Stimulus question 51

Identity Parade

Teaching notes

Key question / task

Here are three sets of algebraic statements.
Can you state which ones are identities for each set?
Try to explain why the other statements are not identities.

Set A: Adding unit fractions

i) $\frac{1}{c} + \frac{1}{d} = \frac{2}{(c+d)}$

ii) $\frac{1}{c} + \frac{1}{d} = \frac{2}{cd}$

iii) $\frac{1}{c} + \frac{1}{d} = \frac{(d+c)}{cd}$

iv) $\frac{1}{c} + \frac{1}{d} = \frac{(c+d)}{6}$

v) $\frac{1}{c} + \frac{1}{d} = \frac{5}{6}$

General supplementary questions:

- What does each equation mean in words?
- What different mathematical techniques can we use to help us?
- Can you test each statement with an example to see if it works? Will one example be sufficient?
- What would a counterexample do?
- Can you work out your own identity for these statements?
- Which examples are obviously not identities? Why is this?

Set B: Adding pairs of fractions

i) $\frac{a}{c} + \frac{b}{d} = \frac{ab}{(c+d)}$

ii) $\frac{a}{c} + \frac{b}{d} = \frac{(a+b)(c+d)}{cd}$

iii) $\frac{a}{c} + \frac{b}{d} = \frac{(c+d)}{ab}$

iv) $\frac{a}{c} + \frac{b}{d} = \frac{(c+d)}{(a+b)}$

v) $\frac{a}{c} + \frac{b}{d} = \frac{ad+bc}{cd}$

Set C: Expanding brackets

i) $(x-1)(x+2) = x^2 + 2x - x - 2$

ii) $(x-1)(x+2) = y$

iii) $(x-1)(x+2) = 4$

iv) $(x-1)(x+2) = x^2 + x - 2$

v) $y = (x-1)(x+2)$

Supplementary question, specific to Set C:

- What are the main differences between the two algebraic statements:
  $(x-1)(x+2) = 4$ and $(x-1)(x+2) = x^2 + x - 2$?

Resources:

- Calculators may possibly be useful, for trying out different values for *a*, *b*, *c* and *d*.

Reasoning: questions to discuss and explore

- Are any of these statements never true?
- Are some statements only true for specific values?
- For set A: What happens to the statements if *c* and *d* are both greater than 1? What if *c* and *d* are both less than 1?
- Can we prove an identity through using lots of numerical examples to show that it is true?

Possible Extensions / Consolidation

- Make up several statements where some are identities and some are not. E.g.

i) $(x + 4)\ (x - 4) = ?$

ii) $(x + 4)\ (x - 4) = ?$

- Consider links to science, e.g. invite learners to work with the formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Commentary / notes:

This is suitable for Higher tier, as this activity is focused on distinguishing between equations and identities.

Adding fractions often presents problems for learners – even with numbers, so the opportunity to revisit the general form of adding fractions will help learners and their teachers uncover and deal with the misconceptions.

There are different objectives here: the manipulation of algebraic symbols; to formulate identities for the addition of two fractions – both unit and otherwise, and for expanding brackets. It is also useful to clear up a common misconception when adding fractions – both numerically and algebraically. Learners who are confident may wish to go straight to manipulating the symbols, rather than use substitution with numerical examples.

Solutions:

For set A, the identity is:	For set B, the identity is:	For set C there are two solutions:
iii) $\frac{1}{c} + \frac{1}{d} = \frac{(d + c)}{cd}$	v) $\frac{a}{c} + \frac{b}{d} = \frac{ad + bc}{cd}$	i) $(x - 1)(x + 2) = x^2 + 2x - x - 2$ and iv) $(x - 1)(x + 2) = x^2 + x - 2$

It may be useful for learners to explore the full set of solutions for **set A**, to strengthen their skills and conceptual understanding of addition of fractions:

i) $\frac{1}{c} + \frac{1}{d} = \frac{2}{(c+d)}$ ***Never true***

ii) $\frac{1}{c} + \frac{1}{d} = \frac{2}{cd}$ ***Sometimes true, e.g. for c = d = 1***

iii) $\frac{1}{c} + \frac{1}{d} = \frac{(d+c)}{cd}$ ***Identity***

iv) $\frac{1}{c} + \frac{1}{d} = \frac{(c+d)}{6}$ ***Sometimes true, e.g. for c = 2, d = 3***

v) $\frac{1}{c} + \frac{1}{d} = \frac{5}{6}$ ***Only true for c = 2, d = 3***

For set B, only the identity (v) is always true.

For set C:

i) $(x-1)(x+2) = x^2 + 2x - x - 2$ ***Identity***

ii) $(x-1)(x+2) = y$ ***An equation***

iii) $(x-1)(x+2) = 4$ ***An equation with a solution for specific values of x.***

iv) $(x-1)(x+2) = x^2 + x - 2$ ***Identity***

v) $y = (x-1)(x+2)$ ***An equation***

GCSE Subject Content		
Foundation	**Intermediate**	**Higher**
		AO3: Construct arguments and proofs using logical deduction. AO3: Reflect on results and evaluate the methods employed. Distinguishing in meaning between equations, formulae, **identities** and expressions. Understanding the basic conventions of algebra. Formation and simplification of expressions involving sums, differences, products and powers Multiplication of two linear expressions; expansion of $(ax + by)(cx + dy)$ and $(ax + by)^2$, where a, b, c, d are integers.

Learner Outcomes and Assessment (*to aid comment-only marking*)	
Reasoning strand **Learners are able to:**	**Assessment Guidance** **Can learners:**
• Explain results and procedures precisely using appropriate mathematical language; • **Generalise in words, and use algebra, to describe patterns that arise in numerical, spatial or practical situations;** • **Justify numerical and algebraic results, making appropriate connections;** • **Explain and justify strategies, methods, reasoning and conclusions in a variety of different ways, including orally, graphically, writing (both in mathematical notation and without);** • **Appreciate the difference between mathematical explanation and experimental evidence.**	• Rearrange equations consistently accurately, to identify identities? • Recognise an identity? • Consider that a single counter-example is sufficient to disprove an algebraic statement? • Explain that algebra explains the general case, i.e. what always happens for any number? • Explain what their symbols mean, and how they are using them to justify the situation? • Use symbols accurately and appropriately to represent a situation?