Stimulus question 74

Circles and Circumscribed Polygons

Teaching notes

Resources:

- Mini-whiteboards or similar individual display resource;
- Paper, pencils, rulers, pairs of compasses;
- Question 74 PowerPoint: 'Circles and Circumscribed Polygons';
- **Question 74 Resource: Triangle and Hexagon**;
- Enclosed teacher script, with supplementary and other probing questions.

Commentary/notes:

This question is suitable for Higher tier only. It links with questions 72: 'Circles, Polygons, Circumference and Perimeters' and 73: 'Circles, Polygons and Area'. You may consider that learners see either or both of these before tackling this question.

The PowerPoint is constructed in four parts:

- The first section contains questions using the information that the radius of the circle is 1 unit;
- The second section generalises what they have found, with the radius having the value r;
- The third section has some extension questions;
- The fourth and final section of the presentation offers hints for working with the triangle, and can be used at any time during this part of the presentation. During planning, teachers may wish to identify which of the hints may be useful to insert into their own presentation to support learners as they tackle the questions.

It is possible to solve these problems in a variety of ways, including using Pythagoras' theorem and/or trigonometry, or through building diagrams. Comparing the different methods used in class will prove invaluable, giving opportunities to evaluate their effectiveness, rigour and efficiency.

Solutions

Radius 1 unit

- The circumscribed square has a perimeter of 8 units and an area of 4 square units;
- The equilateral triangle has a side length of $2\sqrt{3}$ units. (This can be shown through using slides 19-24). The perimeter is $6\sqrt{3}$ units. The area is $3\sqrt{3}$ square units, (where the height is 3 and the base is $2\sqrt{3}$);
- Each side of the hexagon has a length of $(2\sqrt{3})/3$ units. The perimeter is therefore $4\sqrt{3}$ units. Each equilateral triangle within the hexagon has a height of 1 unit (the radius), so the area of each triangle is $(\sqrt{3})/3$ square units. The area of the hexagon is therefore $2\sqrt{3}$ square units.

Radius *r*

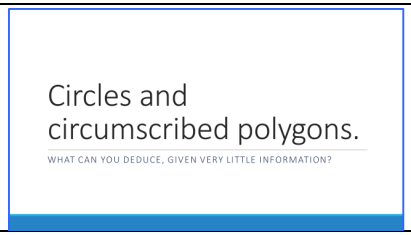
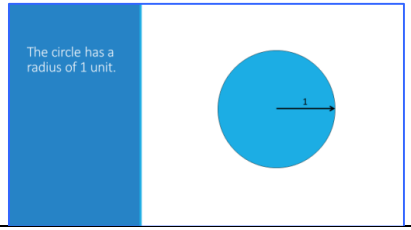
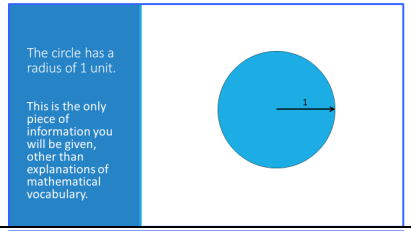
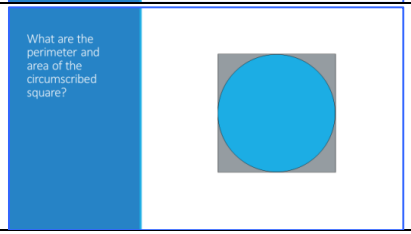
- The circumscribed square has a perimeter of $8r$ units and an area of $4r^2$ square units;
- The equilateral triangle has a side length of $2r\sqrt{3}$ units. (This can be shown through using slides 19-24). The perimeter is $6r\sqrt{3}$ units. The area is $3\sqrt{3}$ square units, (where the height is 3, and the base is $2\sqrt{3}$);
- Each side of the hexagon has a length of $(2r\sqrt{3})/3$ units. The perimeter is therefore $4r\sqrt{3}$ units. Each equilateral triangle within the hexagon has a height of r units (the radius), so the area of each triangle is $(r^2\sqrt{3})/3$ square units. The area of the hexagon is therefore $2r^2\sqrt{3}$ square units.

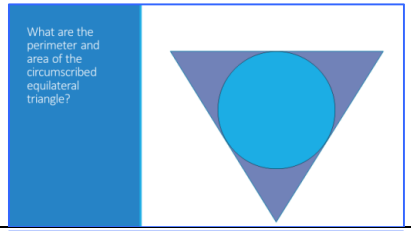
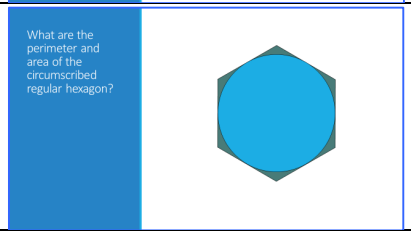
Extension

- The shape can be seen as ¾ of the circle, but it may be more useful for learners to recognise that ¾ is equivalent to 270/360. This will help them to see the link with 269/360, which should be applied to calculating the area and circumference of the full circle. Learners should note that there will be two additional radii to consider when finding the perimeter of the shapes;
- The solutions are $\frac{3}{4}\pi$ square units and $269/360\ \pi$ square units for the areas, $(1\frac{1}{2}\pi + 2)$ units and $(269/180\ \pi + 2)$ units for the perimeters.

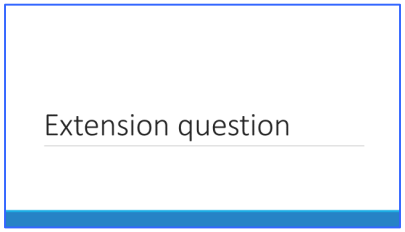
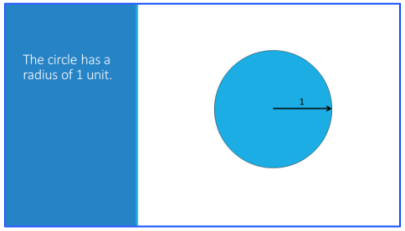
Teacher script (sections 1 and 3)

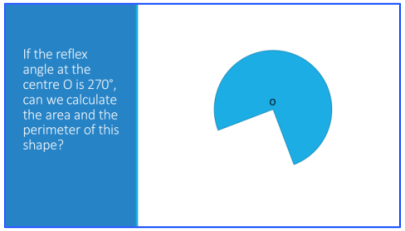
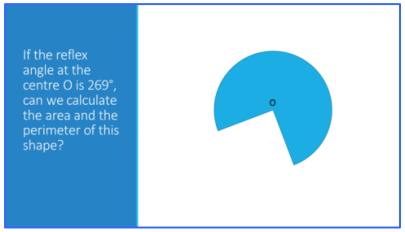
This script is focused only on sections 1 and 3. As the main questions and dialogue are written on the slides, this script only contains further supplementary and reasoning questions (in italics), which are designed to elicit deeper understanding and give explanations of vocabulary. These can be used at the teacher's discretion.

Slide 1		*Keep this slide on the screen until you are ready to start the presentation.*
Slide 2		You will only have a single piece of information to work with.
Slide 3		Although you will only have this information at first I hope you can generate your own information from the diagrams.
Slide 4		• *The square is circumscribed around the circle. Describe what this means.* • *Should the square touch the circle, just miss the circle, or cut through the circle?*

Slide 5		*You may wish to hand out the* **Resource: Triangle and hexagon** *for this question, so that learners may experiment. Slides 17-23 may also support learners' reasoning.* • *Are there any lines we could add to the diagram that may help us?* • *How do your lines help you identify the length of the side of the triangle?*
Slide 6		• *How do your lines help you identify the length of the side of the hexagon?* • *What can we say about the six triangles that form the hexagon?*

Section 3 script

Slide 13		*Keep this slide on the screen until you are ready to start the presentation.*
Slide 14		We are going to consider how we find areas and perimeters of parts of a circle simply by comparing two situations.

Slide 15		• *Is there a straightforward way to solve this problem?* • *In what way is the problem different for area and perimeter?* • *Which of the two is more difficult?* • *There is a common error when considering perimeters of shapes like these – what do you think it might be?*
Slide 16		• *What is similar and what is different about this problem compared with the last?* • *Is there a general rule we can make?* • *Will your rule apply to all angles? How about for a semicircle – does your rule still work?*

GCSE Subject Content		
Foundation	**Intermediate**	**Higher**
		Vocabulary of triangles, quadrilaterals and circles: isosceles, equilateral, scalene, square, rectangle, parallelogram, rhombus, kite, trapezium, polygon, pentagon, hexagon, radius, diameter, tangent, circumference, chord, arc, sector, segment. Calculating: perimeter/area of a square, triangle, circle, semicircle and composite shapes. Using Pythagoras' theorem in 2-D. Using trigonometric relationships in right-angled triangles to solve problems, including those involving bearings and angles of elevation and depression. Calculating a side or an angle of a right-angled triangle in 2-D Manipulating surds; using surds and π in exact calculations. Simplifying numerical expressions involving surds. Lengths of circular arcs. Perimeters and areas of sectors and segments of circles.

Learner Outcomes and Assessment	
Reasoning strand – **Learners are able to:**	**Assessment Guidance –** **Can learners:**
• Select, trial and evaluate a variety of possible approaches and break complex problems into a series of tasks; • Identify, measure or obtain required information to complete the task; • Identify what further information might be required and select what information is most appropriate; • Select appropriate mathematics and techniques to use; • **Develop and evaluate mathematical strategies and ideas creatively;** • Explain results and procedures precisely using appropriate mathematical language; • Use appropriate notation, symbols and units of measurement, including compound measures; • **Generalise in words, and use algebra, to describe patterns that arise in numerical, spatial or practical situations;** • Interpret mathematical information; draw inferences from diagrams.	• Choose from a range of methods to solve the problem and identify reasons for continuing or abandoning them? • Identify that the side length of the polygons is needed to find the perimeter? • Make links between the diameter or radius of the circle and the sides of polygons? • Extend diagrams by drawing appropriate lines, radii, diameters to generate useful information? • Explain their solutions through using conventional mathematical notation and vocabulary? • Derive formulae for the perimeters and circumference? • Evaluate different strategies/methods from other members of the class for solving the problems, for rigour, efficiency and effectiveness?