Contextualised task 7

Bicycle Gears

Teaching Notes

This task focuses on different ways of stating the gear ratios available on bicycles. A number of skills are brought together and many calculations required: students could use a spreadsheet to help progress through the task efficiently.

Task A: Gear inches

Outline

Students learn about the original method of stating gear ratio based on the penny-farthing bike. They calculate different gear ratios and make conclusions from their findings.

You will need:

- Question – 'Gear inches' (see resource sheet)
- Teachers' script
- PowerPoint
- Markscheme

Task B: Metres of development and gain ratio

Outline

Students learn about alternative ways of stating gear ratios. They make further calculations and compare their findings.

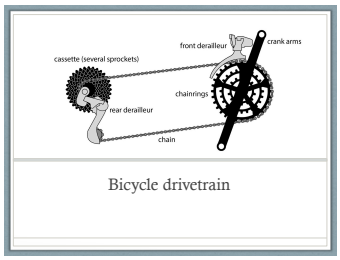
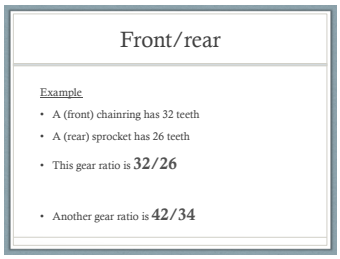
You will need

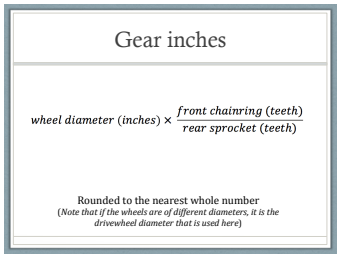
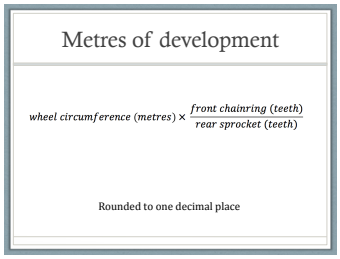
- Question – 'Metres of development and gain ratio' (see resources sheet)
- Teachers' script
- PowerPoint
- Markscheme

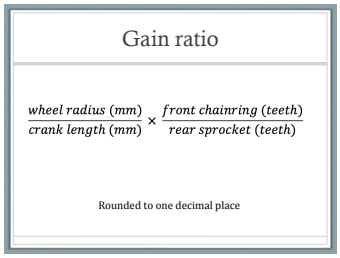
Bicycle gears: Teachers' script for PowerPoint presentation

The text in the right-hand boxes provides a possible script to be read to students. However, it is probably preferable to use your own words and elaboration. When questions are asked, time for discussion in pairs/groups should be provided. Ensure that students are given opportunity to explain their reasoning in response to these questions. All students need to understand the concepts in order to make progress with the task.

Task A only requires slides 1 to 5. Task B requires the remaining slides, although slides 1 to 5 could be reviewed quickly before moving on to slide 6.

Slide 1		*Keep this slide on the screen until you are ready to start the presentation*
Slide 2		*Ask students if they have a bike. What type of bike do they have?* Many types of bike have gears to help riders when they are going uphill and downhill. Mountain bikes usually have more gears than road bikes. A low gear helps when riding uphill. For each turn of the pedals a short distance is travelled. Some people think 'low gear: easier to pedal' When using a high gear, each turn of the pedals moves the bike a lot further forwards. Some people think 'high gear: harder to pedal' How many gears do you have on your bikes? In this task you will investigate ways of describing the gear system on a bike. First, you need to know some useful words.
Slide 3		The set of parts that create the gearing system on a bicycle is called the **drivetrain**. The pedals are connected to **crank arms**. The crank arms turn **chainrings**. In this diagram there are three chainrings. Each chainring has teeth to engage the chain. The bigger the chainring, the more teeth there are. The **cassette** is connected to the rear wheel. It is another set of rings with teeth, but these ones are named **sprockets**. Different combinations of front chainrings and rear sprockets give different **gear ratios**. This means that the wheels turn a different amount for each single turn of the pedals and cranks. **Derailleurs** move the chain from one ring to the next. If there are three chainrings at the front and 9 sprockets at the back, how many gears are there overall? Why? *27, since each of the 3 chainrings can be connected to each of the 9 sprockets. 3 × 9 = 27.*
Slide 4		One way of describing the gear ratio of a bicycle is to state the number of teeth at the front and the number at the back, separated by a 'forward slash'. We could call this gear ratio 'thirty-two, twenty-six' This is very simple to state. But there is a problem because it is very difficult to compare gears using this method

		Advance one click: Which is harder to pedal? 'Thirty-two, twenty-six' or 'forty-two, thirty-four'? *This is a rhetorical question really. The point is that the question is difficult to answer without further calculation or more information.* Other methods of describing gear ratio allow easy comparison of each possible selection.
Slide 5		The original way of describing the gear ratio was **gear inches.** This formula shows how to work out gear inches. It is calculated using three values: the diameter of the wheel in inches, and the number of teeth on the chainring and the sprocket. The value for gear inches is rounded to the nearest whole number. Find the gear inches for the selection 32/26 if the wheel has a diameter of 24 inches. *30* There is a meaning to this value: one turn of the pedals moves the bike as far as one turn of a 30 inch diameter wheel. The origin of the name is from the time of penny-farthing bicycles when drive wheels were very large.
Slide 6		A third way of describing gear ratio is **metres of development**. The formula for metres of development also uses the number of teeth on the chainring and the sprocket. However, wheel circumference in metres is used instead of wheel diameter. The value of metres of development is rounded to one decimal place. What steps would be needed to find the metres of development for the selection 32/26 if the wheel has a diameter of 24 inches? *Use of diameter to work out circumference: ($C = \pi d$)* *Conversion of inches to metres (1 inch is roughly 2.54 cm)* *The solution is 2.4* This is the distance travelled by the bike with one turn of the pedals.

Slide 7		Another popular way to describe gear ratio is **gain ratio** This uses four values: wheel radius, crank length, and – as before – number of teeth at front and back. The wheel radius and crank length must be in the same unit, and this is usually taken to be millimetres. This gives a number that should be rounded to one decimal place. What steps would be needed to find the gain ratio for the selection 32/26 if the wheel has a diameter of 24 inches and the crank length is 175 mm? *Use of diameter to work out radius: (d = 2r)* *Conversion of inches to millimetres: (1 inch is roughly 2.54 cm)* *The solution is 4.3* This number is the result of dividing the distance travelled by the bike by the distance travelled by the pedals.

Task A – Gear inches: Markscheme

The information below is intended as a guide only

Full credit

Creates a table with all 27 front/rear combinations and corresponding gear inches values correctly stated as shown below. Table may be organised by gear inches in preparation for the following questions.

Front gear (chainring)	Rear gear (sprocket)	Front/rear	Gear inches
22	34	22/34	17
22	30	22/30	19
22	26	22/26	22
32	34	32/34	24
22	23	22/23	25
32	30	32/30	28
22	20	22/20	29
42	34	42/34	32
32	26	32/26	32
22	17	22/17	34
42	30	42/30	36
32	23	32/23	36
22	15	22/15	38
42	26	42/26	42
32	20	32/20	42
22	13	22/13	44
42	23	42/23	47
32	17	32/17	49
22	11	22/11	52
42	20	42/20	55
32	15	32/15	55
32	13	32/13	64
42	17	42/17	64
42	15	42/15	73
32	11	32/11	76
42	13	42/13	84
42	11	42/11	99

AND states

- Lowest gear is 22/34
- Highest gear is 42/11
- Tenth lowest gear is 22/17

AND states

- 42/34 and 32/26

- 42/30 and 32/23
- 42/26 and 32/20
- 42/20 and 32/15
- 42/17 and 32/13

as five pairs of combinations that give very similar gear ratios.

AND makes reasonable conclusions based on these findings; e.g. 'there are only really 22 gears available', or 'there is not a lot of benefit of having the big chainring at the front'.

Partial credit

Creates table of 27 combinations AND answers at least one of the further questions correctly

OR

Creates table of 27 combinations (gear inches not rounded to nearest integer) AND answers all further questions correctly

Limited credit

Creates table of 27 combinations with gear inches calculated and rounded correctly

OR

Creates table of 27 combinations with gear inches calculated correctly but rounded incorrectly

OR

Shows clear evidence of correct substitution into formula for gear inches

No credit

Any other response

Task B – Metres of development and gain ratio: Markscheme

The information below is intended as a guide only

Full credit

Adds correct information about metres of development AND gain ratio to table. Table may be organised by any one of the three types of gear ratio, or by logical grouping of front/rear combinations.

Front gear (chainring)	Rear gear (sprocket)	Front/rear	Gear inches	Metres of development	Gain ratio
22	34	22/34	17	1.3	1.2
22	30	22/30	19	1.5	1.4
22	26	22/26	22	1.8	1.6
32	34	32/34	24	2	1.8
22	23	22/23	25	2	1.8
32	30	32/30	28	2.2	2
22	20	22/20	29	2.3	2.1
42	34	42/34	32	2.6	2.3
32	26	32/26	32	2.6	2.3
22	17	22/17	34	2.7	2.4
42	30	42/30	36	2.9	2.6
32	23	32/23	36	2.9	2.6
22	15	22/15	38	3	2.8
32	20	32/20	42	3.3	3
42	26	42/26	42	3.4	3
22	13	22/13	44	3.5	3.2
42	23	42/23	47	3.8	3.4
32	17	32/17	49	3.9	3.6
22	11	22/11	52	4.1	3.8
42	20	42/20	55	4.4	4
32	15	32/15	55	4.4	4
32	13	32/13	64	5.1	4.6
42	17	42/17	64	5.1	4.7
42	15	42/15	73	5.8	5.3
32	11	32/11	76	6	5.5
42	13	42/13	84	6.7	6.1
42	11	42/11	99	7.9	7.2

AND confirms that answer to the first three questions are the same for both new measures of gear ratio.

AND observes that

- there are five pairs of equivalent answers for metres of development, although they are a different five from gear inches
- there are five pairs of equivalent answers for gain ratio, although they are another different five

Partial credit

Adds correct information about metres of development AND gain ratio to table.

OR

Adds correct calculated but incorrectly rounded information about metres of development AND gain ratio to table.

Limited credit

Adds correct information about metres of development OR gain ratio to table.

OR

Adds correct calculated but incorrectly rounded information about metres of development OR gain ratio to table.

OR

Shows clear evidence of correct use of formulae for both metres of development and gain ratio

No credit

Any other response

GCSE Subject Content	
GCSE Mathematics – Numeracy and **GCSE Mathematics**	**GCSE Mathematics only**
Understanding number and place value • Understanding place value and decimal places • Rounding decimals to the nearest whole number or a given number of decimal places • Ordering and comparing whole numbers, decimals, fractions and percentages	
Understanding number relationships and methods of calculation • Using the facilities of a calculator, <u>including the constant function, memory and brackets</u>, to plan a calculation and evaluate expressions • Using calculators effectively and efficiently	
Solving numerical problems • Giving solutions in the context of a problem, <u>selecting an appropriate degree of accuracy</u>, interpreting the display on a calculator, <u>and recognising limitations on the accuracy of data and measurements</u> • <u>Rounding an answer to a reasonable degree of accuracy in the light of the context.</u> Interpreting the display on a calculator	
Understanding and using equations and formulae • Substitution of positive and negative whole numbers, fractions and decimals into simple formulae expressed in words or in symbols	
Understanding and using measures • Standard metric units of length. • Knowledge and use of the relationship between metric units of length • Conversion between the following metric and Imperial units: cm, m – inches, feet; • Calculating the perimeter of a circle	

<u>Key</u>
Foundation tier content is in standard text.
Intermediate tier content that is in addition to foundation tier content is in <u>underlined text</u>.
Higher tier content that is in addition to intermediate tier content is in **bold text**.

Progression in reasoning →			
Identify processes and connections • Prioritise and organise the relevant steps needed to complete the task or reach a solution	Identify a next step towards achieving a solution.	Identify some steps towards achieving a solution, not necessarily in sequence.	Identify a sequence of steps towards achieving a solution. Identify priorities – the tasks that are essential, and those tasks that may embellish the solution if there is time.
Represent and communicate • **Evaluate different forms of recording and presenting information, taking account of the context and audience**	Present a significant number of calculations or solutions in a table.	Recognise and explain that the use of a spreadsheet will be more efficient when so many repeated calculations are required.	Recognise and explain that a spreadsheet is not only helpful when repeated calculations are required, but also that it can be used to sort and compare solutions.
Review • Select and apply appropriate checking strategies	Students check that the set of solutions to each of the three types of gear ratio contains numbers of the same order of magnitude. They go back and recalculate if anomalies occur.	Students check that the order of the 27 gears is identical for each of the three gear ratios.	Students identify and investigate the fact that the rounding functions on a spreadsheet may result in lost information. This may not be immediately obvious, but when each of the three types of gear ratio are calculated and ordered, anomalies may surface.