

- b) The greatest and least of four consecutive numbers are multiplied together; so also are the middle pair.  
Show that the difference of the two products is always 2.

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