

## Contextualised task 35

# The 100 Metre Race

## Teaching Notes

This activity involves interpreting data presented in different forms to compare speed, distance and time. The aim is to find who might win a race between the world's best sprinter (Usain Bolt) and the world's fastest sprint cyclist (René Enders) over a 100 metre race. The final task is to see if either of them could beat an old hatchback car of modest performance from a standing start over the same distance.

A variety of techniques may be employed to solve the problems, including finding the area under the graph (which may use the trapezium rule) for the final task. As with any activity of this kind, where a number of strategies may lead to a reasonable solution, it is not intended to force any one method. It is designed to offer an opportunity for learners to consider a range of strategies and choose the most appropriate.

This activity is probably best suited to splitting into different tasks, which will include an initial search for a way of solving the problem, an examination of the assumptions that are made within certain calculations or graphs, a discussion on the quality of the data we have, and what further information is needed for more accurate working, and an exploration of higher mathematical techniques. The presentation is broken down accordingly, using two separate PowerPoint presentations. This will enable teachers to select the tasks they can offer to any tier: the first two may be tackled by learners at Foundation tier with some support, although arriving at the final solution, by working backwards from the finish, may be counterintuitive for some.

The final task is for higher tier only, but should not be tackled without experiencing the earlier work. It is also best that learners have some experience of calculating the areas under graphs beforehand. This task is not designed to teach this skill as new material, but to give a context for its use.

Finally, but very importantly, the data offered here has limitations. This raises questions about the appropriateness of presenting solutions to fine degrees of accuracy, and this aspect should be discussed by the class. In the end, we can only offer sensible estimates from limited data, and therefore it is likely to raise more questions than answers.

## **Task A: The Runner and the Cyclist**

### **Outline**

#### **Phase 1**

Students are presented with Usain Bolt's world record time for the 100m sprint, and the world record for the cyclist René Enders to cover 250m from a standing start. They are invited to consider how to use this information in order to compare if the runner or the cyclist would reach 100m first in a race.

This part of the activity is suited for about 20 minutes as part of one lesson. The learners' work can then be collected in, and assessed ready for the next lesson, where they should be given the opportunity to improve their work from the feedback given.

#### **Phase 2**

Learners' initial work is returned to them to improve. They have the opportunity to address the questions / guidance offered in the marking, and then to evaluate other work presented on the PowerPoint. Having discussed what further information they might need in Phase 1, some data are now provided at this point: split times for every 10m of Bolt's 100m record and the half-lap split times for a cyclist. This data should help learners refine their calculations and any graphs they draw.

### **You will need**

#### **For phase 1:**

- Teachers' script
- PowerPoint 1 slides 1-4
- A selection of plain, lined, squared and graph paper
- Calculators / rulers / pencils
- Assessment criteria: 'Phases 1 and 2: Learner Outcomes and Assessment'.

#### **Additionally for phase 2:**

- PowerPoint 1 slides 5-9
- **Learner Resource Sheet 1: Split times**

## **Task B: An old hatchback**

### **Outline**

This is only suitable for higher tier. A graph of the performance data for a hatchback car is introduced (using imperial rather than metric measures), and learners are asked to confirm whether or not the car would outperform the runner and cyclist over 100m. As this is a graph of changing speed over time, the problem can be solved using the area under the graph, and the trapezium rule. The teaching notes allow for this task to be presented either for learners to tackle themselves or as a worked example for the teacher to demonstrate, if the learners have a limited experience of this type of problem.



### **You will need**



- Teachers' script
- PowerPoint 2: An Old Hatchback
- **Learner Resource Sheet 1: Split times** (if needed as a reminder)
- **Learner Resource Sheet 2: Hatchback performance**
- **Learner Resource Sheet 3: Some hints**
- A selection of plain, lined, squared and graph paper
- Calculators / rulers / pencils
- Mark scheme provided

## Task A Phase 1: Teachers' script for PowerPoint presentation

The text in the right-hand boxes provides a possible script to be read to students. This is in plain text, with questions for the class shown in bullet points. Possible (correct) student responses are written in bold; instructions are written in italics. However, it is preferable to use your own words and elaboration, and to manage the resources as best suits your class.

When questions are asked, time for discussion in pairs / groups should be provided. Ensure that students are given opportunity to explain their reasoning in response to these questions, to enable them to understand the concepts so they can progress.

Slide 1	<p>The 100 metre race</p> <p>THE WORLD'S GREATEST SPRINTER, USAIN BOLT, AGAINST THE WORLD'S FASTEST SPRINT CYCLIST, RENÉ ENDERS. WHO WINS?</p> 	<p><i>Keep this slide on the screen until you are ready to start the presentation</i></p>
Slide 2	<p>Usain Bolt vs Rene Enders over 100 metres: who wins the race?</p> <div data-bbox="376 842 544 959"> <p>THE RUNNER: USAIN BOLT</p> <p>Men's sprint 100m World Record is 9.58 seconds <a href="http://news.bbc.co.uk/2/hi/sport1/hi/athletics/3910433.stm">http://news.bbc.co.uk/2/hi/sport1/hi/athletics/3910433.stm</a> Alternative: <a href="http://www.youtube.com/watch?v=lyjGKofLMM">http://www.youtube.com/watch?v=lyjGKofLMM</a></p> </div> <div data-bbox="555 842 723 959"> <p>THE CYCLIST: RENÉ ENDERS</p> <p>The men's cycling world record for a 250m lap from a standing start is 16.984 seconds, by René Enders of Germany. <a href="https://www.youtube.com/watch?v=Yv1u1qUuM0I">https://www.youtube.com/watch?v=Yv1u1qUuM0I</a> (This video is of the German team, which includes Enders. A video of the world record ride is not available).</p> </div> 	<p><i>You may wish to show the video clip of Usain Bolt's world record in Berlin in 2009. There are two options for this.</i></p> <p>A standard lap in a velodrome is 250 metres.</p> <p>The video clip for the cyclist is of Germany vs New Zealand in the World championships of 2013. The German team includes Rene Enders, and also shows clearly the riders striving to achieve 17 seconds for the first lap: 250 metres. We do not have the video for the world record attempt.</p> <p><i>If you can rewind and pause the video at the moment the riders have completed the first lap – one or two questions will help students understand what is going on.</i></p> <ul style="list-style-type: none"> <li>• The lap time for Germany is 17.323 seconds. Which rider is given that time? <b>(The third and final rider in the team)</b></li> <li>• What is the +0.038 time given at the bottom of the screen? <b>(The extra time taken by the final NZ rider)</b></li> <li>• What is 0.038 seconds in hundredths or thousandths? <b>38 thousandths of a second, or 3 hundredths and 8 thousandths of a second.</b></li> </ul>

		<ul style="list-style-type: none"> <li>Why does a rider peel away each lap? <b>It's the final rider who completes three laps, and they are being helped by being behind the others which reduces the wind resistance. It is also possible that each member of the team specialises in one aspect of the race: the front rider for the start and generating initial pace, the second rider for the best time over two laps, and the third has the greatest endurance for all three laps.</b></li> </ul>
Slide 3	<p>Explore some ways to solve the problem.</p> <div> <div> <p>How could you use this information to decide who will win in a 100 metre race from a standing start?</p> <p>What calculations could you make?</p> <p>What diagrams could you draw?</p> </div> <div> <p>Usain Bolt's 100 metre World Record is 9.58 seconds</p> <p>Rene Enders' 250 metre (one lap) cycling World Record is 16.984 seconds, from a standing start.</p> </div> </div> 	<ul style="list-style-type: none"> <li>So, if Usain Bolt and Rene Enders meet in a 100 metres showdown: who will win?</li> <li>How could we work it out?</li> </ul> <p>I will give you a few minutes (<i>specify your own time suitable for the class – anything up to 10 minutes</i>) to see how you get started. Some clues are in the questions on the left of the slide. We will then share some ideas.</p> <p><i>As you circulate around the class, you will notice good examples of working which learners may wish to share with the class.</i></p> <p><i>You will also pick up others who may be stuck. Ask them to consider</i></p> <ul style="list-style-type: none"> <li><i>What calculations can they make from the information?</i></li> <li><i>What diagrams could they draw? What would the diagrams look like?</i></li> </ul> <p><i>You may wish to hold up some examples of different calculations or graphs from the class to discuss.</i></p> <p><i>You may wish to give the class a few more minutes to finish their work after this discussion if seeing some examples prompts them to improve it.</i></p>
Slide 4	<p>Explore some ways to solve the problem.</p> <div> <div> <p>What assumptions are you making?</p> <p>What are the limitations of the data we have?</p> <p>What further information do you need?</p> </div> <div> <p>Usain Bolt's 100 metre World Record is 9.58 seconds</p> <p>Rene Enders' 250 metre (one lap) cycling World Record is 16.984 seconds, from a standing start.</p> </div> </div> 	<p>Ask each question from the slide:</p> <ul style="list-style-type: none"> <li>What assumptions are we making?</li> </ul> <p><b>E.g. some ideas could be: a straight line graph or an average speed does not take into account acceleration, but assumes both go straight into a steady speed.</b></p> <p><b>No fluctuations in speed.</b></p> <p><b>No reaction time at the start.</b></p> <p><b>If a curve is drawn, there will be an assumed rate of acceleration, as we do not have the data.</b></p> <ul style="list-style-type: none"> <li>What are the limitations of the data we have?</li> </ul>

		<p>E.g. no split times, no details about how they accelerate, no information about when they achieve maximum speed.</p> <ul style="list-style-type: none"> <li>▪ What further information do we need?</li> </ul> <p><i>This can be managed as a class discussion, or as a written task.</i></p> <p><i>There should be questions that learners raise, but if not may be useful for the class:</i></p> <ul style="list-style-type: none"> <li>• Who reaches top speed first?</li> <li>• Does Bolt accelerate more quickly?</li> <li>• If so, will Bolt secure enough of a lead on Enders to win the race?</li> </ul> <p><i>If it is undertaken as a written task, give a few more minutes to complete the work, ready for assessment. The work may be handed in for the teacher to mark but an opportunity also arises here for peer-assessment of the calculations / graphs / diagrams and answers to the questions on this slide. Use the criteria below: <b>'Phases 1 and 2: Learner Outcomes and Assessment'</b> so that adults / students can offer comments for improvement.</i></p> <p><i>Take notes of the information requested. It is likely that some will ask for the information that will come in the next phase of the task, and some will ask for other information which is not provided here. Suggest a homework to research the data!</i></p>
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Phases 1 and 2: Learner Outcomes and Assessment <i>(To aid comment-only marking)</i>	
Reasoning strand – Learners are able to:	Assessment Guidance: Can learners -
<ul style="list-style-type: none"> <li>▪ Transfer mathematical skills across the curriculum in a variety of contexts and everyday situations</li> <li>▪ Select, trial and evaluate a variety of possible approaches and break complex problems into a series of tasks</li> <li>▪ Select appropriate mathematics and techniques to use</li> <li>▪ Identify what further information might be required and select what information is most appropriate</li> <li>▪ Explain results and procedures precisely using appropriate mathematical language</li> <li>▪ Select and construct appropriate charts, diagrams and graphs with suitable scales</li> <li>▪ Interpret graphs that describe real-life situations, including those used in the media, recognising that some graphs may be misleading</li> <li>▪ <b>Evaluate different forms of recording and presenting information, taking account of the context and audience</b></li> <li>▪ Select and apply appropriate checking strategies</li> <li>▪ Interpret mathematical information; draw inferences from graphs, diagrams and data, including discussion on limitations of data</li> </ul>	<ul style="list-style-type: none"> <li>• Get started by choosing a calculation to do (e.g. average speed, or distance travelled after 100m for the cyclist)?</li> <li>• Get started by drawing a suitable distance-time graph, showing the progress of each of the athletes over their races?</li> <li>• Identify how they may represent acceleration on their graphs, and what implications that has about the shape, gradient and direction of their graph?</li> <li>• Suggest or create a different diagram to represent the situation?</li> <li>• Identify the assumptions made in the calculations / graphs / diagrams?</li> <li>• Suggest improvements they could make in their calculations / graphs?</li> <li>• Interpret their graph, to identify the winner?</li> <li>• Identify the limitations of the data they have, and suggest what further information would be useful to make their work more accurate?</li> <li>• Evaluate the class's presentations of the data, suggesting improvements and better solutions?</li> <li>• Use reliable methods to check calculations, and the accuracy of their graphs or diagrams?</li> </ul>

## Phase 2: Teachers' script for PowerPoint presentation

This phase begins at slide 5, but you may wish to recap by going through the previous slides.

<p>Slide 5</p>	<p>The 100 metre race: Evaluating solutions</p> <hr/>	<p><i>Keep this slide on the screen until you are ready to start the presentation.</i></p> <p><i>At this point, you may wish to hand back the work that students have presented for phase 1, with assessment comments provided by you or peers.</i></p> <p><i>Give time for them to work on their own improvements, either individually or in pairs.</i></p> <p><b><i>It is likely that the members of the class have provided some useful examples for discussion and class evaluation. You may wish to present these in addition or instead of the examples provided on the slides.</i></b></p>
<p>Slide 6</p>	<div data-bbox="371 643 497 852"> <p><b>The Calculation</b></p> <p>What assumptions have been made in this calculation?</p> <p>Are these assumptions reasonable?</p> <p>What further information do you need to make better calculations?</p> </div> <div data-bbox="512 727 710 798"> <p>If Rend Enders completed 250m in 16.984 seconds, then to cover 100m he would take</p> <math display="block">(16.984 \div 250) \times 100 \text{ seconds.}</math> <p>This equals 6.7936 seconds</p> </div>	<p><i>Ask the questions on the right of the slide, giving time for learners to consider each, perhaps by discussing their answers in pairs before addressing the whole class.</i></p> <ul style="list-style-type: none"> <li>• What assumptions have been made in this calculation?</li> <li>• Are these assumptions reasonable?</li> <li>• What further information do you need to make better calculations?</li> </ul> <p><i>Further questions may be</i></p> <ul style="list-style-type: none"> <li>• What actually is being calculated here?</li> <li>• Is this calculation going to give us a correct answer?</li> <li>• Can we use 'average speed' calculations later if we had <u>more</u> or <u>better</u> information?</li> </ul>
<p>Slide 7</p>	<div data-bbox="371 1070 497 1272"> <p><b>The Graph</b></p> <p>How would you read the graph to tell you who would win the race?</p> <p>What assumptions have been made in drawing this graph? Are these the same assumptions as in the calculation?</p> <p>How can it be improved?</p> </div> <div data-bbox="512 1094 698 1243"> <p>Graph to show the progress of Bolt's and Enders' world record sprints</p> </div>	<p><i>Again, ask the questions on the right of the slide, giving time for learners to consider each, perhaps by discussing their answers in pairs before addressing the whole class.</i></p> <ul style="list-style-type: none"> <li>• How would you read the graph to tell you who would win the race?</li> <li>• What assumptions have been made in drawing this graph? Are these the same assumptions as in the calculation?</li> <li>• How can it be improved?</li> </ul> <p><i>Further questions could be:</i></p>



		<ul style="list-style-type: none"><li>• Could this graph be easily drawn by hand? Could it easily be done using ICT?</li><li>• What calculations had to be done to generate this graph using Excel?</li><li>• Is this graph a fair representation or model of the reality?</li></ul>																						
Slides 8 and 9	<div><table><thead><tr><th>Distance run (metres)</th><th>Time (seconds)</th></tr></thead><tbody><tr><td>10</td><td>1.89</td></tr><tr><td>20</td><td>2.88</td></tr><tr><td>30</td><td>3.78</td></tr><tr><td>40</td><td>4.64</td></tr><tr><td>50</td><td>5.47</td></tr><tr><td>60</td><td>6.29</td></tr><tr><td>70</td><td>7.10</td></tr><tr><td>80</td><td>7.92</td></tr><tr><td>90</td><td>8.75</td></tr><tr><td>100</td><td>9.58</td></tr></tbody></table><p>These are estimated times at each 10m stage for Usain Bolt's world record 100m run in Berlin on the 16<sup>th</sup> August 2009.</p></div> <div><p>Qualification standards for cyclists</p><p>The standards are those published by the Australian National Track Cycle Team for their selection process.</p><p>These are the standards that a male cyclist has to achieve to be considered for the team for the World Championships and the Olympic Games.</p><p>Men's Standing 250m 17.40 sec Last half lap split time of 6.75sec</p><p><i>Additional information:</i> A cyclist reaches top speed in this event at about 90 metres.</p></div>	Distance run (metres)	Time (seconds)	10	1.89	20	2.88	30	3.78	40	4.64	50	5.47	60	6.29	70	7.10	80	7.92	90	8.75	100	9.58	<p>I am now going to share some more information about the 100m race, and about cyclists. It may not be all the information you want, but it's all we have right now!</p> <p><b><i>Hand out Learner Resource Sheet: Split times</i></b></p> <p>I want you to use this information, and what you have learned about presenting it, to come up with a clearer way of showing who will win the race. You may still have to do some calculations, and you may have to create more graphs or diagrams.</p> <p><i>You may wish to direct the class to work in pairs (or groups) and give a deadline for this piece of work – perhaps 10 or 15 minutes.</i></p> <p><i>Some supporting /guidance questions may be:</i></p> <ul style="list-style-type: none"><li>• What does split time mean?</li><li>• How can we use the half-lap split time? What do we know about a whole lap?</li><li>• What calculations can we do using this?</li><li>• Can we assume that once cyclists have hit top speed they stay at that rate for the rest of the lap?</li><li>• How accurate do we need to be in our answer?</li><li>• How good is the data we are working with? Does this affect the degree of accuracy we should use?</li></ul> <p><i>You may wish to hand out to students again the assessment criteria sheet 'Phases 1 and 2: Learner Outcomes and Assessment', and invite them now to assess how they have now improved their work.</i></p>
Distance run (metres)	Time (seconds)																							
10	1.89																							
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## Solutions and Mark scheme: Phases 1 and 2

The suggestions below are only that. Different possibilities will arise from your group, giving the opportunity to evaluate their methods.

### Phase 1:

A good solution is not going to be found with the initial data, so it is important that we focus our attention on deciding what information may be needed. Some learners may offer the kinds of calculations or graphs that are given in phase 2, both of which suggest that Enders would win the race fairly easily. However, this would assume that there is no acceleration – that both athletes reach top speed immediately.

Other learners may develop more complex responses – e.g. they make assumptions themselves about the acceleration, and how it can be presented. They may therefore attempt a graphical solution – but be uncertain about the direction of the curve, if the speed is changing. When assessing learners' work, this could be a point to pick up.

So we are looking for learners to identify some more information about the progress of the athletes' races – e.g.

- split times,
- data about how the athletes accelerate,
- when the athletes reach top speed,
- what their top speed may be.

### Phase 2: Using better data

It is during phase 2, when there are more detailed data to work with that we would expect to see reasonable calculations or graphs that would lead to a solution, e.g.

A cyclist reaches top speed at 90 metres, and combined with the information that they are expected to cover 125 metres in 6.75 seconds, we can suggest their estimated split times from 90m to 250m. (See spreadsheet extract below, and associated graph).

- Assuming that the final 6.75 seconds are all at top speed, this would be 18.52 m/s.
- If the cyclist is riding at top speed from 90 metres, then we can subtract the time taken for the final 150 meters of his 250 lap, to give us a time for 100m.
  - $6.75 \text{ seconds} \times 150 / 125 = 8.10 \text{ seconds}$ . This is the time for the final 150m.
- $16.894 - 8.10 \text{ seconds} = 8.884 \text{ seconds}$  at 100m. (A longer spreadsheet version of this is included below, with a graphical representation of the data).

**Accuracy:**

Most world records timings are presented to the nearest 100<sup>th</sup> of a second, but Enders' world record is given in thousandths, and as the figures are presented to the learners both to 2d.p. and to 3d.p. accept either in their presentation of the work.

However, there must be some conversation about the degree of accuracy we can use, when there is incompleteness about the data we are using for the cyclist. It is possible that learners will argue that Enders should be a better rider than the qualification standard for the Australian team (as he is the world record holder), and that he would ride faster than them over the last 150m. He may therefore have a 100m time that *is less* than 8.884 seconds, but not by the full 0.416 seconds that his world record time betters the Australian 17.4 seconds standard for a full lap. Learners may subtract a reasonable proportion of the 0.416 seconds, and as soon as they do, then they are working with less accuracy. So there is good reason to accept a figure such as 8.7 seconds, if it is supported by an argument about the quality of data they are working with.

**Mark scheme*****Full Credit***

- States that Enders will win the race, and gives an accurate time for him to complete the 100m. Offers an answer to a suitable degree of accuracy (i.e. 2 or 3 decimal places, as presented in the original world record data), but also states that the cycling data is not sufficiently accurate to give a definitive solution to this degree of accuracy.
- Gives the winning time as 8.884 seconds, *or less*, but no lower than about 8.65 seconds. This will enable them to present an argument that Enders' world record time is just over 0.4 seconds inside the Australian standards, and so considers subtracting a proportion of this from the time above. Accept a reasonable argument and a reasonable time, such as one between 8.65 to 8.88 seconds.
- May present their working in any of a variety of ways, but must draw conclusions and state their answers clearly.

***Partial Credit***

- States that Enders will win the race, but rounds to the nearest second or does not form an argument about the quality of the data.

OR

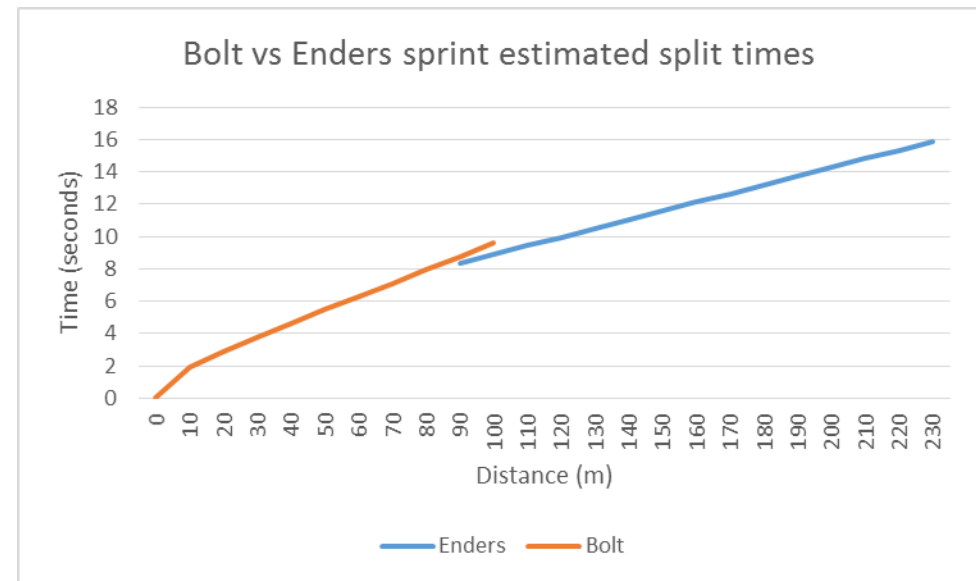
- Working is appropriate and accurate, but solution is not clearly stated.

***No credit***

Any other response.

### Spreadsheet and graph

	Estimated Split Times (seconds)	
Distance (m)	Enders	Bolt
0		0
10		1.89
20		2.88
30		3.78
40		4.64
50		5.47
60		6.29
70		7.1
80		7.92
90	8.344	8.75
100	8.884	9.58
110	9.424	
120	9.964	
130	10.504	
140	11.044	
150	11.584	
160	12.124	
170	12.664	
180	13.204	
190	13.744	
200	14.284	
210	14.824	
220	15.364	
230	15.904	
240	16.444	
250	16.984	



## **Task B: An old hatchback**

### **Outline**

This is only suitable for higher tier. Firstly, a graph of the performance data for a hatchback car is introduced using an imperial measure (mph), and learners are invited to consider how the information should be adapted to help them compare the athletes' performance. They are then offered data that has been converted to metres per second.

Learners are asked to confirm whether or not the car would outperform the runner and cyclist over 100m.

As this is a graph of changing speed over time, the problem can be solved by finding the area underneath the graph, possibly using the trapezium rule.


The learning resources include a 'hints' sheet to offer some scaffolding for learners.


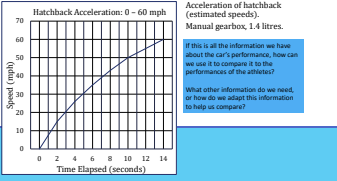
### **You will need**

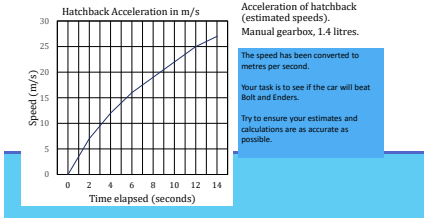
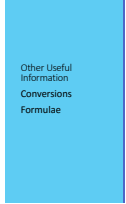
- Teachers' script
- Learners' previous work from phases 1 and 2.
- PowerPoint 2 slides: 'An Old Hatchback'
- **Learner Resource Sheet 1: Split times** (if needed as a reminder)
- **Learner Resource Sheet 2: Hatchback performance**
- **Learner Resource Sheet 3: Some hints**
- A selection of plain, lined, squared and graph paper
- Calculators / rulers / pencils
- Mark scheme provided

## Task B: Teachers' script for PowerPoint presentation

The text in the right-hand boxes provides a possible script to be read to students. This is in plain text, with questions for the class shown in bullet points. Possible (correct) student responses are written in bold; instructions are written in italics.

Slide 1	<div>The 100 metre race</div> <div>THE WORLD'S GREATEST SPRINTER, USAIN BOLT, AND THE WORLD'S FASTEST SPRINT CYCLIST, RENÉ ENDERS...</div> <div></div>	<p><i>Keep this slide on the screen until you are ready to start the presentation.</i></p> <p><i>Ensure that learners have their previous work on finding who wins between Bolt and Enders.</i></p> <p><i>You may wish to organise your class into pairs / groups for this activity, and ensure that learners have the equipment they need: calculators, paper / graph paper, pencils rulers.</i></p> <p>We are going to see if both Usain Bolt and René Enders will beat...</p> <p><i>Bring in the next slide</i></p>
Slide 2	<div>... against this old hatchback over 100 metres. Will the hatchback win?</div> <div></div> <div></div>	<p>This old car!</p> <p>Will Enders beat this car?</p> <p>Will Bolt beat this car?</p>
Slide 3	<div>Usain Bolt and René Enders over 100 metres: their world records.</div> <div><div><div>THE RUNNER: USAIN BOLT</div><div>Men's sprint 100m World Record is 9.58 seconds.</div></div><div><div>THE CYCLIST: RENÉ ENDERS</div><div>The men's cycling world record for a 250m lap from a standing start is 16.994 seconds, by René Enders of Germany.</div><div>This suggests that he will cover the first 100m in 8.88 seconds.</div></div></div> <div></div>	<p>This is just to remind you of the work we have done so far. A reasonable time for René Enders over 100m is about 8.88 seconds.</p> <p><i>You may wish to hand out <b>Learner Resource sheet 1: Split times</b> to remind learners of the data.</i></p>

<p>Slide 4</p>	<p>Estimate the performance of this car.</p> 	<ul style="list-style-type: none"> <li>Can anyone suggest how long this car would take to reach 60mph from a standing start? Which of these suggestions is the most reasonable?</li> </ul> <p><i>Some learners may ask about the engine, or how old. Suggest 1.4 litres, and over 15 years old.</i></p> <p>We have a graph to look at on the next slide, to see who is closest.</p>
<p>Slide 5</p>		<ul style="list-style-type: none"> <li>Can anyone read the graph for me, to tell me what the 0-60 performance is for this car? <b>(For this activity, we are working with an average of about 14 seconds.)</b></li> </ul> <p><i>Ask the questions on the slide:</i></p> <ul style="list-style-type: none"> <li>If this is all the information we have about the car's performance, how can we use it to compare it to the performances of the athletes?</li> <li>What other information do we need to make these comparisons?</li> </ul> <p>Work together in pairs. I will give you about two minutes to come up with some solutions.</p> <p><i>Discuss students' ideas.</i></p> <p><i>If it has not already come up in the discussion, establish with the group that if they are trying to see if the car beats the athletes over 100m, we want information in metres. This means that there needs to be some conversion from mph to m/s.</i></p> <p>Here we have a graph with the conversion already made. It is also on the next slide.</p> <p><b>Hand out the Learner Resource sheet 2: 'Hatchback performance'.</b></p>

<p>Slide 6</p>		<p>It's time to have a try at this question.</p> <p>Have a go in your groups, trying any strategy you can, making suitable calculations. You have paper /graph paper and calculators if you need them.</p> <p><i>Set a suitable deadline for your class, saying that you will expect them to feedback some ideas / methods /strategies for solving the problem (rather than the answer at this stage).</i></p> <p><i>Later, if you notice that some learners need support, you may wish to hand out</i></p> <p><b>Learner Resource Sheet 3: Some Hints</b></p> <p><i>Towards the end of the task, ask learners to discuss the quality of the data they have, and then add their own ideas about what level of accuracy is appropriate in this situation. Considerations may be:</i></p> <ul style="list-style-type: none"> <li><i>• Data was presented to the learners both to 2d.p. and 3d.p.</i></li> <li><i>• Why should an estimate be given to 2d.p. or 3d.p.?</i></li> <li><i>• If we are making <u>estimates</u> from the data, what would be an appropriate degree of accuracy?</i></li> <li><i>• If all we are doing is trying to find out if the car or Enders wins, do we have to give that to 3d.p.?</i></li> </ul>
<p>Slides 7 - 9</p>	 <p>1 mile = 1609.344 metres 1 hour = 3600 seconds</p> <p>The area of a triangle = <math>\frac{1}{2}</math> base <math>\times</math> height</p> <p>The area of a trapezium = <math>\frac{1}{2} (a + b) \times</math> height where a and b are the lengths of the parallel sides.</p>	<p>These slides are unscripted. They are only offered for teachers if they wish to use them for discussion / support for the class.</p>





### Solutions and possible working

These are only suggestions: learners may offer a variety of strategies which give a reasonable estimate of the distance covered by the car after certain times in its journey.

**Learners may begin by considering an estimate** of the distance covered after a time that they know will beat Enders' time of 8.88 seconds. The graph shows that after about 8.7 seconds the car will be travelling at approximately 20m/s. The area of the triangle from start to this point would be  $\frac{1}{2} \times 8.7 \times 20 = 10 \times 8.7 = 87$  metres. There remains an area under the graph not covered by the triangle, so this may be a good starting point.

**Learners may then wish to take each part of the graph at a time.** This will be more accurate.

- After 2 seconds, the car has travelled 7 metres.

*The following parts use the formula for the area of a trapezium.*

- After 4 seconds, the car has travelled 19 more metres.  $(\frac{1}{2} (7 + 12)) \times 2$ . Running total  $7 + 19 = 26$  metres.
- After 6 seconds, the car has travelled  $(\frac{1}{2} (12 + 16)) \times 2 = 28$  more metres. Running total  $26 + 28 = 54$  metres.
- After 8 seconds, the car has travelled  $(\frac{1}{2} (16 + 19)) \times 2 = 35$  more metres. Running total  $54 + 35 = 89$  metres.

To ensure that the car beats Enders, we need to look at times less than 8.88 seconds. After 8.5 seconds the car has reached about 19.7 m/s.

- After 8.5 seconds the car has travelled a further  $(\frac{1}{2} (19 + 19.7)) \times 0.5 = 38.7 / 4 = 9.7$  metres, a running total of 98.7 metres.

- Calculating for 8.7 seconds and 20 m/s:  $(\frac{1}{2} (19 + 20)) \times 0.7 = 13.65$  more metres after 8 seconds: 102.65 metres.
- A good estimate therefore, is about 8.6 seconds.  $(\frac{1}{2} (19 + 19.8)) \times 0.6 = 11.64$  more metres after 8 seconds, giving a total of 100.64 metres after 8.6 seconds.

**Applying the trapezium rule (up to 8 seconds)** where we have considered the width of each trapezium to be 2, and the side lengths of each trapezium are given by the speeds at each 2-second interval, the estimated area under the graph after 8 seconds would be:

$$= \frac{1}{2} (2) (0 + 2(7 + 12 + 16) + 19)$$

$$= 2(35) + 19$$

$$= 89.$$

**Learners could, *and should* argue that the data is limited**, and what we have is probably not accurate enough to come to a final, definitive conclusion. They might suggest that the car just beats Enders, but it could be argued that we do not have his split times, and only the data from comparable (but slightly slower) cyclists from the Australian team qualification. Both car and Enders will beat Usain Bolt.

## Mark Scheme

### *Full Credit*

Clear working to find the area under the graph or *some other means* to lead to a suggestion that the car will reach 100m after about 8.6 seconds, and therefore *is likely* to beat Rene Enders, but perhaps only just.

Other means could also include:

- Making suitable calculations of distance covered from average speeds in each section of the graph.
- Adding this information to the graph of Bolt and Enders, to establish the progress of the car over time.

States that the data is limited, and therefore accuracy to 2 or 3 decimal places is unnecessary. Offers a time of 8.6 seconds.

### *Partial Credit*

Correctly calculates at least four sections under the graph.

Offers a reasonable solution, but does not suggest that the data is limited.

Correctly calculates all sections, but does not offer a clear final solution.

### *No credit*

Any other response.

Progression in reasoning			
<b>Identify processes and connections</b> <ul style="list-style-type: none"> <li>select, trial and evaluate a variety of possible approaches and break complex problems into a series of tasks</li> </ul>	<p>Start the problem by looking at some calculations, e.g. comparing mph with m/s.</p> <p>Perhaps makes some estimates of distance covered over given numbers of seconds.</p>	<p>Break the problem into parts – e.g. identify that they will compare distances or speeds, and that they will need to convert one set of information to compare with the other – either with speeds or distances.</p> <p>Identify further information that may be needed or useful, beyond that on the first slide – i.e. a breakdown of Bolt's 100m at split times, and / or a breakdown of how the car accelerates.</p> <p>Organise work clearly.</p>	<p>Identify a way of comparing the car speeds at various times, with the split times of the sprinter. This could be by creating a speed /time graph; a distance / time graph; a table of estimations; or other means where both sets of data are shown.</p>
<b>Represent and communicate</b> <ul style="list-style-type: none"> <li>select and construct appropriate charts, diagrams and graphs with suitable scales</li> </ul>	<p>Choose suitable scales for any graphs, to aid clarity and ease of interpretation?</p>	<p>Draw suitable tables / charts / graphs that show comparison – e.g. own conversion graph between mph and m/s?</p> <p>Interpret their charts appropriately.</p>	<p>Evaluate their methods of calculating and presenting their findings, with regard to clarity and efficiency.</p>
<b>Review</b> <ul style="list-style-type: none"> <li>select and apply appropriate checking strategies</li> <li>interpret mathematical information; draw inferences from graphs, diagrams and data, including discussion on limitations of data</li> </ul>	<p>Ensure the calculations they make are checked for accuracy. Identify what further working may be needed to solve the problem.</p>	<p>Check that they have answered the problem, and that their results are sensible.</p>	<p>Discuss what limitations there may be in terms of accuracy from the data given.</p>

GCSE Content	
GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only
<b>Understanding number relationships and methods of calculation</b> <ul style="list-style-type: none"> <li>Using the facilities of a calculator, including the <u>constant function, memory and brackets</u>, to plan a calculation and evaluate expressions.</li> <li>Knowing how a calculator orders its operations.</li> </ul>	
<b>Solving numerical problems</b> <ul style="list-style-type: none"> <li>Giving solutions in the context of a problem, <u>selecting an appropriate degree of accuracy</u>, interpreting the display on a calculator, and <u>recognising limitations on the accuracy of data and measurements</u>.</li> <li><u>Rounding an answer to a reasonable degree of accuracy in the light of the context.</u></li> <li>Interpreting the display on a calculator.</li> <li>Knowing whether to round up or down as appropriate.</li> </ul>	
<b>Understanding and using functional relationships</b> <ul style="list-style-type: none"> <li>Construction and interpretation of conversion graphs.</li> <li>Construction and interpretation of travel graphs.</li> <li>Construction and interpretation of graphs that describe real-life situations.</li> <li><b>Constructing and using tangents to curves to estimate rates of change for nonlinear functions, and using appropriate compound measures to express results, including finding velocity in distance-time graphs and acceleration in velocity-time graphs.</b></li> <li>Interpreting the meaning of the area under a graph, including the area under velocity-time graphs and graphs in other practical and financial contexts.</li> <li>Using the trapezium rule to estimate the area under a curve.</li> </ul>	

### Key

Foundation tier content is in standard text.

Intermediate tier content which is in addition to foundation tier content is in underlined text.

Higher tier content which is in addition to intermediate tier content is in **bold text**.