Stimulus question 53

Understanding Dimension

Teaching notes

Key question 1

Gethin, Lewis and Jac are discussing the volumes of cuboids, cylinders and prisms.

Gethin says that you need to find the area of the top face of the cuboid, and multiply that by the height to find the volume.

Lewis says that you need to find the area of the front face of the cuboid, and multiply that by the width to find the volume.

Jac states that you need to find the area of the side face of the cuboid, and multiply that by the width to find the volume.

Who is correct?

Supplementary questions:

- Can you identify each of the faces referred to in this question?
- Can you identify the height, width and length?
- Can more than one person be correct?
- What are we doing when we find an area of a face of a cuboid?
- How does this compare with finding a volume of a cuboid?

Key question 2

Gethin suggests that you could do something similar with a triangular prism. He says you can find the area of one of the faces, then find half the height from that face to the apex opposite, and multiply the two together to find the volume.

Does that work? If so, show how, and if not explain why.

Resources:

- Resource sheets;
- Cuboids and triangular prisms for learners to use in discussion.

Reasoning: questions to discuss and explore

- We have shown that we can find a volume by finding an area of any face (using two of the dimensions), and then multiplying by the third dimension. Does this only apply to cuboids?
- How does this situation link with the commutative rule for multiplication?
- (For the prism question 2): Is there an easier way of thinking about the volume of a triangular prism?

Commentary / Solutions

Note this question links with Question 39: 'Dimensions'.

Quite often, when learners are only exposed to the formulae for areas and volumes of 2D and 3D shapes, they miss the links to the fact that multiplication is commutative, expressed variously as:

$(H \times W) \times L = V$

$H \times (W \times L) = V$

$(H \times L) \times W = V$

The implication of this is that to find the volume of any cuboid, we can find the area of one face, and then multiply by the other dimension.

Care must be taken when generalising this rule to other prisms and to cylinders, which is highlighted in the second question.

When discussing this question, learners should be made aware of the need for an agreed labelling of the dimensions. It may be assumed that the length is the longer (horizontal) side, and the width the shorter side of the base, but this should be clarified in class. If this is agreed, then Gethin and Lewis are both correct, and Jac is not.

The second question provides a twist on an accepted form of thinking about the volume of a triangular prism. Normally expressed as ½ base × height of the triangular cross-section, then multiplied by the length, this can be expressed as ½ b × h × l. However, this can also be shown as (b × l) × ½h, which is the area of a rectangular face, and then multiplied by half the height of the triangle. Gethin's alternative perspective is correct.

Task 2 The cards for task 2 are designed for learners to match, creating formulae for lengths, areas and volumes. Some cards are left blank for the learners to complete the task. Some expressions give a meaningless answer. **Supplementary questions:** • What formulae do you know about finding an area? How could these help with solving these card problems? • What formulae do you know about finding a volume? How could these help? • What algebraic skills are we using here? ***For Task 3:*** • What units do we use in speed/Density/Population Density? How can using units help us to decide how to rearrange the formulae?	**Resources:** • Card sets 1 and 2 for task 2; • Card sets 3 and 4 for the Compound Measures extension task. **Reasoning: questions to discuss and explore** • If we add lengths what do we end up with? • If we multiply lengths, what do we end up with? • Can you identify what will happen if we subtract lengths? • Can you identify what would happen if we divided lengths? • What would we get if we divided one length by another, or one area by another, or one volume by another? **Extension/consolidation:** • Card set on Compound Measures uses the same idea.

GCSE Subject Content		
Foundation	**Intermediate**	**Higher**
	AO1: Recall and use mathematical facts and concepts. AO3: Construct arguments and proofs using logical deduction. AO3: Reflect on results and evaluate the methods employed. Distinguishing between formulae for length, area and volume by considering dimensions. Using compound measures including speed, density and population density.	

Learner Outcomes and Assessment (*to aid comment-only marking*)	
Reasoning strand – **Learners are able to:**	**Assessment Guidance –** **Can learners:**
• Identify, measure or obtain required information to complete the task; • Select appropriate mathematics and techniques to use; • Estimate and visualise size when measuring and use the correct units; • **Develop and evaluate mathematical strategies and ideas creatively;** • **Consider connections between mathematical skills and contextualise these;** • Explain results and procedures precisely using appropriate mathematical language; • Use appropriate notation, symbols and units of measurement, including compound measures; • Select and apply appropriate checking strategies; • Interpret answers within the context of the problem and consider whether answers are sensible; • Verify and justify results or solutions; • **Justify numerical and algebraic results, making appropriate connections.**	• Make the links between the commutative law for multiplication and the formulae for area and volume – i.e. recognise that the formulae for areas are embedded in the formulae for volumes? • Identify how to use units in deciding how formulae are rearranged? • Explain, check and justify their results, by using units, or by rearranging formulae? • Use real cuboids/prisms, to explain how the formulae for areas and volumes work?