Communication, language and literacy: classroom culture, collaborative problem-solving, and reading and writing in mathematics

Classroom culture: 'a conjecturing atmosphere'

> 'In the schools where teaching is good or better, teachers require pupils to reason and explain orally, using correct mathematical terms'. (Estyn 2013)

This suggests that the most effective classrooms are those where the children are engaged in constructing meaning in a 'mutually supportive' working environment. John Mason (1988) argues that 'mathematical thinking is best supported by adopting a conjecturing attitude', stressing that 'an attitude or atmosphere of conjecturing frees you from the dreadful fear of being wrong … we should bless our mistakes as golden opportunities'. Building this kind of working environment doesn't happen by accident – co-operation can be taught; appreciating each other's contributions can be an explicit classroom value and it can be seen that allowing each other time to work things through is important for everyone. Learners in these classrooms therefore carry a responsibility for everyone's learning, not just their own. If we create this atmosphere for learning, it can become our greatest ally.

'What if…?' is a powerful question to ask, to begin to generate a conjecturing atmosphere. 'What if we changed something?' 'What if we applied that idea to this problem?' When it is our learners who are asking these questions, then we can be confident they are on their way to becoming mathematicians.

Collaborative working

All learners must be given many opportunities to explain, to reason, and to use the vocabulary of mathematics. To maximise these opportunities, they have to engage in conversations in pairs or groups. There is simply not the time in a lesson for the teacher to mediate all the mathematical dialogue.

> 'Through explaining, both partners in the pairing come to realise what it is they are thinking, clarify their thinking and understanding and sometimes come to new thoughts or concepts'. (Vygotsky, 1962).

In the report 'Good Practice in Mathematics at KS4', Estyn summarises the mathematical outcomes that learners experience through these collaborations:

'In the majority of schools … pupils work well together in pairs and groups to:

- Discuss, predict, reason and form opinions in classroom discussions;
- Plan, communicate and evaluate outcomes, particularly in problem-solving and investigative activities; and
- Analyse, synthesise and evaluate information from a range of sources.

In many schools, pupils work well in pairs and groups, often using mini whiteboards to share ideas and communicate findings'. (Estyn, 2013)

It is essential to plan effectively for collaboration, and consider:

- The nature and challenge of the task on which the groups are working, (including what mathematical ideas are developed, the connections with other aspects of mathematics, and the range of strategies they may need to develop and discuss);
- The purposes of the collaboration, and therefore the roles of members of groups.

This planning will involve considerations of:

- Open questions;
- Questions focused on methods rather than answers;
- Activities that expose and deal with misconceptions;
- Seating arrangements (for paired and group work, rather than individual work);
- Resources, e.g. large paper for posters or shared work; mini-whiteboards for classroom collaboration (and formative assessment); ICT (such as dynamic geometry and graphing software, spreadsheets); sets of cards to organise;
- Acceptable outcomes (e.g. a poster, notes on a mini-whiteboard, contributions to class discussion and not necessarily a neat page of written exercises).

In planning for collaboration, Swan (2006) identifies five types of activity to encourage different ways of reasoning and learning:

- **Classifying mathematical objects:** devising their own classifications for mathematical objects, and/or applying classifications devised by others.
- **Interpreting multiple representations:** working together matching cards that show different representations of the same mathematical idea.
- **Evaluating mathematical statements:** e.g. deciding whether given statements are always, sometimes or never true.
- **Creating problems:** writing their own examination questions, or other problems for others to solve.
- **Analysing reasoning and solutions:** comparing different methods for doing a problem, organising solutions and/or diagnosing the causes of errors.

Planning the structure of groups, and individual roles for learners

A few examples are given here to facilitate effective paired/group work in mathematics classrooms, through structuring the groups and defining the roles each member plays.

Talker, questioner, recorder

Arrange pupils to work in groups of three.
Each pupil takes on one of these roles:

- *Talker:* who does the explaining, describing and reasoning;
- *Questioner:* who clarifies ideas or points made, and prompts or interviews the talker;
- *Recorder*: who makes notes and reports back at the end of the discussion, either to the whole class or another group.

Expert

This type of group work usually requires a large task that can be split into several parts. Group sizes will depend on the number of sections of the task. More than one group may work on the same part of the whole task, which will allow for comparison of ideas.

- Allocate a section of the task to each group, so that each member becomes an expert on that part of the work.
- Reform the groups, so that there is now an expert on each part of the task in the new groups.
- In the new groups, the whole task may be tackled, using the expertise that each individual brings.

Spokesperson

Ask each group to appoint a spokesperson or, to maintain fairness, select them randomly. They should be the ones who report the findings of the group back to the class. This does not have to be a repetitive or long process, if:

- The first group gives full feedback and others offer only additional points that have not been covered;
- The first group makes one point, and then each group in turn is asked to offer something new.

Group structures

Working in pairs – enables learners to:

- Refine their ideas, overcome mistakes, and use the vocabulary correctly before addressing the whole class;
- Establish greater confidence in their own explanations;
- Give fuller, more considered contributions, rather than one-word answers;
- Establish a classroom culture where everyone thinks and contributes;
- Access the curriculum through mutual support (an effective form of differentiation).

Joining pairs together – creates opportunities to:

- Explain and compare their ideas or strategies to solve a problem;
- Evaluate or assess another pair's work. Often it will be at this point where new ways of working or of solving the problems are discussed;
- Allow teachers to circulate through the classroom to notice ideas raised that would be useful to other groups or misconceptions that arise.

Two other types of group management are common:

Jigsaw groups:

- Groups of students work on different parts of a problem – specifically problems that have many aspects, such as surveying different groups of people, making a plan of the school field (requiring a lot of measurements), or finding the cost of furnishing all the rooms of a house;
- Groups then share their information with the rest of the class.

Rainbow groups:

- Can be established through assigning numbers or colours to each member of each small group (say 4 people), who then discuss how to solve a problem;
- After their discussion, new groups are formed by putting together the students with the same colour or number;
- The findings or ideas from each small group can be shared among the new larger group who evaluate the ideas and improve on them;
- Students return to their original groups of 4 with these new, improved ideas.

Summary

Whatever group or pair structure is used, it is always important to keep the purposes of the group work in mind: developing thinking and reasoning, building self-esteem, and enabling all to access the curriculum and understand what they are doing.

For further reading on building group-working skills, there are articles from nrich:
http://nrich.maths.org/6933
http://nrich.maths.org/7908

This article includes a reference to the work of Jo Boaler:
http://nrich.maths.org/7011

Literacy in mathematics: Reading

All classrooms should have mathematics dictionaries available for learners to access, to build their own independence in reading and writing.

Learning and doing mathematics provides a unique and specific opportunity for using and developing many of the literacy skills detailed in the Literacy and Numeracy Framework (LNF) and in the new Programmes of Study for Language at KS3 and KS4.

Here are some guidelines that teachers could offer learners as they read mathematical texts:

1) Take time over reading mathematical ideas, instructions and questions
    - Reading mathematics is not a left-to-right, moving-down-the-page experience. Sometimes, the texts need to be read several times, moving backwards and forwards through the text to find the links between relevant information;
    - Stop, reflect and if necessary take notes.
2) Understand the whole problem
    - Learn what the mathematical vocabulary *precisely* means – use knowledge of word roots and families;
    - Go beyond finding key words: look at the whole problem before identifying which mathematical operation or technique to use.
3) Be an active reader
    - Reorganise or represent the information in a different way – such as using diagrams/charts or models;

- Add to the information you are given – drawing extra lines, making calculations, identifying angles;
- Act out or discuss the texts;
- Build your own examples/work on the examples that are given;
- Identify the information or the facts you have – write a list, highlight;
- Identify some rules or strategies you know that may be relevant to the materials you are reading;
- Start a plan – e.g. by breaking down the problem into small steps;
- Write down what information you need or any questions you have;
- Use a mathematical dictionary.

Literacy in mathematics: Writing

In mathematics we use language concisely and accurately to convey meaning efficiently. Elegance in mathematical reasoning can often be described through its simplicity. Mathematics thus makes a significant but perhaps particular contribution to learners' literacy, through the need for precise description, justification, explanation and proof. These processes are the means by which learners come to understand their mathematics, providing a purpose for developing their technical fluency.

Mathematics teachers therefore have a specific responsibility in the development of literacy. There are many short writing tasks we can use to build these skills, such as:

- Creating definitions: e.g. of shapes, different types of graph/chart/types of number etc;
- Giving instructions: e.g. using a specific technique (e.g. solving a problem using Pythagoras' theorem, or trigonometry), explaining how to construct a histogram;
- Simplifying: e.g. starting with a long set of instructions, which the learners could abbreviate using symbols; simplifying long algebraic expressions;
- Constructing a chain of reasoning or proof (as in identifying the size of angles using circle theorems);
- Interpreting and comparing sets of data, presented in a variety of ways – in graphs, charts or tables.

Developing the drafting process

The drafting process is a key element in improving learners' writing in mathematics. The aims would be to develop learners' precision in using the language of mathematics, and their capacity to construct a convincing argument, clear explanation or definition.

Both of these are authentic aims for mathematicians at any level. They can be addressed in a variety of ways, using:

- Word-processing or PowerPoint presentations;
- Other communication devices – e.g. define a square in a single text message (140 characters);
- Mini-whiteboards;
- Posters or leaflets;

- Partners in the classroom – reading, evaluating and discussing written work, to refine definitions and explanations, instructions and proofs. Learners could pass their written pieces to each other, to see how well they understand each other's arguments. They can then review each other's work together and improve the writing.

We should take time building learners' own definitions, instructions and chains of reasoning. These tasks may be set in class – perhaps to close a lesson, and the teacher could collect the pieces of writing, and either copy some examples to display on the classroom whiteboard for the next class discussion, or assess the writing, and write comments for learners on how to improve their work. The class could then work individually or together to improve their writing.

In working in these ways, we are showing our learners that:

- We cannot expect to master everything – including writing – at the first go (Mason, 1988);
- The first written draft does not have to be perfect – indeed, it is unlikely to be so;
- The quality of our work improves through the process of review and redraft.

Using writing frames

Writing frames help learners who experience difficulties with mathematics to organise their thinking. These frames act as prompts to thinking about how they are going to solve problems, rather than test recall of knowledge. However, a writing frame need not become a complex form-filling exercise. Here is one that a class developed themselves to help with their work on mathematics tasks:

- *My task is …*
- *The information I have is …*
- *The information I need is …*
- *My working is …*
- *My results show that …*
- *My conclusions are …*

A frame such as this makes learners aware of what is expected of them. As they become more confident the frame should be adjusted or discarded altogether.

Writing frames can help students by:

- Showing how to get started on a problem;
- Offering a structure and style to their writing;
- Encouraging learners to move to higher level thinking, such as giving evidence to support their findings, and evaluating their work;
- Building self-esteem and further motivation to write, as the frames help to clarify the whole process.

However, we should be mindful of:

- Limiting creativity, and inhibiting 'thinking outside box'. There are many different ways to solve mathematics problems, and we should be wary of inhibiting the different strategies and methods that learners may generate by structuring writing frames in particular ways;
- Dependence – the goal is to dispense with the frames, and for learners to write independently and confidently. One stage in this process is for learners to

construct their own frames, which may help them internalise the structure of their work;
- Frames are fluid and transitory. They may need to be adapted – there is no 'one size fits all', and it should be remembered that the goal is for learners to work without them in the end.

Examples of writing frames

Two examples of writing frames accompany these notes. The first (Problem-solving writing frame) is linked to the problem-solving tools included within this pack: the 'Problem-solving' PowerPoints, the A3 sheet: 'Developing Numerical Reasoning: open-ended questioning for solving problems' and the 'Steps to tackle a task' resource. It uses the same structure in these resources to guide thinking about solving the problems. The second frame is focused on data-handling tasks.

Developing literacy in mathematics: OCW

OCW (Organising, Communicating and Writing accurately)

For many years now Mathematics GCSE papers have included questions where marks are awarded for the Quality of Written Communication (QWC). From 2017, the GCSE Mathematics and GCSE Mathematics-Numeracy examinations will both contain questions focused on Organising, Communicating and Writing accurately (OCW). As in previous years, two marks are awarded in addition to the marks for the question itself.

Note that the OCW questions in the examination paper are always indicated, both at the beginning of the paper and at the start of the question, where these words are shown:

> *'You will be assessed on the quality of your organisation, communication and accuracy in writing in this part of the question'.*

The notes for organisation, communication and accuracy from the WJEC specifications (2014) state:

"Two marks will be awarded on each examination paper, at each tier, for the assessment of 'communicating, organising and writing accurately'. These marks will be in addition to the marks allocated to the mathematics. One mark will be awarded for communicating and organising. The other mark will be awarded for writing accurately (incorporating language, grammar, punctuation, spelling and mathematical notation). These questions will be clearly indicated on each examination paper.

Mark schemes for all units include the following specific criteria for the assessment of written communication (including mathematical communication):

- Accuracy of spelling, punctuation and grammar; clarity of meaning;
- Selection of a form and style of writing appropriate to purpose and to complexity of subject matter;
- Organisation of information clearly and coherently; use of specialist vocabulary where appropriate"

In questions assessing OCW, learners will be expected to:

- Construct a reasoned argument, e.g. in a proof (algebraic or geometric), or a chain of reasoning in algebraic or geometric problems;
- Present work in an ordered way that allows the examiner to follow the work without difficulty. For example, showing clear steps in constructing, manipulating and/or solving equations and formulae; or conducting sequences of calculations; or writing in full the description of transformations (e.g. centre of rotation, angle and direction of turn);
- Present statistical diagrams with clear labelling, scales and/or axes, or geometric diagrams with conventional labelling;
- Explain work where evidence has to be gathered with a clearly stated decision or outcome.

Assessment of OCW is thus likely to follow these guidelines:

2 marks

- Presents relevant material in a coherent and logical manner, using acceptable mathematical form, and with few if any errors in spelling, punctuation and grammar.

1 mark

- Presents relevant material in a coherent and logical manner but with some errors in use of mathematical form, spelling, punctuation or grammar
  OR
  Evident weaknesses in organisation of material but using acceptable mathematical form, with few if any errors in spelling, punctuation and grammar.

0 marks

- Evident weaknesses in organisation of material, AND errors in use of mathematical form, spelling, punctuation or grammar

Note that the mark for organisation and communication and the mark for written accuracy are awarded separately, and therefore on some occasions, may be awarded in two separate questions on an examination paper.

Common errors made by learners include:

- Writing a story giving a narrative of the events (which are often clear from the working), rather than, for example, explain why particular evidence was sought, or how a formula is constructed or manipulated, or what their results showed.
- Writing very little or nothing to explain the steps of working.
- A series of equals signs in sequences of calculations, e.g.

  $$4 \times 2 = 8 \times 5 = 40 \div 2 = 20.$$

  Calculations are best written separately, probably on separate lines. (4 times 2 does not equal 20, which may not be intended in this calculation, but that is what is being stated).
- Inaccurate or incomplete use of geometrical language, e.g. “opposite angles” for “<u>vertically</u> opposite angles” or failing to give a full description of transformations. Other examples may be using "edge" instead of "circumference", "angles in a circle" rather than "angles around a point" or "a quadrilateral that touches all sides of the circle" rather than "a cyclic quadrilateral".

Example question and learner response

A typical question in GCSE Mathematics-Numeracy may involve money calculations with percentage increase or decrease, such as sales, discounts, depreciation or interest. Below is an example of such a question suitable for Intermediate Tier, where a calculator is allowed, and a possible response from a candidate. A mark scheme is also included.

Harri's Car

Harri bought a car three years ago for £2800.
He sells it to a friend for 45% less than he paid for it.

He wants to take advantage of a sale which says '30% off marked price' on a car that is advertised at £3750.

Harri will use the money he has from selling his old car towards buying the new one.

How much extra will Harri have to pay towards the new car in the sale?
You must show all your working.

Markscheme

	Mark	**Comment**
(Loss) 0.45 × 2800	M1	
(Selling price =) 2800 – 0.45 × 2800	m1	OR M2 for 0.55 × 2800
(£)1540	A1	
(New car costs =) 3750 – 0.3 × 3750 or 0.7 × 3750	M1	
(£)2625	A1	
(Extra money needed)(= 2625 – 1540) (£)1085	B1	FT 'their 1540' provided M1 awarded for loss, and 'their 2625' provided M1 awarded for new car cost *SC1 for (£)2210 (discount for sale price not considered)*
Organisation and communication	OC1	
Accuracy of writing	W1	

Marking codes

- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy.

Commentary on candidate's response:

Harri bought a car three years ago for £2800.

He sells it to friend for 45% less than he paid for it.

He wants to take advantage of a sale which says '30% off marked price' on a car that is advertised at £3750.

Harri will use the money he has from selling his old car towards buying the new one.

How much extra will Harri have to pay towards the new car in the sale?

You must show all your working.

45 % of £2800 = £1540 (2800 x 0.55)

70 % of £3750 = £2625 (3750 x 0.7)

£1540

- £2625

= £1085

He will have to pay £1085 extra

Although this candidate has ended up with a correct answer using an appropriate strategy for calculation the percentage decrease each time, there are clear errors in the mathematical form.

The first error is in stating that 45% of £2800 = £1540 (where that is 55% of £2800). The next line is correct however.

The next error is in setting out the subtraction.

It is possible therefore that this candidate will lose the mark for organisation and communication, but there are no spelling errors, the £ symbol is included correctly, so this mark will be gained.

References

Estyn, '*Good Practice in Mathematics at KS4',* October 2013

Mason, J. H. (1988), *Learning and Doing Mathematics,* Basingstoke/London, MacMillan

Swan, M. (2006), *Collaborative learning in mathematics: A challenge to our beliefs and practices,* National Research and Development Centre for adult literacy and numeracy (NRDC) and the National Institute of Adult Continuing Education (NIACE). A CD of resources and video clips is available with the book.

Vygotsky, L. S. (1962), *Thought and Language,* Cambridge, MA: MIT Press