3.5 The NSP self-review, development and planning toolkit: Learning and teaching: numeracy

Introduction

There are two sub-sections ('components') on learning and teaching. One deals with literacy (sub-section 3.4) and one with numeracy (sub-section 3.5).

Each component is divided into two parts.

- The first covers the review and development of a learning and teaching policy (that is, setting the school's overall approach)
- The second covers learning and teaching strategies (that is, what happens in the classroom).

Feedback suggests that some schools will want to deal with literacy and numeracy separately. However, other schools, particularly smaller schools and primary schools, may want to look at literacy and numeracy together.

Some of the issues, such as achieving a consistent approach to literacy and numeracy across the school, are the same. Others, which relate more directly to the content of the separate literacy and numeracy components of the LNF, are different.

Consequently, in order to give schools flexibility, the component dealing with literacy is separate from the one dealing with numeracy. However, the broad structure of self-review and planning is similar for both components, so that they can be brought together by schools wishing to do so.

Part 1: Policy

Overview

A whole-school approach to the LNF involves planning for the teaching of numeracy in the whole curriculum and embedding practices in each year group and subject, to support learners in developing and applying numeracy skills in different contexts.

> One of the most influential factors in continuing to improve numeracy in the most successful secondary schools is having a well-established numeracy group within the school, representing most, if not all departments. This group makes sure that there is good communication and that all departments receive information, training and reminders of ways of using numeracy well. Members of the group act as numeracy 'champions' within each subject and drive the initiative within that subject. The most effective groups also include a member of the senior leadership team.
>
> In the best practice, primary and secondary schools have clear policies to make sure there is a consistent approach to numeracy across the school and that numeracy regularly features on the agenda of staff meetings. All staff are aware of the numeracy policy and the levels of pupils' numeracy skills[62].

The school numeracy policy should outline the principles for numeracy across the curriculum and summarise the approaches to be used. It should be reviewed and amended to reflect progress and changing circumstances.

Since the LNF is statutory from September 2013, all schools will need to have a numeracy policy which reflects the implementation of the framework. For schools that have an existing numeracy policy, the starting point for the process is to review and update that document. Schools where there currently is no numeracy policy need to decide on the structure and content of that policy. This policy has aspects in common with a policy for mathematics but they are not the same. A numeracy policy concerns teaching of the skills of developing numerical reasoning, using number skills, using measuring skills and using data skills across the curriculum, whereas a policy for mathematics relates to the statutory requirements for mathematics and includes aspects such as probability and algebra.

What works? Evidence about good practice

A policy is an overview document of principles which reflect and guide practice. It sets out objectives and enables staff and governors to review progress in reaching them.

[62] *Improving numeracy in key stage 2 and key stage 3* (Estyn, 2010)

A whole-school numeracy policy aims to:

- develop, maintain and improve standards of numeracy across the school;
- promote consistent classroom approaches to developing learners' mental calculation skills, teaching basic calculation strategies, and developing efficient written methods for routine calculations and for the appropriate use of calculators;
- enable learners to transfer skills, knowledge and understanding between subjects;
- ensure careful tracking and monitoring of learners' progress, using assessment for learning;
- support consistent assessment, recording and reporting of learning.

Schools will be at various stages and on different paths to implementing the numeracy component of the LNF. Rather than waiting until all aspects of the LNF are fully embedded in a school, it is important to capture the principles of teaching numeracy in a policy which can be updated as developments take place. It is essential that the policy reflects actual classroom practice.

In schools where a working group is established to write the draft policy it is important that all staff are given the opportunity to review and feed back on the document. This can help to ensure that all staff are kept up to date with developments and have ownership of any processes adopted.

The following documents outline one possible approach to writing a numeracy policy. Schools can select which steps of the process are applicable to them. Guidelines and exemplars are provided which can be adapted according to a school's needs.

Review and development process: learning and teaching – literacy/numeracy

Developing a policy for literacy/numeracy across the curriculum
School/cluster approach

A. Preparing the way
- Identify working group to write the policy
- Convene initial meeting

B. Team development and support
- Initial meeting held
- Purpose and outcomes agreed
- Timelines set
- Ways of working agreed

C. Getting under way
- Identified steps of work carried out
- Draft policy written

D. Keeping up the momentum
- Draft policy agreed by cluster group
- Draft policy presented to all staff and discussed
- Amendments made based on feedback
- Review date for policy agreed

E. Progress so far
- Governors approve final draft version of policy
- Plans for further developments

Key:

Preparing the way column:

Red = alternative approach where cluster group decide to work together to produce a common whole-school literacy policy. This encourages consistency in strategies used across the curriculum within schools, between schools and between key stages, e.g. FP to KS2 and KS2 to KS3.

What this might look like … column:

Black = Exemplar for both primary and secondary schools

Green = Primary school exemplar

Blue = Secondary school exemplar

Red = Cluster exemplar

How much progress has been made with LNF implementation?

In reaching judgements, use the following assessment criteria, which are consistent with those in the Progress Map:

Red = not in place

Amber = work has started but consolidation is needed

Green = in place and monitored regularly

Step-by-step guidance

Learning and teaching: numeracy policy

NSP model		School review			
A. Preparing the way	What this might look like …	**School** *(purpose, lead person, activity, outcome)*	R	A	G
A.1 A member of the SLT is appointed as manager with overall responsibility for developing the numeracy policy. Cluster group representative for each school is likely to be a senior manager. Need to elect a chair for the working group.	Since the LNF is a statutory requirement which influences a whole school, the curriculum leader on the management team is made responsible for overseeing the development of the numeracy policy in Primary school A. In Cluster B it is decided to have the curriculum deputy headteacher of the secondary school leading, with the assistant headteacher of one of the primaries as co-leader.				
A.2 The senior leader Identifies a cross-curricular working group, probably with the numeracy coordinator in charge of drafting the policy. The senior manager and numeracy	In Primary school C, with only three teaching staff, all three are involved in the development of the numeracy policy. In Primary school D the group includes someone from each key stage as well as the coordinator for mathematics/numeracy, and they appoint the deputy headteacher to help with the work and ensure continuity if				

coordinator agree the composition and size of the working group before approaching staff members. In clusters, staff need to consider: • schools' representation; • expertise in numeracy; • seniority of representation.	the coordinator is absent or leaves. In Secondary school E a numeracy coordinator is appointed who works closely with members of the mathematics department and representatives from subject departments. Secondary school F opts to base its working group on the school faculties. The head of each faculty – Humanities, Language, Science, Technology and Creative – is asked to attend or appoint a representative to the working group.				
A.3 Initial meeting is convened. Before the meeting the working group members are asked to familiarise themselves with relevant documentation, e.g. school learning and teaching policy, current whole-school numeracy policy, any existing departmental numeracy polices and the LNF. Cluster approach is the same.	In Primary school G different members of the group are asked to compare specific documents, e.g. the school's learning and teaching policy with the framework, or the numeracy policy and the mathematics policy, or the current numeracy policy with the section on numeracy in the school SEF/SDP. The group members are asked to be ready to report at the first meeting. In secondary School H the group also look at the mapping of curriculum opportunities which have taken place earlier and the subject schemes of work where numeracy is included.				

B. Team development and support	What this might look like …	School *(purpose, lead person, activity, outcome)*	R	A	G
B.1 At the first meeting of the numeracy working group (see Annex 3.4.i, page 187) consider: 1. how the group will operate, for example: • how often the working group will need to meet, where and when, e.g. designated PPA time, twilight sessions, INSET days; • how outcomes of the meetings will be recorded, e.g. minutes or agreed action points; • who will be responsible for writing the policy, e.g. one person or several members of the working group with one person collating the document.	In Primary school I all three members of staff meet during a twilight session to plan how they will write their school's numeracy policy. They decide to meet weekly for three weeks. The discussions enable the numeracy coordinator to write a first draft for the agreed sections of the policy. These are reviewed at the next meeting before discussion of further sections begins. A working group at Secondary school J holds its first meeting during the morning of an INSET day. After a brief recap on the LNF the group considers the existing whole-school numeracy policy and whether it provides an accurate overview of activity within the school in relation to the LNF. A work plan is created to keep track of the proposed work. Dates are set for completion of draft sections of the policy. Future meetings, held as twilight sessions, are planned to review and amend the written drafts.				

2. the approach taken to the work, for example: • review what was learned from the pre-reading comparing the various documents; • decide whether the format and sections of the existing policy are appropriate; • decide which aspects need to be rewritten to reflect the implementation of the LNF; • anticipate any difficulties which may arise, e.g. timing, relating to other school priorities. The above applies equally to the work of the cluster group. Careful consideration will be needed to determine whether it is necessary for every section of the policy to be common to					

all schools. This might not always be possible and cluster might agree that some sections be kept specific to individual schools.					

C. Getting under way	What this might look like …	**School** *(purpose, lead person, activity, outcome)*	R	A	G
C.1 The group agrees the steps to be carried out by members of the working group. These include: • agreeing the structure of the policy (Annex 3.5.i, page 249, can help with this); • reviewing current practice for each aspect of policy;	Primary school K identifies that one of their school priorities, mental mathematics, is not evident in their policy, so uses a staff meeting to agree what the approach should be, which is then incorporated into the policy. Primary school L teachers have found that they are not currently reporting to parents/carers in relation to the LNF, so need to review their tracking, assessing and reporting arrangements. This review is noted in the policy and a date is set for updating it with the new procedures.				

• identifying areas in need of development to incorporate the LNF from review. Further work will need to be planned and undertaken on these; • writing the draft policy. A coded version of the LNF is available to support identification of progression and cross referencing. The group members carry out the writing tasks which are brought back to the group for review. The numeracy coordinator collates the policy and ensures consistency of style and content. Cluster approach is the same.	The focus of the numeracy working group meeting in Secondary school M is to discuss the roles and responsibilities of all those involved in developing numeracy within the school. The main points for discussion are identified and the existing policy reviewed. Agreed outcomes from the discussions enable the numeracy coordinator to draft those sections of the policy. These are reviewed in the next meeting and amendments are made. The process is repeated for other sections of the policy.				

D. Keeping up the momentum	What this might look like …	**School** *(purpose, lead person, activity, outcome)*	R	A	G
D.1 Cluster group agree draft version of the policy before it is shared with staff members in individual schools. In a meeting of the school where all parts of the curriculum are represented the numeracy coordinator explains the rationale for the draft policy, what it is based on and the implications for teaching across the curriculum. Staff given the opportunity to feedback on the policy. This can: • update and reinforce understanding of the processes adopted to implement LNF amongst teachers and TAs	In Primary school N a draft numeracy policy is shared with all teachers and TAs during a whole school meeting. All staff are given the opportunity to provide oral feedback during the meeting. Feedback given is discussed and amendments are made to the draft policy in light of the outcomes of the discussions. A final draft policy is then produced. In Secondary school O the numeracy coordinator gives a presentation on the draft policy in a whole-school staff meeting. Each member of staff is given a copy of the policy and a feedback form for noting suggested amendments (see Annex 3.4.iii, page 193). Comments on returned forms are collated and discussed in the next meeting of the working group. Responses to each comment are noted and any necessary amendments made to the policy. The collated feedback form plus responses (see Annex 3.4.iii, page 193) together with the amended policy are discussed in a further staff meeting and a final draft policy is agreed.				

• encourage ownership of the policy across the school. The last draft of the policy is checked by the school leadership team and approved.					
D.2 A review date is set for the policy. As the process of implementing the LNF is likely to be ongoing over the next two years, the policy will need to be reviewed and updated regularly during this period. Schools decide the best approach to take. Cluster approach is the same.	With work on mapping the LNF ongoing in the school and aspects such as tracking, assessment and reporting still in their infancy, staff at Primary school P decide that they will update the policy annually. In Secondary school Q (a large school) where the work of mapping opportunities and ensuring progression across the curriculum is ongoing, it is decided to update various aspects of the policy termly, e.g. tracking and monitoring, assessment and reporting. It is felt that updating sections to reflect the work completed during a term keeps the work in manageable chunks. Sharing updates with the staff keeps them abreast of developments.				

E. Progress so far	**What this might look like …**	**School** *(purpose, lead person, activity, outcome)*	R	A	G
E.1 Final draft policy is presented to the governors for approval, together for plans for development, such as: • a focus on progression in basic number skills; • ensuring consistency in teaching simple calculations; • ensuring suitable challenges for different groups of learners. This is also an ideal opportunity for updating all governors about numeracy in the school. Once governors have approved the policy a final version should be circulated to all staff. The policy may need to be approved more than once during the period that the implementation	The headteacher of Primary school R presents the numeracy policy to the board of governors for approval. She makes the most of the opportunity to explain the processes adopted for implementing the LNF in the school and is able to share data and anecdotal feedback as to the impact of the LNF to date. The governors are keen to look at data over time and request further updates on this. Future planned developments in numeracy are also shared and there is discussion of the likely impact of these developments. Following a detailed discussion the governors approve the policy. The numeracy coordinator in Secondary school S is invited to give a presentation to the governors on the draft numeracy policy. She shows how the school's vision of the implementation of the LNF will improve numeracy standards. Given the need for further development and consolidation, the policy is a draft one which will be reviewed biannually and presented for approval to the governors each time.				

of the LNF is ongoing. Cluster approach is the same.					

Part 2: Strategy

Overview

> Too often there is no agreed whole school approach to building pupils' numeracy skills or performing basic calculations. This leads to a lack of consistency in using numeracy skills across classes and departments that confuses pupils.
>
> *Numeracy in key stages 2 and 3: a baseline study* (Estyn, 2013)

Effective planning is essential, and without clarity about what is taught and how it should be taught, the lack of consistency in using numeracy skills across classes and departments can confuse learners.

Effective teaching strategies need to address the common weaknesses identified by Estyn[63]. All teachers need to:

- ensure learners:
  - know and can remember basic number facts
  - have effective mental calculation strategies
  - have confidence in using efficient written methods for routine calculations
  - can work with fractions, decimals and percentages;
- be able to:
  - judge whether learners' numeracy skills are above or below expectations for their age
  - plan meaningful opportunities for learners to use numeracy skills in their subject;
- use methods and strategies consistent with those used elsewhere in the school.

What works? Evidence about good practice

Schools showing good practice ensure that:

- curriculum plans and schemes of work provide learners with the necessary skills. This may include joint planning of numeracy across departments to encourage reinforcement of number skills and numerical reasoning as well as their application in different contexts;
- sufficient time is allocated to improving learners' skills in mental and written calculations;

[63] *Improving numeracy in key stage 2 and key stage 3* (Estyn, 2010)

- planning includes a focus on progression and challenge (see Sections 3.2 and 3.3);
- there is a consistent approach to teaching numeracy in all subjects, across year groups and key stages.

Schools aiming for consistency will:

- have discussed approaches used for ensuring consistency across all four strands of the numeracy component of the LNF in, for example:
  - the use of mathematical language
  - the teaching of basic mental calculations
  - the teaching of effective written calculations
  - the use of calculators;
- have developed guidelines as a result of these discussions which are shared with all staff;
- provide training to ensure that all staff have the required level of skills and knowledge for teaching numeracy;
- monitor the use of the agreed approaches;
- evaluate the impact on learners' progress.

A lack of consistency in the use of mathematical language and the teaching of mental and written calculation strategies across the four strands of the NNF can create confusion and result in numeracy being perceived as more difficult than necessary by many learners. This, in turn, can quickly lead to a lack of confidence and even eventually a dislike of dealing with numbers.

Few learners possess the ability to make the link themselves between the same topic taught in different subjects across the curriculum. This, together with exposure to inconsistent use of terminology and strategies, creates barriers to the transfer of skills from one subject to another.

Developing consistency in numeracy does not necessarily mean that only one method of calculation should be offered to learners. Although this is one possible approach, welcomed by some practitioners, in many cases it will not be the best approach to adopt for a learner. Different methods will be better understood by different learners and therefore favoured, as they are found easier to apply. Where several methods are introduced, careful consideration needs to be given to the

range, order and pace at which they are introduced. It is imperative that learners be given time to consolidate whatever method they choose to use. During this time of consolidation, they may well need the support of their teacher. This has training implications for staff.

The question of whether skills should first be taught in mathematics before being used in other subjects can be contentious. Some practitioners will favour this model while others may believe it would be impractical to wait until a skill has been taught in mathematics before drawing on it elsewhere. There may also be benefits in demonstrating practical applications of the skill.

The key is to ensure:

- consistency of approach;
- teaching of skills by teachers who themselves are sufficiently competent and confident.

Review and development process: learning and teaching

Developing consistent, effective strategies for number skills

A. Preparing the way

- Appoint senior manager to lead working group
- Appoint members of working group

B. Team development and support

- Agree ways of working, including remit and timescale

C. Getting under way

- Gather information about current numeracy strategies used across the curriculum
- Collate information gathered and evaluate whether the school is teaching the full range of strategies, across the curriculum, accurately, effectively and consistently. Identify good practice

D. Keeping up the momentum

- Develop proposals that build on identified good practice and address weaknesses in terms of strategies to be used, guidelines, materials and professional development
- Consult staff and governing body

E. Progress so far

- Introduce strategies and related materials to use across the curriculum to the whole school
- Agree how and when strategies implemented should be monitored and evaluated for effectiveness

Key:

Preparing the way column:

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What this might look like … column:

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How much progress has been made with LNF implementation?

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Green = in place and monitored regularly

Step-by-step guidance

Learning and teaching: numeracy strategies

NSP model		School review			
A. Preparing the way	What this might look like …	**School** *(purpose, lead person, activity, outcome)*	R	A	G
A.1 Senior manager responsible for numeracy is appointed to oversee a working group set up to look at the consistency of numeracy strategies used across the curriculum. This person may lead the working group themselves or delegate that role to the numeracy coordinator.	The deputy head in Primary school A has responsibility for numeracy in the school. Together with the numeracy coordinator, he leads the working group investigating the consistency of numeracy strategies across the curriculum. The assistant headteacher in Secondary school B (a large school) oversees the strategic planning for numeracy. Numeracy development is carried out by the numeracy coordinator (a member of the mathematics department) and his deputy, a member of the science department.				
A.2 Senior manager and numeracy coordinator (this may be same person in some schools) decide membership of the working group, e.g. a representative from each	Curriculum leads in Primary school C are invited to be members of the working group. The headteacher in Secondary school D requests that heads of department from subject areas having most numeracy content – e.g. science, geography, design				

year group, key stage, heads of departments or faculties.	and technology and PE – participate in the working group. An invitation is also issued to heads of any of the other departments who might have an interest in developing numeracy within their subject.				
A.3 First meeting of the working group is convened by senior manager or numeracy coordinator to discuss and agree ways of working. It is agreed that the working group would take an evidence-based approach.	The first meeting of the working group in Primary school E is held during a twilight session, attended by all five staff members. The numeracy coordinator has prepared the agenda. Following discussion with the senior manager the numeracy coordinator in Secondary school F arranges date, time, location and agenda for the first working group meeting. He also prepares a summary of numeracy performance nationally and within the school using recent Estyn reports, the PISA report and school data from numeracy assessments. The senior manager makes initial contact with those directed to participate in the working group and sends out the agenda and summaries to be read before the meeting.				

B. Team development and support	What this might look like …	**School** *(purpose, lead person, activity, outcome)*	R	A	G
B.1 Review of evidence by the working group leads to a decision about the remit. This might cover some or all of the following aspects of numeracy: • teaching of basic number facts; • teaching of mental and written methods of calculation; • working with fractions, decimals and percentages; • use of mathematical language; • consistent use of calculators; • consistency of teaching; • teachers' own competence and confidence;	Consortium personnel carry out a review of numeracy practice in all primary and secondary schools within an authority. The reviews note strengths and weaknesses of provision. Schools are able to use this feedback to help them decide which areas of practice they need to focus on. Primary school G had just undergone an Estyn inspection. The report comments on the lack of consistency in written methods for routine calculations used in different key stages. The headteacher asks the numeracy working group to investigate further, in order to gather more information about the current situation, and also to decide what actions would need to be taken to bring about improvements. The working group were in the process of developing an action plan outlining how they intended to do this.				

• guidance on good practice.					
B.2 Group agree way of working and responsibilities. A programme of meetings, with milestones, for completing intermediate tasks and producing a report on current activity is drawn up. Members decide to split the task of gathering information. The chair of the working group volunteers to pull the information together into a draft report with recommendations.	In Primary school H the numeracy working group has identified the need to improve the teaching of basic number facts. They decide to gather information about current practice by conducting structured interviews with all teachers to question them about the methods they use and to interview a sample of learners to test their knowledge and attitudes towards numeracy. Different members of the working group are made responsible for designing the two interview questionnaires, selecting the sample of learners and organising a date and time, as well as conducting the interviews. A date is set by which the findings from all research need to be passed on to the person elected to collate the information and write a draft report. Secondary school I want to look at the current situation regarding the consistent use of mental addition and subtraction strategies throughout the school. They decide to observe a selection of cross-curricular lessons and conduct individual interviews with one teacher from each				

	year group. They also select ten learners of mixed ability to interview. Some of the interview questions are designed to test the mental addition and subtraction strategies of the learners.				

C. Getting under way	What this might look like …	**School** *(purpose, lead person, activity, outcome)*	R	A	G
C.1 Working group members provide their information to the chair of the working group in the agreed format. Information about current activity may be gathered from a variety of sources including all or some of the following: • an audit of numeracy strategies currently used across the curriculum; • lesson observations which have a numeracy focus or lesson observations with a specific numeracy focus for this project	The numeracy coordinator in Primary school J decides to review a sample of Year 2 and Year 3 learners' work to evaluate the consistency of teachers' use of 'jottings' used as a method of scaffolding mental calculations. Transition from the Foundation Phase to Key Stage 2 is a stage that requires monitoring to ensure consistency of approach. Initial evidence gathered from Secondary school K has indicated that there are too few opportunities to rehearse strategies for working with decimals and fractions. The numeracy working group decides to conduct an audit across all departments to gain an accurate picture of what is currently happening. They design an audit questionnaire which all departments are asked to complete, to a deadline. The				

(see exemplar lesson observation form Annex 3.5.ii, see page 255); • review of learners' work involving looking at a sample of books; • interview with individual teachers, e.g. curriculum leads, year group teachers or heads of department, to gain clarification about numeracy strategies that they use; • interviews with individual learners during which it is possible to, e.g. ascertain their attitude towards numeracy and ask specific questions to test their use of selected strategies; • summary of relevant guidance on good	group also decides to consider evidence from lesson observations. Although lesson observations carried out to date refer to numeracy content they have not focused on specific teaching strategies, so do not provide any useful information. It is decided that members of the maths department will observe three lessons in each subject area the following term, focusing specifically on this issue. Teachers are invited to participate in the lesson observation exercise.				

practice from Estyn and other sources.					
C.2 Information gathered is collated. (Annex 3.5.iii on page 260 can be used to collate the evidence.)	In Secondary school L the numeracy coordinator collates all questionnaire responses; the evidence from the lesson observations and from secondary written sources in one document.				
C.3 A report is written: • summarising the findings; • evaluating whether the school is teaching the full range of strategies, accurately, effectively and consistently across the curriculum; • identifying good practice.	The chair of the working group in Primary school M collates the feedback from the audit and book review and writes a report. The report concludes there is evidence of a consistent approach as a result of: • the introduction of a comprehensive numeracy booklet which gives detailed outlines of the strategies that can be used in each year group; • accurate use of mathematical language; • effective whole-school training; • portfolios of work showing the use of numeracy strategies across the curriculum.				

D. Keeping up the momentum	**What this might look like …**	**School** *(purpose, lead person, activity, outcome)*	R	A	G
D.1 Working group meets to: • discuss the draft report; • identify what steps need to be taken to improve numeracy teaching strategies; • prepare recommendations, which build on good practice, relating to identified issues, for example: – strategies to use to teach number facts – use of mathematical language – teaching mental and written calculations – staff development.	The working group in Secondary school N discusses the draft report. Having identified weaknesses in numeracy teaching, the group decide to ask the head of the mathematics department to develop proposals for improving numeracy teaching strategies. The working group approve his recommendations, which include: • designing posters and whiteboard resources to introduce the main strategies in the four strands of the NNF; • allocating a set of these posters and CD of the whiteboard resources to each teacher so that identical messages will be received by learners in all classrooms; • arranging a series of INSET days involving practical training and modelling of the use of strategies and appropriate resources; • following the training, individuals to be offered the opportunity to observe good practice within maths lessons.				
D.2 Consultation over the plan takes place.	The plans put forward by the maths department in school O are presented to all teachers and teaching assistants. They are				

• Teachers and teaching assistants are asked to contribute their ideas about the best ways to proceed. • Numeracy coordinator or senior manager presents the plan to the governors for their comments. Amendments made and implementation is the next focus.	invited to comment. A teaching assistant suggests that a bilingual dictionary of mathematical terms would be very useful for staff, learners and parents/carers. This idea was incorporated into the plan. The school governors are informed of the plan and also invited to comment. A parent governor welcomes the plan as she has become very concerned about her son's negative attitude towards numeracy. She believes that the root cause of his attitude is a lack of understanding of basic concepts.				
D.3 Plans are made to implement recommendations. These could include: • the development of materials to ensure understanding of approaches tonumeracy by non-specialist teachers and consistency across the curriculum (see Annex 3.5.iv, page 262); • addressing any	Cluster group P decides that they will work together to develop materials to help ensure consistency in teaching strategies for number across all their schools. Numeracy coordinators and a member of the mathematics department of all the schools in the cluster meet to agree the best strategies. They produce a booklet which provides step-by-step explanations of the strategies chosen. The booklet is available in paper form for all staff and learners and is placed on the intranet of all the schools so that it is accessible for parents. Promotion of the booklet takes place through:				

identified training needs; • sharing of good practice.	• training for staff; • a numeracy fun day for learners which offers opportunities to practise the skills; • a presentation at a parents' evenings supported by a leaflet which is sent to all homes. In Primary school Q, several TAs have noted in an informal discussion with the SENCO that they would welcome support to develop their data skills. It is decided to allocate time to enable TAs to undertake a series of training sessions. The content is shared with teaching staff so they can use the TAs' knowledge effectively.				

E. Progress so far	**What this might look like …**	**School** *(purpose, lead person, activity, outcome)*	R	A	G
E.1 Training is delivered to all staff. Research has shown that continuing professional development has more influence on practice than one-day training. The best training model would be a package delivered over several	Primary school R identifies the need for consistency in the methods used for addition, subtraction, multiplication and division. Continuing professional development is arranged over three extended twilight sessions over two school terms. Model of possible training: **Session 1** Importance of consistency is explained and discussed				

sessions, e.g. twilight or training days which would involve: • presentation of information; • intersessional task(s) involving trialling some aspect of the work; • feedback session(s).	Modelling of addition and subtraction methods **Intersessional task** is set, e.g. trialling methods of addition and/or subtraction in class before the next session **Session 2** Feedback given on intersessional task Modelling of multiplication and division methods **Intersessional task** is set, e.g. trialling methods of multiplication and/or division before the next session **Session 3** Feedback given on intersessional task. Topics introduced for discussion with the view of arriving at outcomes which inform practice are: – identify the most important factors for ensuring consistency – how can consistency in use of strategies across the curriculum be monitored? – how can the effect of using consistent strategies, across the school, on learner outcomes be evaluated?				

E.2 Agree when and how strategies implemented should be monitored, for example: • review use of strategies on agreed date i.e. repeat audit/lesson observations/review of learners' work; • collate information and arrive at conclusions. Agree when and how strategies implemented should be evaluated for effectiveness, for example: • track learners' progress in relevant strands of the LNF to determine impact of strategies on learner outcomes.	Primary school S has addressed the issue of a lack of consistency in mental calculation strategies and the use of calculators by: • producing and distributing support materials; • delivering a series of training sessions to all staff. Monitoring and evaluation, planned for six months after the training, comprises lesson observations and interviews with learners. Secondary school T decides to evaluate the impact of the introduction of mental calculation strategies by investigating the progress made by learners who regularly practise them in activities across the curriculum. The numeracy coordinator identifies a target group and records the group members' performance in the relevant strand of the NNF before the strategies are implemented. The coordinator tracks the group's progress in these strands on two occasions during the academic year following implementation of the strategies.				

Annex 3.5: Learning and teaching: numeracy

3.5.i: Numeracy policy: suggested structure

	School notes
Introduction • Why numeracy is a priority in this school. • Why now? • Relationship to the SEF. • Relationship to policies for mathematics.	Brief introduction which gives an indication of why the school is reviewing its policy.
Definition of numeracy Numeracy is an essential life skill and learners need to be able to apply this skill across the curriculum in different subject areas, and in real life contexts. Mathematics is a part of numeracy, but to be numerate means you are able to apply some of these mathematical skills in many more contexts than in mathematics lessons and across several subject areas. It is therefore our expectation that all teachers will be teachers of numeracy. To help distinguish between numeracy and mathematics, we have defined numeracy in the following way for the purposes of this programme: 'Identifying and applying numerical reasoning skills in order to solve a problem, and carrying out the numerical procedures which enable people to work out and show their solutions.' (*National Numeracy Programme*, September 2012)	School adapts this for its aims, giving emphasis needed.

Aims	
• To ensure that numeracy is a priority within the school. • To foster a positive attitude towards numeracy. • To help learners to become confident in numeracy and able to apply and communicate their results across the curriculum and in real life. • To make sure that learners master basic number skills thoroughly and have effective strategies to recall essential number facts quickly and accurately. • To give learners a firm basis of knowledge and skills so that they are numerate, able to participate in numerical discussions and able to work flexibly and logically. • To encourage learners to work as members of a group and to find appropriate strategies. • To develop a consistent approach across all key stages and subject areas to teaching basic calculation strategies, developing learners' mental calculation skills, efficient written methods for routine calculations and appropriate use of calculators. • To provide more opportunities for learners to use numeracy skills, particularly number skills and numerical reasoning in subjects across the curriculum. • To ensure activities offered across the curriculum support consistency and progression in learners' numeracy skills. • To make sure that numeracy activities across the curriculum are suitably challenging for all learners. • To effectively assess and track a learners' progress in numeracy skills.	Key aims for the LNF can be found on page 4 of the Welsh Government's *National Literacy and Numeracy Framework* (January 2013). School aims for its learners.

Roles and responsibilities What each of these does: • the Senior Leadership Team • numeracy coordinators • heads of departments/ subject coordinators • Outstanding Teachers of Numeracy • teachers • governors • parents/carers.	School makes clear how responsibility for numeracy is embedded across the school. The Welsh Government's *Curriculum planning guidance* (January 2013) outlines the roles and responsibilities for: • senior managers (see page 24); • numeracy coordinators or teams (pages 24–25); • heads of departments/subject coordinators (page 25); • all teachers (page 26). **Governors should:** • be fully aware of the NNF; • appoint a governor to have specific responsibility for numeracy; • be clear of the differences between numeracy and mathematics and change the negative thoughts and feelings that some may have towards numeracy; • be fully aware of the need to improve learners numeracy levels, and of the benefits of doing so; • be aware of the strategies and resources used to promote the effective implementation of the NNF; • attend some of the whole-school training days devoted to numeracy; • be provided with opportunities to observe good practice in the delivery of numeracy across the curriculum; • contribute to the school's efforts to inform parents and involve them in their children's' learning of numeracy. **Parents/carers should:** • be aware of the importance of improving their children's numeracy skills and of the benefits of doing so; • be clear on the differences between numeracy and mathematics and change the negative thoughts and feelings that some may sometimes have towards numeracy; • be aware of the strategies and resources used to promote better understanding of numeracy; • be aware of the school point of contact for numeracy and be encouraged to attend sessions to receive guidance for home delivery.

Numeracy across the curriculum The numeracy component of the LNF offers a set of objectives that are relevant across the curriculum and are to be embedded in all subjects. Consistency in teaching numeracy will enable learners to develop strategies and be able to adapt them to different contexts. ***Developing numerical reasoning*** This strand focuses on three elements: • Identify processes and connections; • Represent and communicate; • Review. ***Using number skills*** • Use number facts and relationships; • Fractions, decimals, percentages and ratio; • Calculate using mental and written methods; • Estimate and check; • Manage money. ***Using measuring skills*** • Length, weight/mass, capacity; • Time; • Temperature; • Area and volume; angle and position. ***Using data skills*** • Collect and record data; • Present and analyse data; • Interpret results.	School covers these areas, giving brief indication of the school approach. This is to ensure: • common understanding of the aspects of numeracy; • consistent teaching strategies and approaches are used by all staff.
Curriculum planning • Differentiation – sufficient challenge for all learners; • Consolidation – learners can confidently adapt and independently use relevant numeracy skills; • Progression – all learners reach or exceed the expectations; • Across the curriculum – common approaches.	School provision which maximises all learning of numeracy.

Assessment, recording and reporting Assessment **for** learning. • How tracking of individual learners' progress, marking of work and recording are undertaken. • How this data, together with data from national tests, is used to support learners' progress. Assessment **of** learning. • How judging against expectations is carried out and when. • How progress is reported to parents/carers.	School approaches and systems outlined.
Monitoring and evaluating implementation • Identifying impact and areas for further work. • Deciding action plan for this.	
Training The quality of teaching has been identified as a key to success for learners. All teachers across the curriculum should aim to become accomplished teachers of numeracy. To achieve this they need to ensure: • their own knowledge of numeracy enables them to teach their learners effectively; • a greater understanding of different approaches to teaching numeracy; • consistency in the use of mathematical language, methods and strategies across the school. This section should refer to: • identifying training priorities and how these will be met, e.g. sources of finance, time for training, providers of training; • the action plan for delivery of training.	

Timeline Plan for implementation of teaching, assessing and evaluating progress in numeracy across all subjects.	Welsh Government's 'Timeline of implementation and support' can be found on page 10 of the *Curriculum planning guidance* (January 2013).

3.5.ii: Lesson observation form

Purpose

The sample observation form that follows can be used:

- by individual teachers to reflect on their practice;
- for voluntary peer observation and feedback ;
- by volunteer teachers wishing to contribute to audits of teaching strategies in their school.

Instructions for use

This form has three main sections.

- **Section 1:** For recording details of the lesson, teacher and learners being observed.
- **Section 2:** To record a brief overview of the lesson objectives, numeracy objectives, content of the lesson and an overview of the numeracy observed.
- **Section 3:** Enables reference to specific numeracy activities, considered good practice, which may have occurred during the lesson.

The observer should provide constructive feedback to the teacher on the lesson observed either at the end of the lesson or at another mutually convenient time.

Lesson observation form
Focus: Numeracy
SECTION 1
Subject:
Year group:
Class:
Date of lesson:
Time of lesson:
Teacher observed:
Number of learners:
SECTION 2
Summary of lesson objectives:
Summary of numeracy objectives:
Outline of lesson content:
Outline of numeracy observed:

SECTION 3[64]		
Which of the following occurred?	**Y/N**	**Briefly outline activity**
1. Was reference made to other lessons where numeracy skills used would have been covered?		
2. Did the teacher explain mathematical concepts when skills had not been previously covered?		
3. Were questions asked to assess learners' understanding of mathematical terms?		
4. Were mathematical terms explained and used consistently?		
5. Were learners required to work out simple calculations mentally?		
6. Were calculators used appropriately?		

[64] Questions in Section 3 are derived from the Basic Skills Cymru *How to … review basic skills learning and teaching provision. A guide for senior managers* (Welsh Assembly Government, 2009).

7. When calculators were used was guidance given and checks made of learners' understanding?		
8. Were learners encouraged to estimate answers prior to working out a calculation and use estimate to check answer?		
9. Were learners permitted to use a range of calculation methods?		
10. How were all learners encouraged to answer questions, e.g. use of whiteboards or number fans?		
11. Were resources used to support the learning of mathematical concepts?		
12. Were learners asked to explain their methods and ways of working?		
13. If difficulties were encountered were strategies offered to help overcome them?		

14. Were numeracy lesson starters used to help prepare the learners for lessons involving significant numeracy?		
15. Was the use of measuring equipment modelled?		
16. Was there a focus on suitable units of measure including conversion of units?		
17. Was support provided to construct graphs, diagrams, charts and tables?		
18. Was support provided to interpret graphs, diagrams, charts and tables?		

Observed on: **by:**

Signature of observer:

Feedback given to: **on:**

Signature of teacher:

3.5.iii: Collating evidence about numeracy teaching

Purpose

This template can be used to collate findings from the research by the numeracy working group. These will form the basis of a report for discussion.

Instructions for use

- Note key bullet points from each source in the first part of the document.
- List key issues and a summary of good practice identified in Part 2.
- The writer may choose to suggest some draft recommendations or wait for them to emerge as part of the working group's discussions.

Part 1	
Source of evidence	**Findings**
For example, External written sources	
For example, School audit	
For example, Lesson observation	
For example, Review of learners' books	
For example, Individual interviews	

Part 2
Report
Key issues
Examples of good practice
Recommendations

3.5.iv: Resources for addition, subtraction, multiplication and division

This annex contains four parts dealing with teaching mental and written methods of addition, subtraction, multiplication and division. The parts can be used as follows.

- As the starting point for a discussion within a numeracy working group or whole-school staff meeting when consideration is being given to improving consistency of mental and written methods for the four operations within the school.
- As the basis for development of the section on the four operations in a school numeracy booklet. The booklet could be made available in hard copy format or placed on the school internet or both. Decisions would need to be taken as to the most accessible and effective format for learners, teachers, teaching assistants and parents/carers. An alternative approach to a numeracy booklet would be to include age-appropriate extracts in learners' homework diaries for reference.
- To design posters. Displaying posters of consistent design and consistent content, where appropriate, throughout a school can help learners make connections between numeracy taught in different year groups or different subjects.
- As exemplar material for a series of INSET sessions for all staff. To ensure that the sessions were interactive, discussion points and practical exercises would need to be added.
- As a basis for annotating examples of learners' work, linking them to the age-related expectation statement.

Portfolios of these annotated examples of learners' work can be used to demonstrate good practice and to help ensure consistency in age-related expectations across the school.

Addition

This annex includes:

1. a brief overview of the most common mental calculation strategies for addition;

2. a step-by-step overview of a route for the development of addition methods from simple mental calculations involving pictures, number lines and practical resources to the written method of addition involving 'carrying'.

This route provides appropriate strategies for a range of addition sums from two one-digit numbers to the addition of a list of numbers with any number of digits as well as numbers with up to two decimal places.

Common mental calculation strategies

Mental recall of number bonds

6 + 4 = 10 $\square$ + 3 = 10

25 + 75 = 100 19 + $\square$ = 20

Use near-doubles

6 + 7 = double 6 + 1 = 13

Add using partitioning and recombining

34 + 45 = (30 + 40) + (4 + 5) = 79

Count on or back in repeated steps of 1, 10, 100, 1000

86 + 57 = 143 (by counting on in tens and then in ones)

460 – 300 = 160 (by counting back in hundreds)

Add the nearest multiple of 10, 100 and 1000 and adjust

24 + 19 = 24 + 20 – 1 = 43

458 + 71 = 458 + 70 + 1 = 529

Use the relationship between addition and subtraction

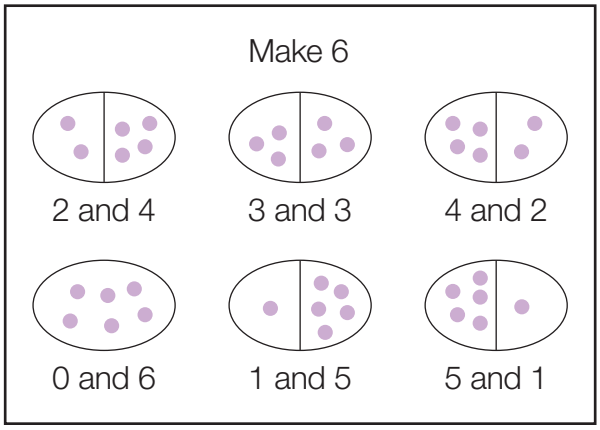
36 + 19 = 55 19 + 36 = 55

55 – 19 = 36 55 – 36 = 19

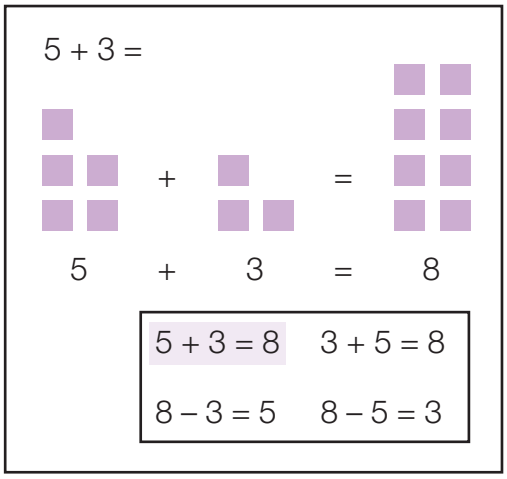
Many mental calculation strategies will continue to be used. They are not replaced by written methods.

Step 1

Learners are encouraged to develop a mental picture of the number system in their heads to use for calculation.

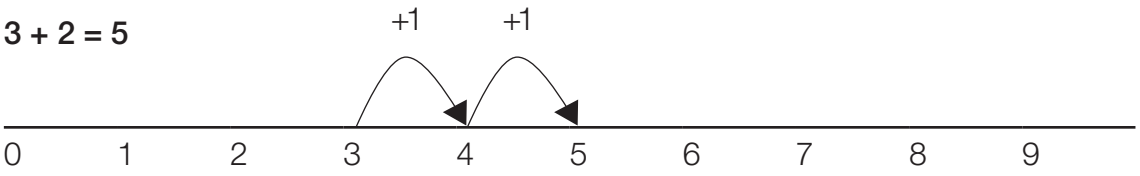


They develop ways of recording calculations using pictures, etc.



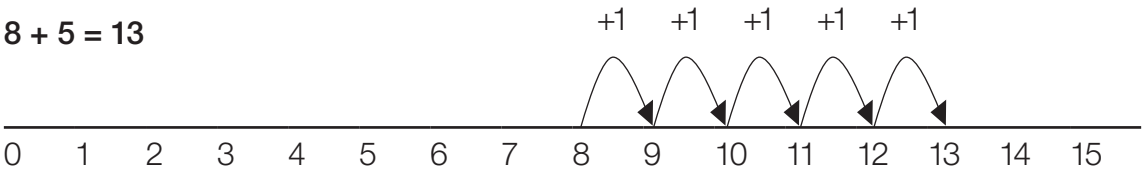
They use number lines and practical resources to support calculation and teachers **demonstrate** the use of the numberline.

3 + 2 = 5



Learners then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

8 + 5 = 13



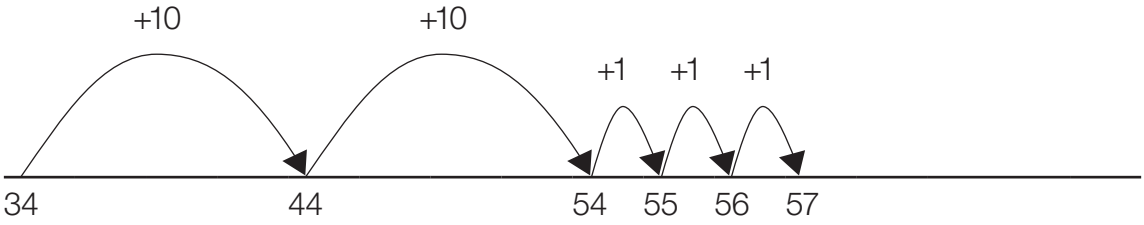
Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

Step 2

Learners will begin to use 'empty number lines' themselves, starting with the larger number and counting on.

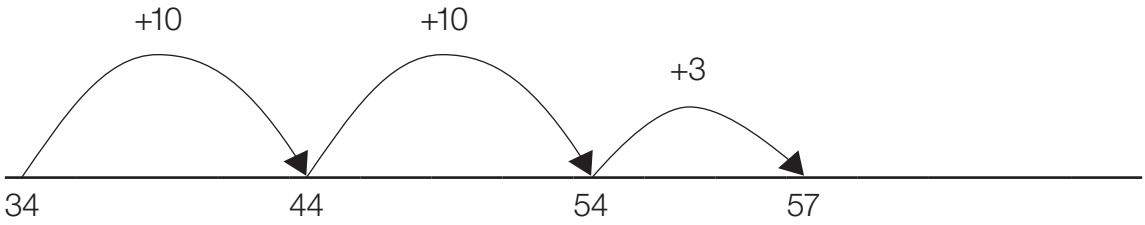
- First counting on in tens and ones.

34 + 23 = 57



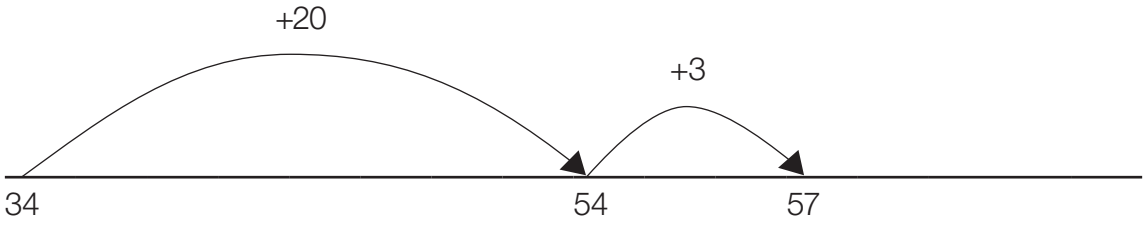
- Then helping learners to become more efficient by adding the units in one jump (by using the known fact 4 + 3 = 7).

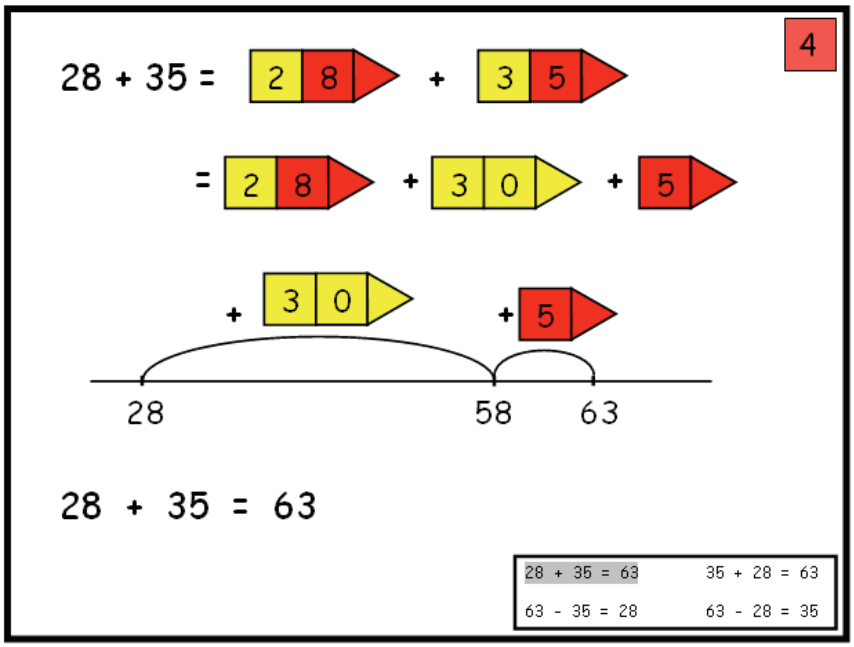
34 + 23 = 57



- Followed by adding the tens in one jump and the units in one jump.

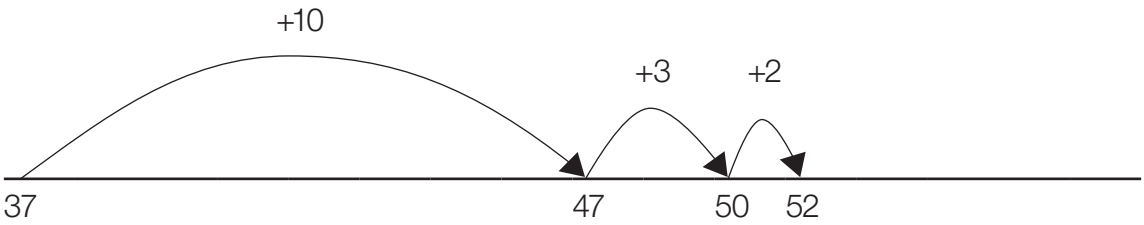
34 + 23 = 57





- Bridging through ten can help learners become more efficient.

37 + 15 = 52

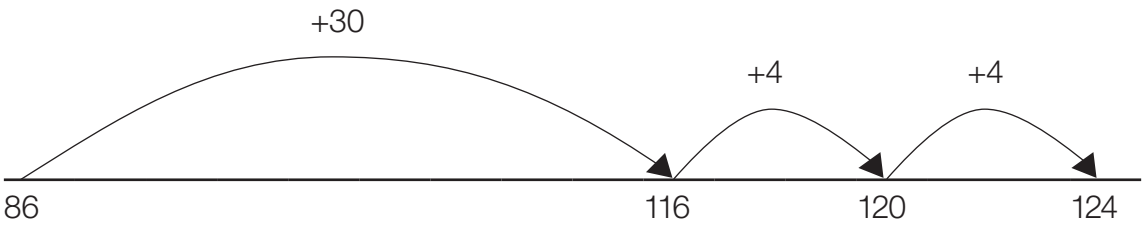


Step 3

Learners will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

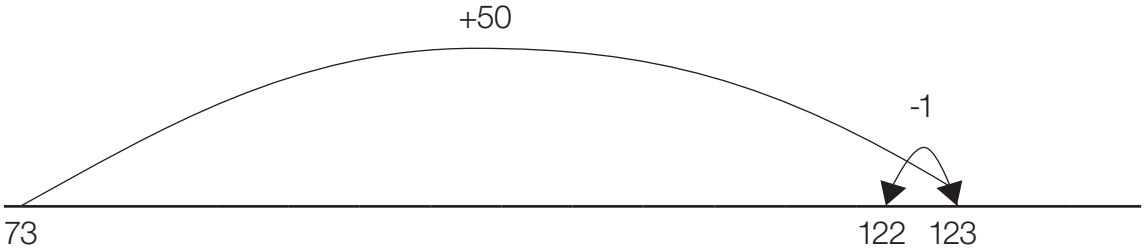
- Count on from the largest number irrespective of the order of the calculation.

38 + 86 = 124



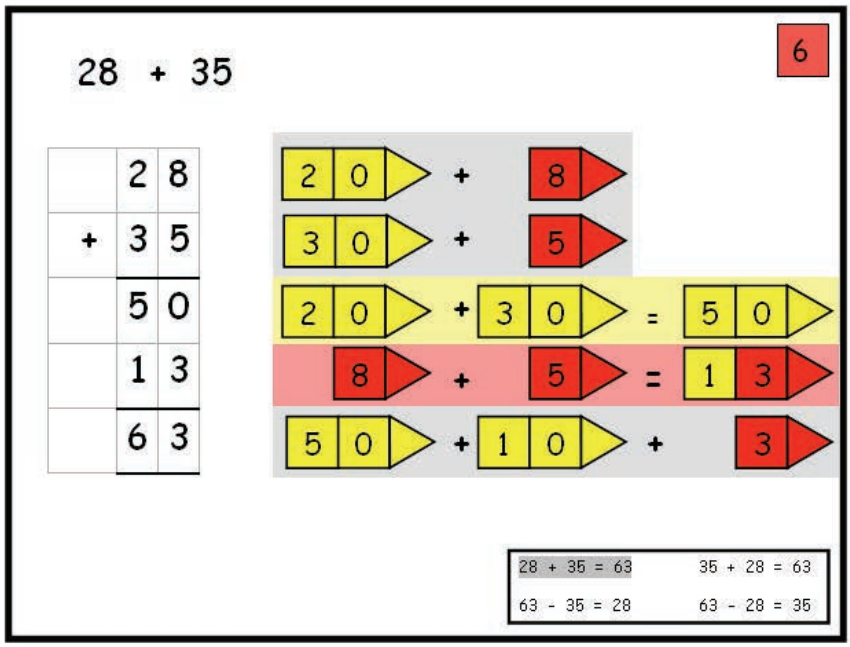
- Compensation

49 + 73 = 122

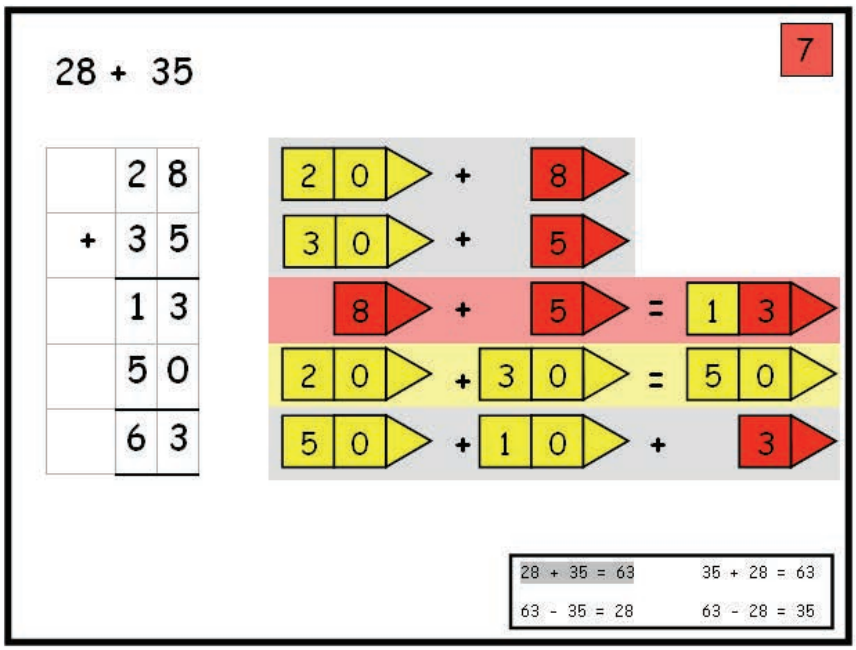


Learners will begin to use informal pencil and paper methods (jottings) to support record and explain partial mental methods building on existing mental strategies.

Phase 1: Adding most significant digits first, and then moving to adding least significant digits.



Phase 2: Moving to adding the least significant digits first in preparation for 'carrying'.

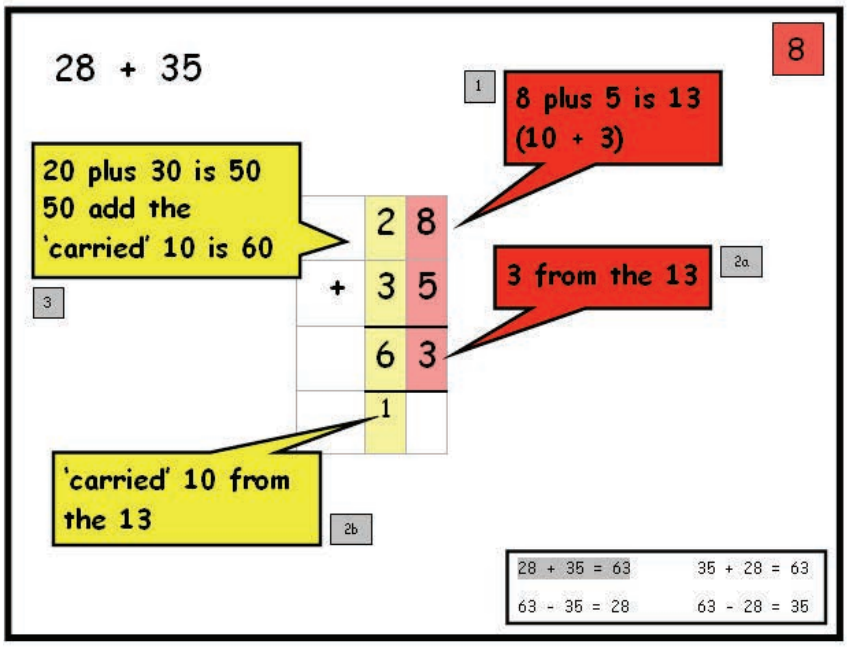


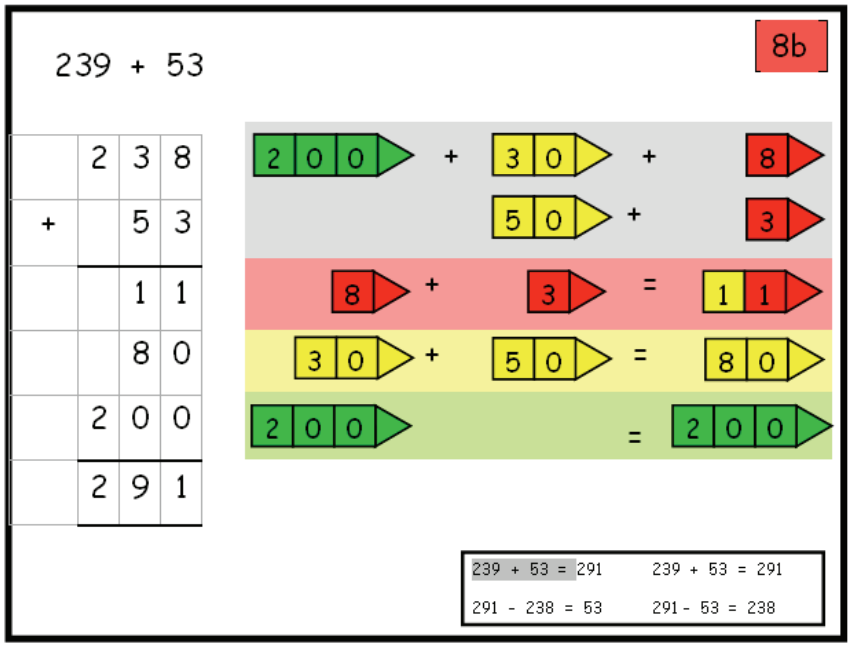
Adding the least significant digits first, without using place value cards.

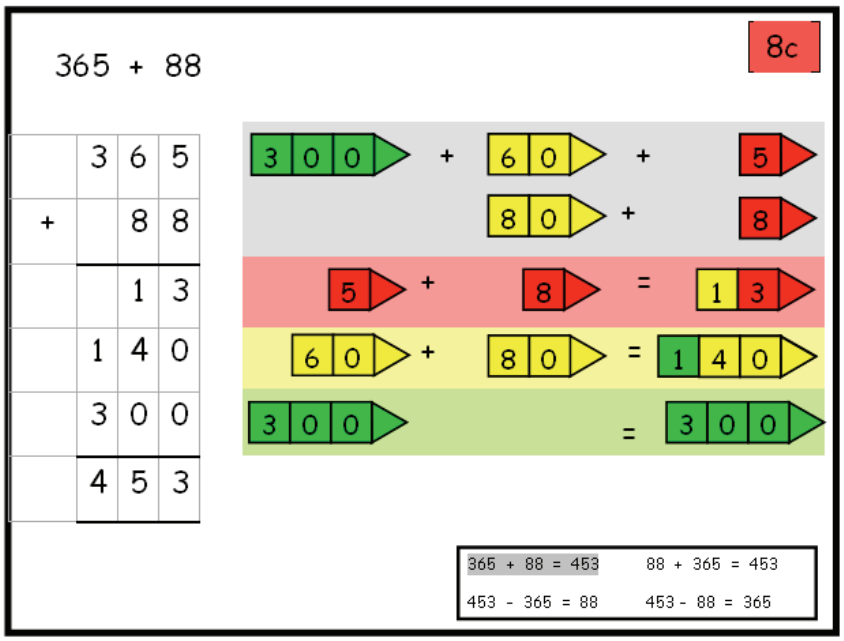
	2	8	
	3	5	
+	1	3	(8 + 5)
	5	0	(20 + 30)
	6	3	

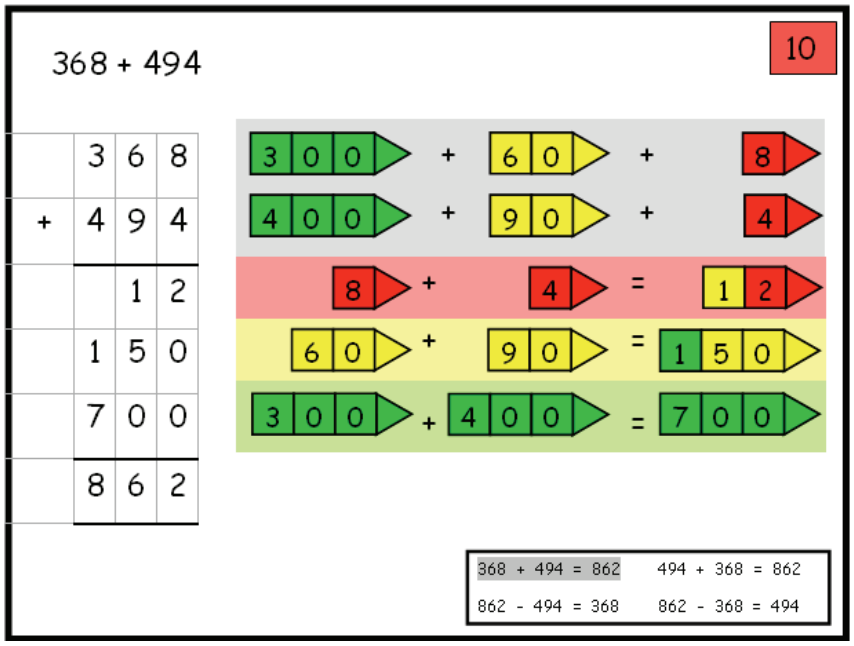
Step 4

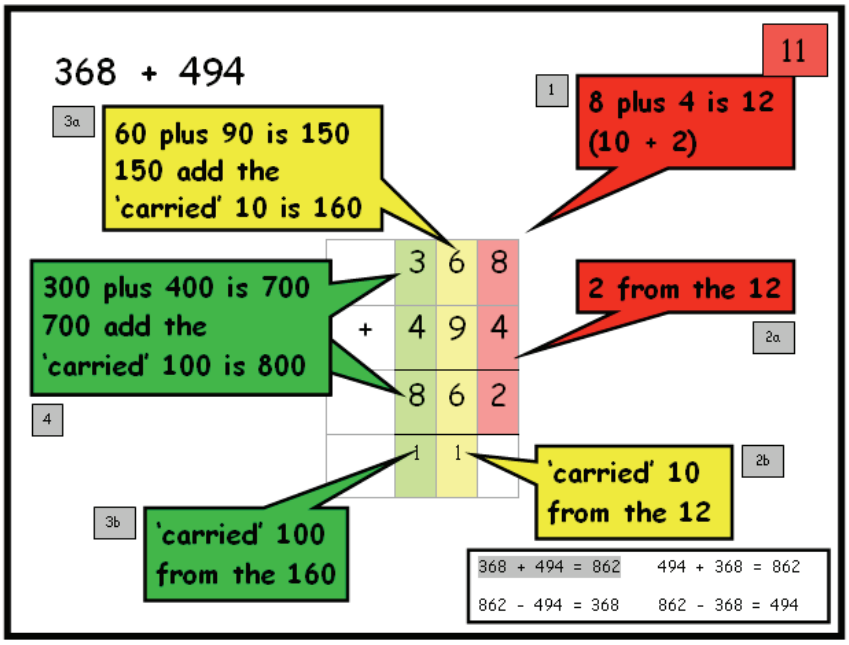
From this, learners will begin to 'carry' below the line, using the correct mathematical vocabulary.




239 + 53
8b
2 3 8
+ 5 3
1 1
8 0
2 0 0
2 9 1
200 + 30 + 8
50 + 3
8 + 3 = 11
30 + 50 = 80
200 = 200
239 + 53 = 291
239 + 53 = 291
291 - 238 = 53
291 - 53 = 238


365 + 88
8c
3 6 5
+ 8 8
1 3
1 4 0
3 0 0
4 5 3
300 + 60 + 5
80 + 8
5 + 8 = 13
60 + 80 = 140
300 = 300
365 + 88 = 453
88 + 365 = 453
453 - 365 = 88
453 - 88 = 365





Using similar methods, learners will:

- *add several numbers with different numbers of digits;*
- *begin to add two or more three-digit sums of money, with or without adjustment*

from the pence to the pounds;

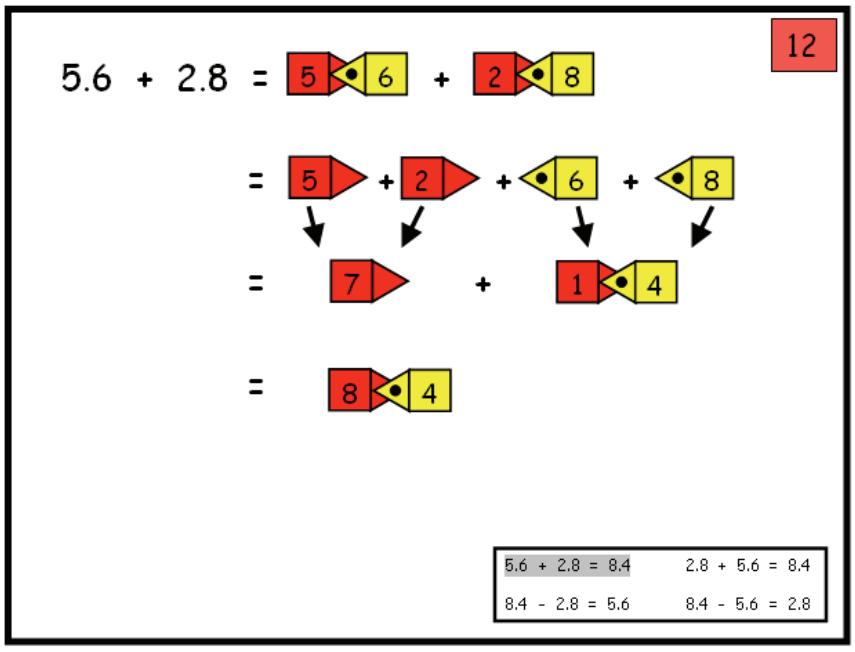
- *know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.*

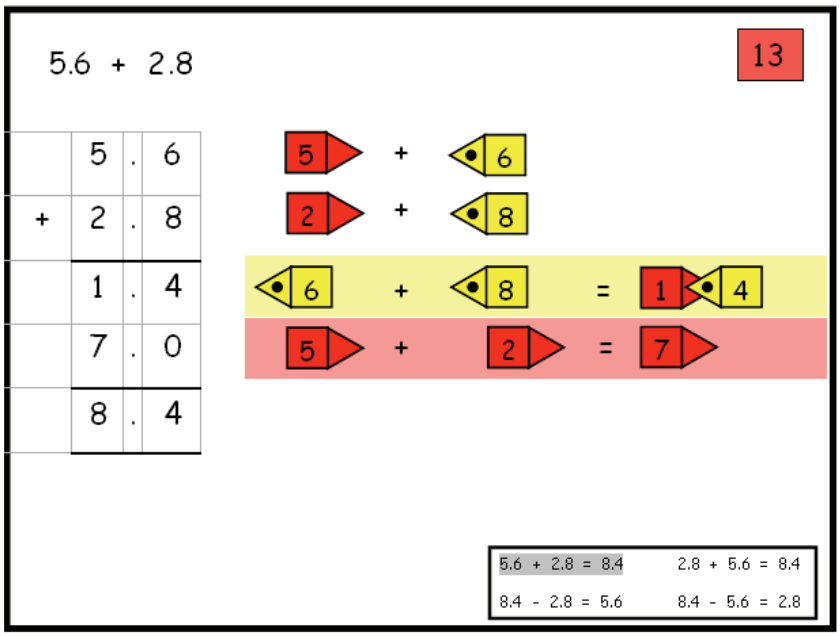
Step 5

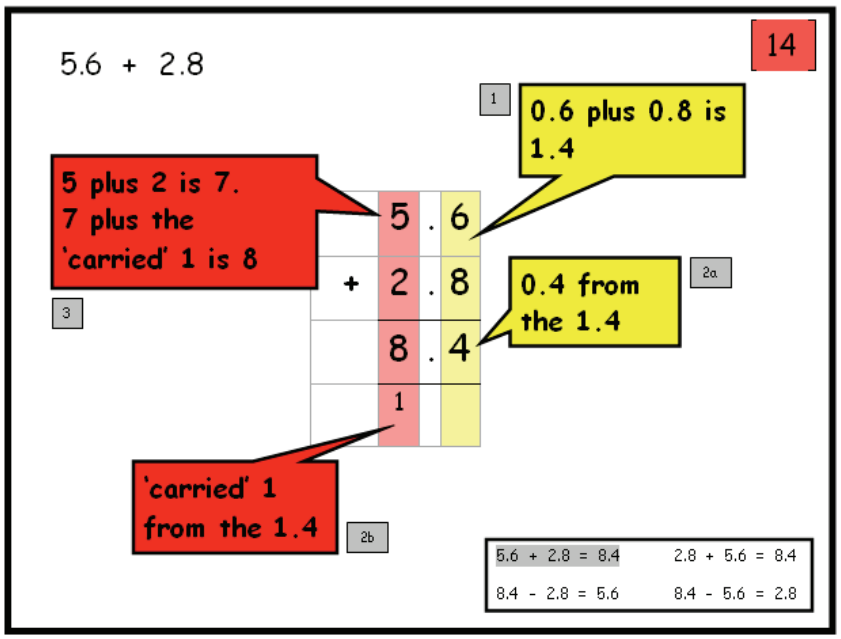
Learners should extend the 'carrying' method to numbers with at least four digits.

587	3587
+ 475	+ 675
1062	4262
11	111

Learners should extend their methods to decimal numbers.







Using similar methods, learners will:

- *add several numbers with different numbers of digits;*
- *begin to add two or more decimal fractions with up to three digits and the same number of decimal places;*

- *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m – 280 cm.*

Step 6

Learners should extend the 'carrying' method to numbers with any number of digits.

7648	6584	42
+ 1486	+ 5848	6432
9134	12432	786
111	111	3
		+ 4681
		11944
		121

Using similar methods, learners will

- *add several numbers with different numbers of digits;*
- *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;*
- *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 401.2 + 26.85 + 0.71.*

Learners need to have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.
Learners should not be made to go on to the next stage if:

1. they are not ready;
2. they are not confident.

Learners should be encouraged to approximate their answers before calculating.
Learners should be encouraged to check their answers after calculation, using an appropriate strategy.
Learners should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Subtraction

This annex includes:

1. a brief overview of the most common mental calculation strategies for subtraction;

2. a step-by-step overview of a route for the development of subtraction methods from simple mental calculations involving pictures, number lines and practical resources to a written method using decomposition.

This route provides appropriate strategies for a range of subtraction sums from two one-digit numbers to the subtraction of two numbers with any number of digits, as well as numbers with up to two decimal places.

Mental calculations

Mental recall of addition and subtraction facts

10 – 6 = 4 17 – ☐ = 11

20 – 17 = 3 10 – ☐ = 2

Find a small difference by counting up

82 – 79 = 3

Count on or back in repeated steps of 1, 10, 100, 1000

86 – 52 = 34 (by counting back in tens and then in ones)

460 – 300 = 160 (by counting back in hundreds)

Subtract the nearest multiple of 10, 100 and 1000 and adjust

24 – 19 = 24 – 20 + 1 = 5

458 – 71 = 458 – 70 – 1 = 387

Use the relationship between addition and subtraction

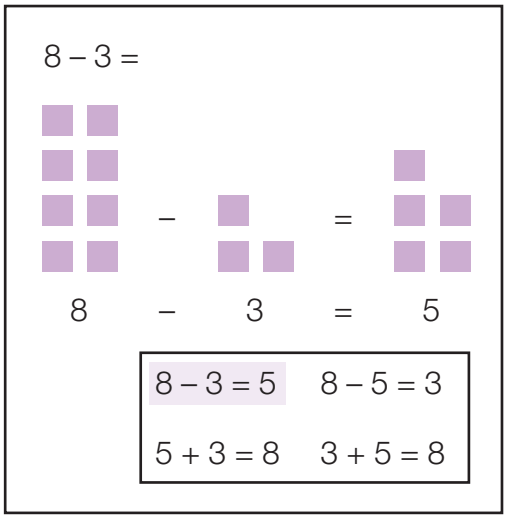
36 + 19 = 55 19 + 36 = 55

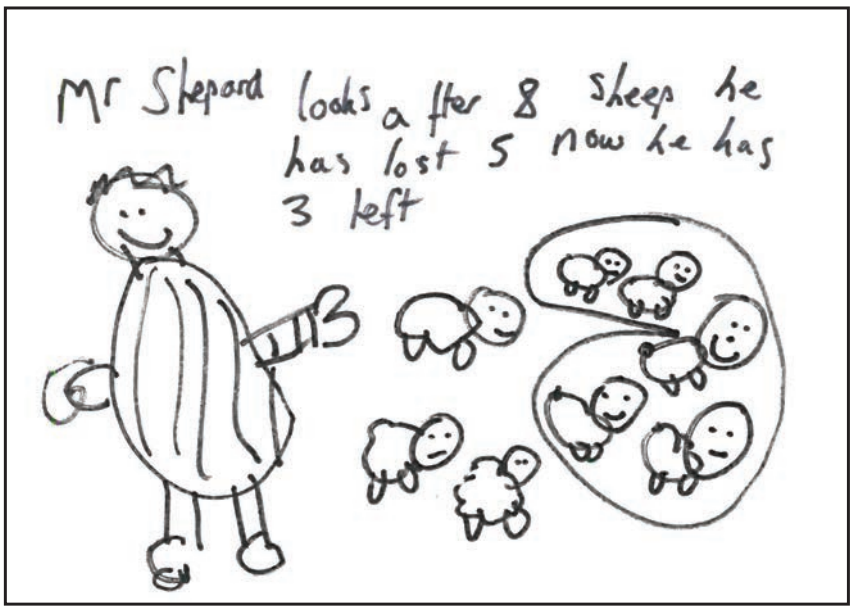
55 – 19 = 36 55 – 36 = 19

Many mental calculation strategies will continue to be used. They are not replaced by written methods.

Step 1

Learners are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.

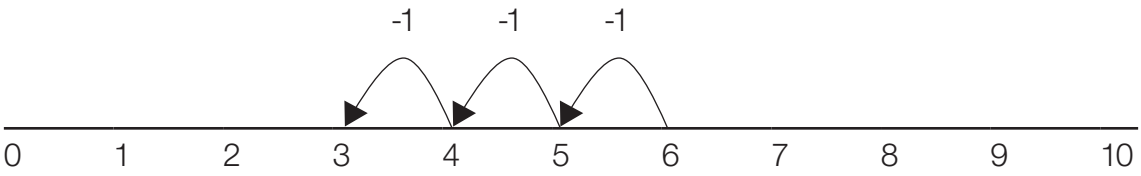




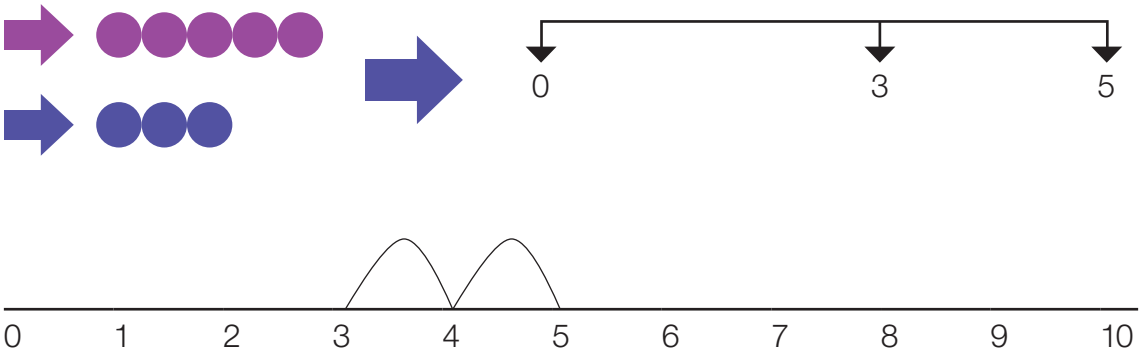
They use number lines and practical resources to support calculation.

Teachers **demonstrate** the use of the number line.

6 – 3 = 3

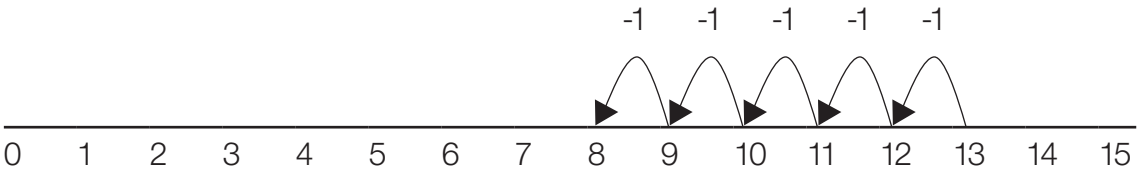


The number line should also be used to show that 5 – 3 means the 'difference between 5 and 3' or 'the difference between 3 and 5' and how many jumps they are apart.



Learners then begin to use numbered lines to support their own calculations, using a numbered line to count back in ones.

13 – 5 = 8



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

13 – 5 = 8

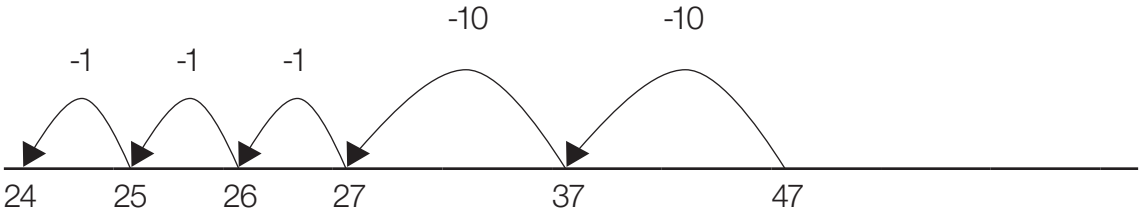
Step 2

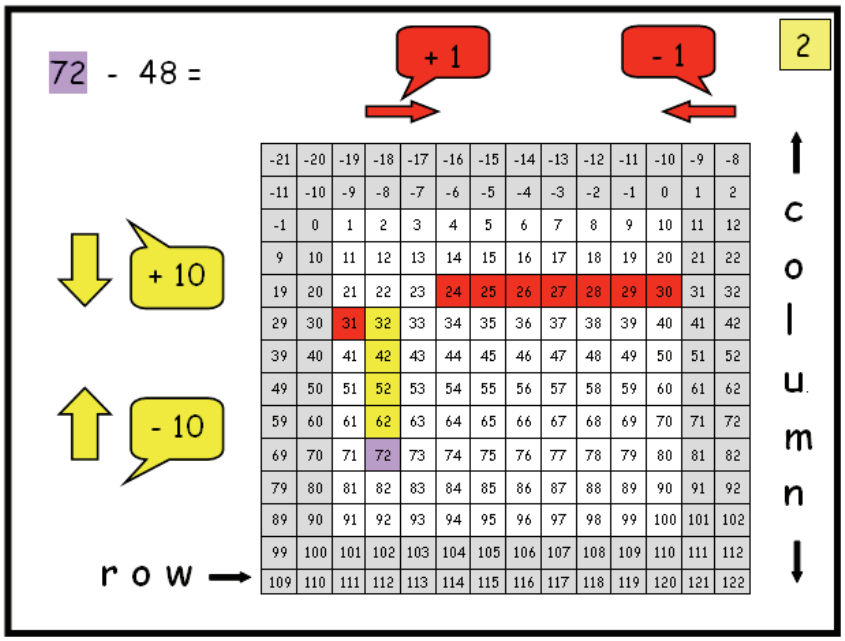
Learners will begin to use extended hundred squares and empty number lines to support calculations.

Counting back

- First counting back in tens and ones.

47 – 23 = 24

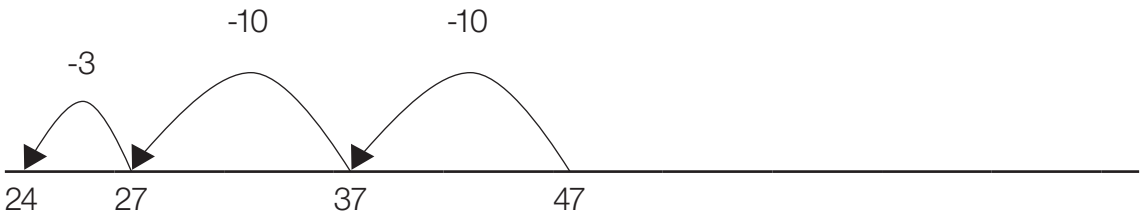




-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8
-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2
-1	0	1	2	3	4	5	6	7	8	9	10	11	12
9	10	11	12	13	14	15	16	17	18	19	20	21	22
19	20	21	22	23	24	25	26	27	28	29	30	31	32
29	30	31	32	33	34	35	36	37	38	39	40	41	42
39	40	41	42	43	44	45	46	47	48	49	50	51	52
49	50	51	52	53	54	55	56	57	58	59	60	61	62
59	60	61	62	63	64	65	66	67	68	69	70	71	72
69	70	71	72	73	74	75	76	77	78	79	80	81	82
79	80	81	82	83	84	85	86	87	88	89	90	91	92
89	90	91	92	93	94	95	96	97	98	99	100	101	102
99	100	101	102	103	104	105	106	107	108	109	110	111	112
109	110	111	112	113	114	115	116	117	118	119	120	121	122

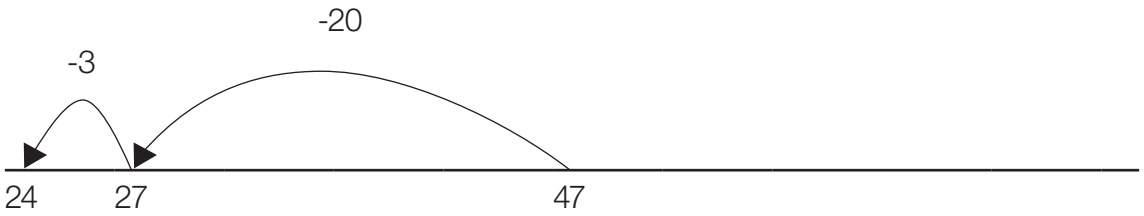
- Then helping learners to become more efficient by subtracting the units in one jump (by using the known fact 7 – 3 = 4).

47 – 23 = 24



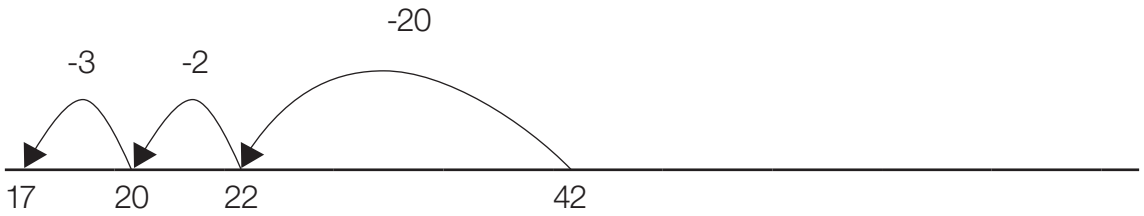
- Subtracting the tens in one jump and the units in one jump.

47 – 23 = 24



- Bridging through ten can help learners become more efficient.

42 – 25 = 17



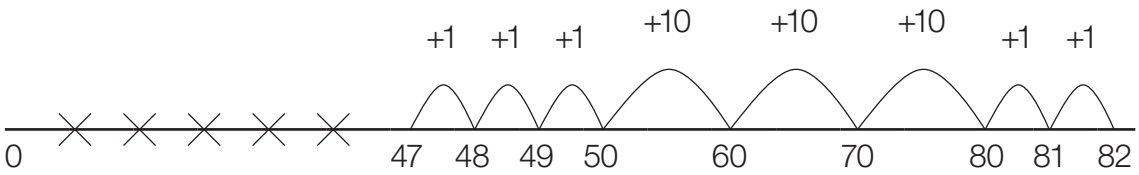
Counting on

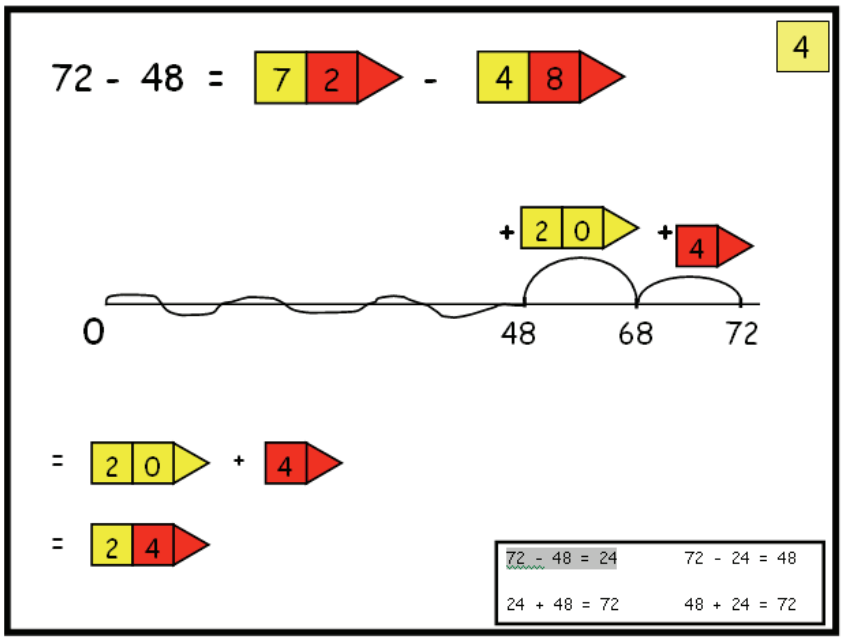
If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so learners can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

82 – 47





Help learners to become more efficient with counting on by:

- subtracting the units in one jump;
- subtracting the tens in one jump and the units in one jump;
- bridging through ten.

Step 3

Learners will continue to use empty number lines with increasingly large numbers.

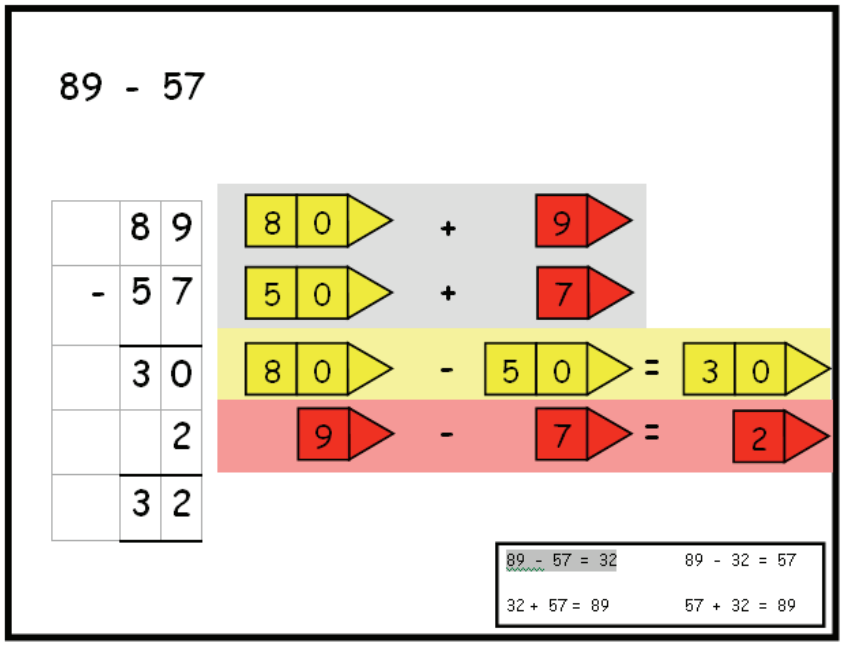
Learners will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods, building on existing mental strategies.

Partitioning and decomposition

This process should be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

NOTE When solving the calculation 89 – 57, learners should know that 57 **does NOT EXIST AS AN AMOUNT**, it is what you are subtracting from the other number. Therefore, when using resources, learners would need to count out only the 89.

89	=	80 + 9
- 57		50 + 7
		30 + 2 = 32



Initially, the learners will be taught using examples that do not need the learners to exchange.

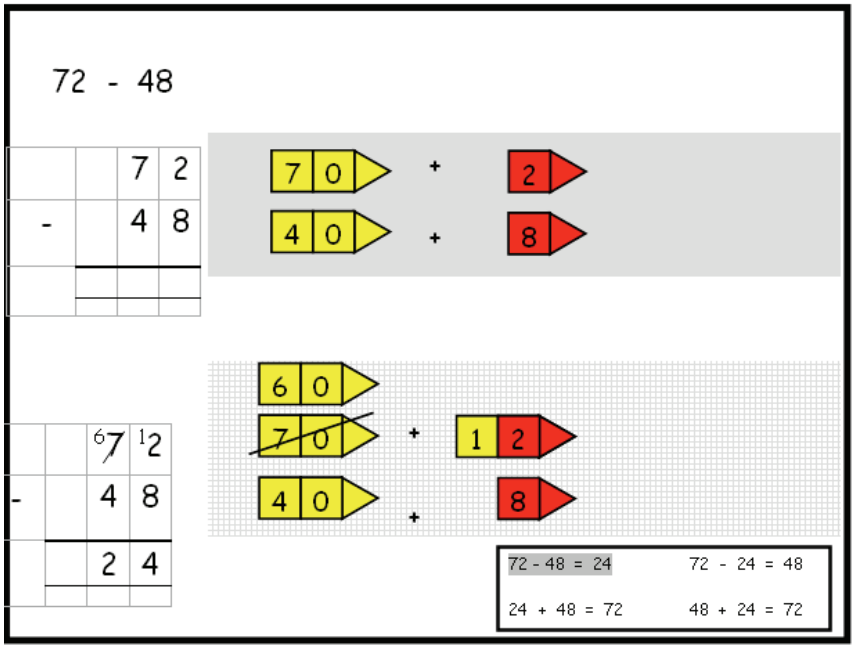
From this the learners will begin to exchange.

72 (The calculation should be read as, e.g. take 8 from 2.)
- 48

Stage 1	**Stage 2**
70 + 2	60 + 12
- 40 + 8	- 40 + 8
	20 + 4 = 24

This would be recorded by the learners as:

60 1
~~70~~ + 2
- 40 + 8
20 + 4 = 24



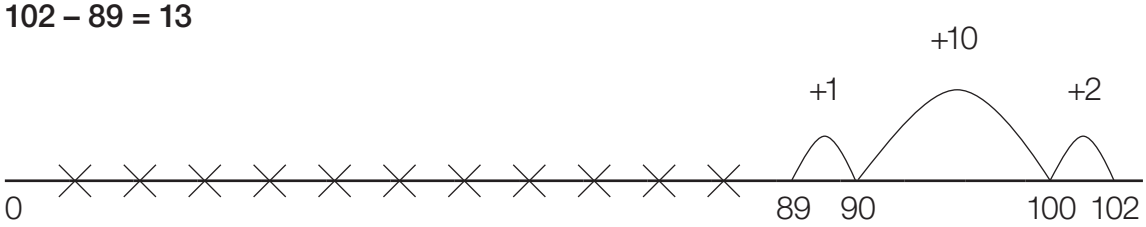
Learners should know that units line up under units, tens under tens, and so on.

If the teaching staff feel that the use of addition signs within a subtraction calculation will cause confusion, then they can be replaced with arrows, as in the example below. This needs to be agreed as part of the whole-school policy and applied consistently throughout the school.

$$\begin{array}{rcl} 89 & = & 80 \rightarrow 9 \\ -\ 57 & & 50 \rightarrow 7 \\ \hline & & 30 \rightarrow 2 \end{array} = 32$$

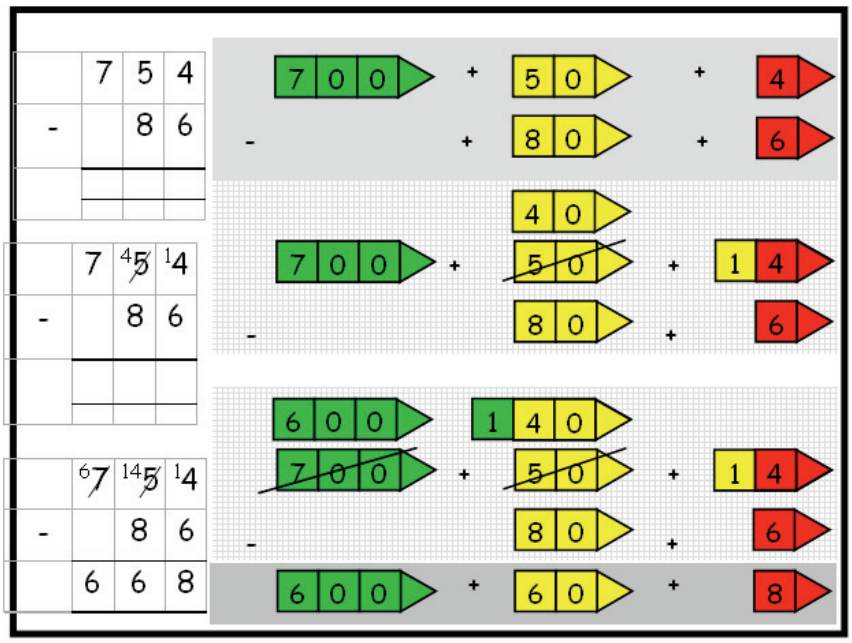
Where the numbers involved in the calculation are close together or near to multiples of 10, 100, etc, counting on using a number line should be used.

102 – 89 = 13



Step 4

Partitioning and decomposition



754
− 86

Stage 1 700 + 50 + 4
− 80 + 6

Stage 2 700 + 40 + 14 *(adjust from T to U)*
− 80 + 6

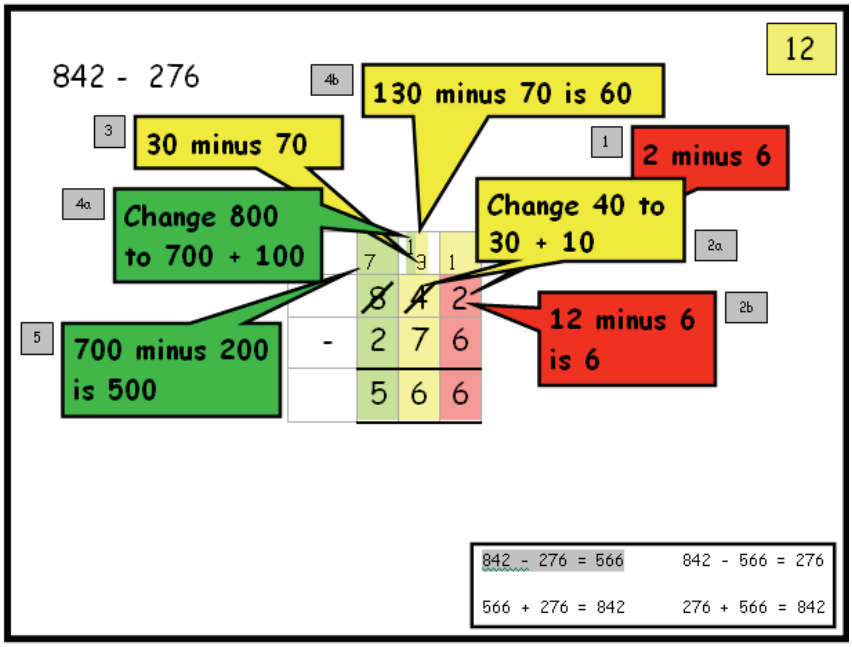
Stage 3 600 + 140 + 14 *(adjust from H to T)*
− 80 + 6
600 + 60 + 8 = 668

This would be recorded by the learners as:

$$\begin{array}{rr} & \begin{array}{rrr} 600 & 140 & 1 \end{array} \\ & \cancel{700} + \cancel{50} + 4 \\ - & 80 + 6 \\ \hline & 600 + 60 + 8 \end{array} = 668$$

Decomposition

$$\begin{array}{rr} & \begin{array}{r} 7131 \end{array} \\ & 842 \\ - & 276 \\ \hline & 668 \end{array}$$



Learners should:

- *be able to subtract numbers with different numbers of digits;*
- *using this method, also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;*
- *know that decimal points should line up under each other;*

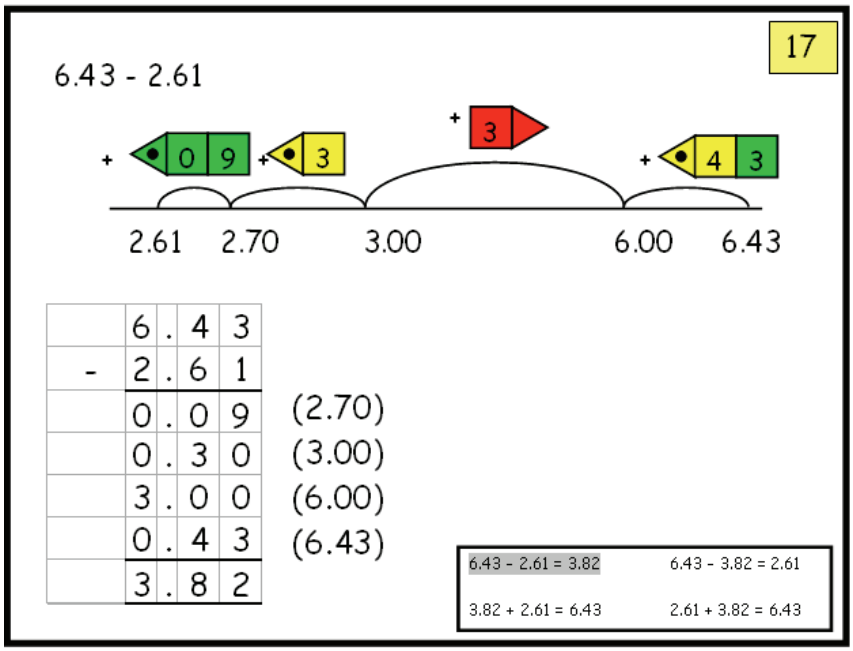
• *be able to use a number line to help in making the calculation.*

£8.95 =	8 + 0.9 + 0.05	
– £4.38	4 + 0.3 + 0.08	*leading to*

			1
=	8 + 0.8 + 0.15	*(adjust from T to U)*	8.85
	4 + 0.3 + 0.08		– 4.38
–	4 + 0.5 + 0.07		= £4.57

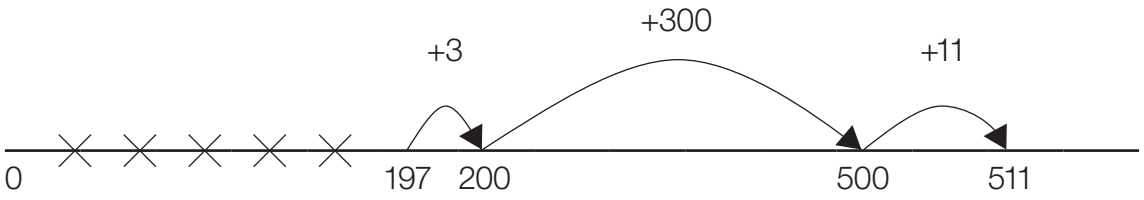
Alternatively, learners can set the amounts to whole numbers, i.e. 895 – 438 and convert to pounds after the calculation.

Using a number line to help in making the calculation:



Where the numbers involved in the calculation are close together or near to multiples of 10, 100, etc, counting on using a number line should be used.

511 – 197 = 314



Step 5

Partitioning and decomposition

Stage 1

$$\begin{array}{rcl} 754 & = & 700 + 50 + 4 \\ -\ 286 & & -\ \ 200 + 80 + 6 \end{array}$$

Stage 2

$$\begin{array}{rl} 700 + 40 + 14 & \textit{(adjust from T to U)} \\ -\ \ 200 + 80 + \ 6 & \end{array}$$

Stage 3

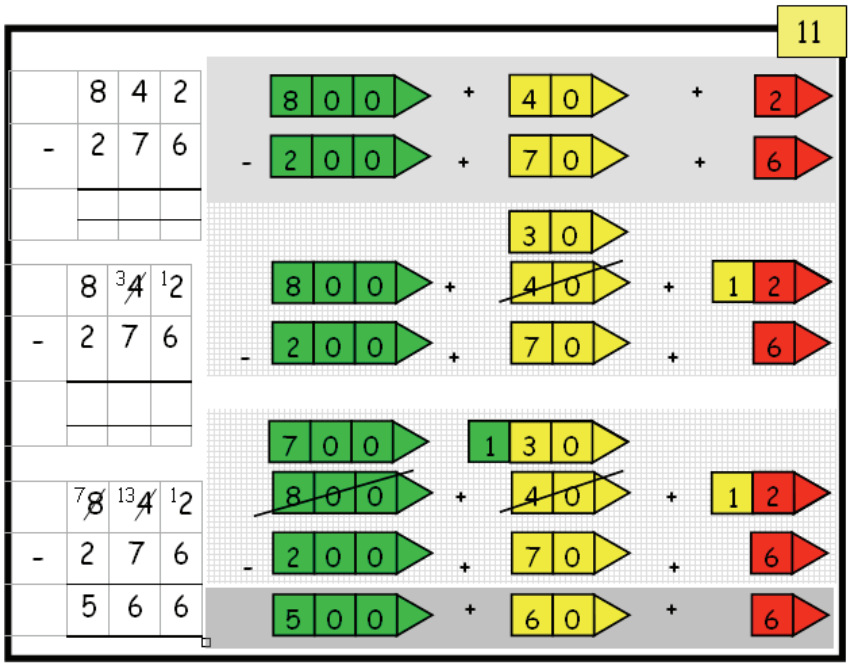
$$\begin{array}{rl} 600 + 140 + 14 & \textit{(adjust from H to T)} \\ -\ 200 + \ 80 + \ 6 & \\ \hline 400 + \ 60 + \ 8 & = 468 \end{array}$$

This would be recorded by the learners as:

$$\begin{array}{rl} & \begin{array}{ccc} 600 & 140 & 1 \end{array} \\ & \cancel{700} + \cancel{50} + 4 \\ -\ \ & 200 + 80 + 6 \\ \hline & 400 + 60 + 8 \end{array} \quad = 468$$

Decomposition

6141
~~7~~54
– 286
468

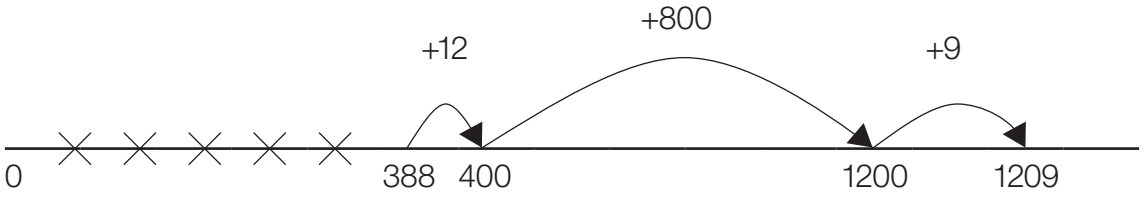


Learners should:

- *be able to subtract numbers with different numbers of digits;*
- *begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;*
- *know that decimal points should line up under each other.*

Where the numbers involved in the calculation are close together or near to multiples of 10, 100, etc, counting on using a number line should be used.

1209 – 388 = 821



Step 6

Decomposition

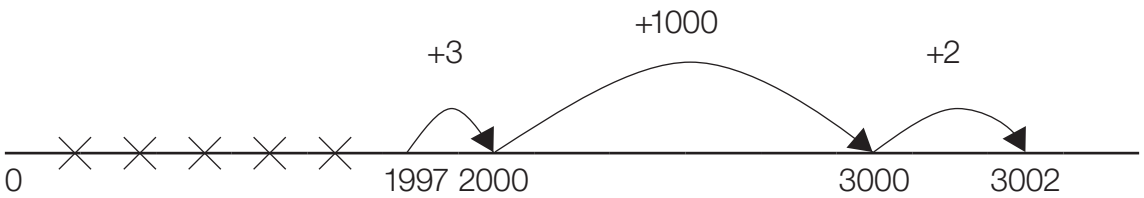
$$\begin{array}{rr} & {\scriptstyle 5131} \\ & \cancel{6}467 \\ - & 2684 \\ \hline & 3783 \end{array}$$

Learners should:

- *be able to subtract numbers with different numbers of digits;*
- *be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;*
- *know that decimal points should line up under each other.*

Where the numbers involved in the calculation are close together or near to multiples of 10, 100, etc, counting on using a number line should be used.

3002 – 1997 = 1005



Learners need to have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Learners should not be made to go on to the next stage if:

1) they are not ready;

2) they are not confident.

Learners should be encouraged to approximate their answers before calculating.

Learners should be encouraged to check their answers after calculation using an appropriate strategy.

Learners should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Multiplication

This annex includes:

1. a brief overview of the most common mental calculation strategies for multiplication including:

- doubling and halving;
- understanding multiplication as repeated addition;
- multiplication facts up to 10 x 10;
- recognising the relationship between tables: families of tables;
- using knowledge of tables to derive other facts;
- multiplying by 10 and 100;
- partitioning;
- using factors.

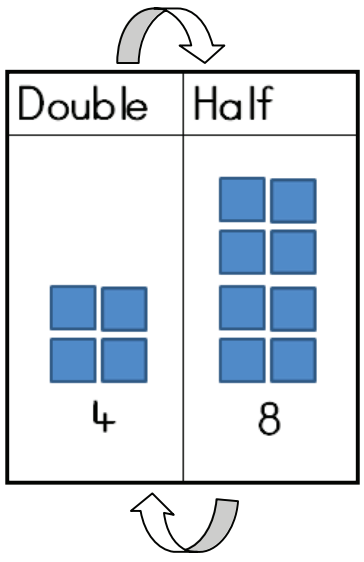
2. a step-by-step overview of one route by which multiplication can be learnt from counting equal groups of objects through to using the Grid Method (also known as the Area Method) for short and long division of whole numbers and decimals.

Mental calculations

Step 1

Doubling and halving

Apply the knowledge of doubles and halves to known facts, e.g. 4 x 4 is double 2 x 4.



Step 2

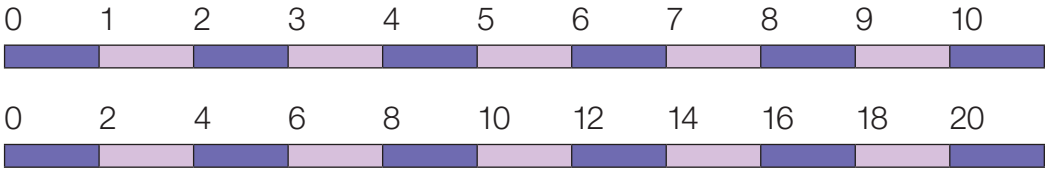
1 times table

2 times table

1 times table

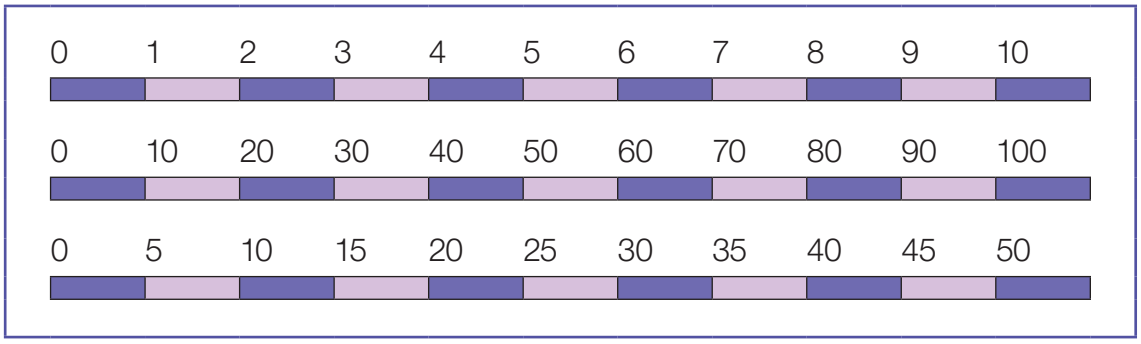
10 times table

5 times table



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	2	=		0
	1	x	2	=		2
	2	x	2	=		4
	3	x	2	=		6
	4	x	2	=		8
	5	x	2	=	1	0
	6	x	2	=	1	2
	7	x	2	=	1	4
	8	x	2	=	1	6
	9	x	2	=	1	8
1	0	x	2	=	2	0



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	1	0	=		0
	1	x	1	0	=	1	0
	2	x	1	0	=	2	0
	3	x	1	0	=	3	0
	4	x	1	0	=	4	0
	5	x	1	0	=	5	0
	6	x	1	0	=	6	0
	7	x	1	0	=	7	0
	8	x	1	0	=	8	0
	9	x	1	0	=	9	0
1	0	x	1	0	=	1	0	0

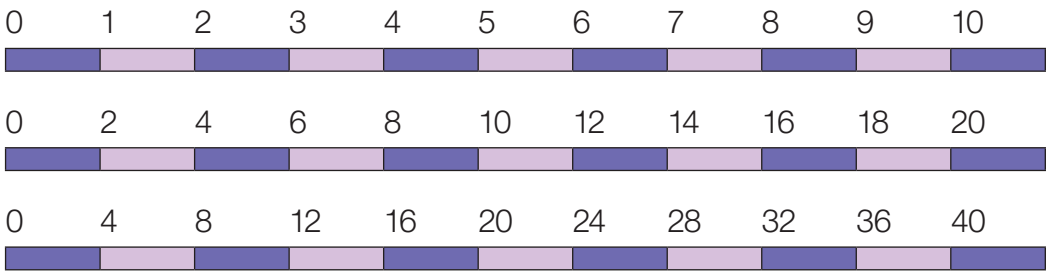
	0	x	5	=		0
	1	x	5	=		5
	2	x	5	=	1	0
	3	x	5	=	1	5
	4	x	5	=	2	0
	5	x	5	=	2	5
	6	x	5	=	3	0
	7	x	5	=	3	5
	8	x	5	=	4	0
	9	x	5	=	4	5
1	0	x	5	=	5	0

Step 3

1 times table
2 times table
4 times table

1 times table
10 times table
5 times table

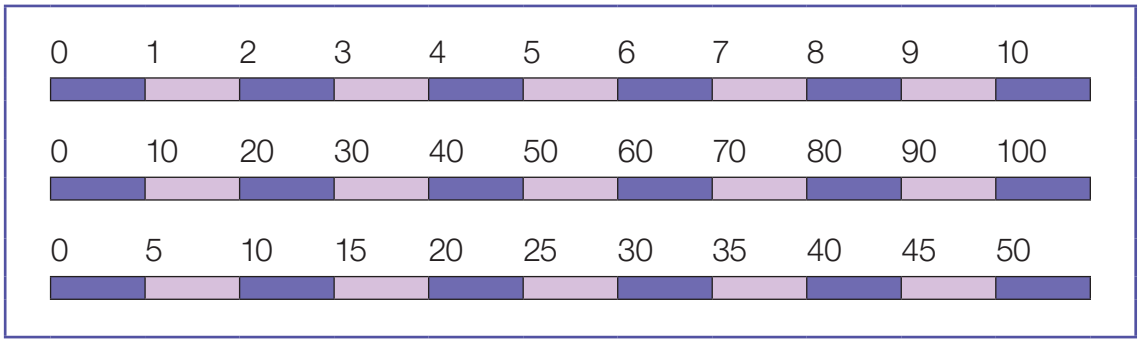
1 times table
3 times table



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	2	=		0
	1	x	2	=		2
	2	x	2	=		4
	3	x	2	=		6
	4	x	2	=		8
	5	x	2	=	1	0
	6	x	2	=	1	2
	7	x	2	=	1	4
	8	x	2	=	1	6
	9	x	2	=	1	8
1	0	x	2	=	2	0

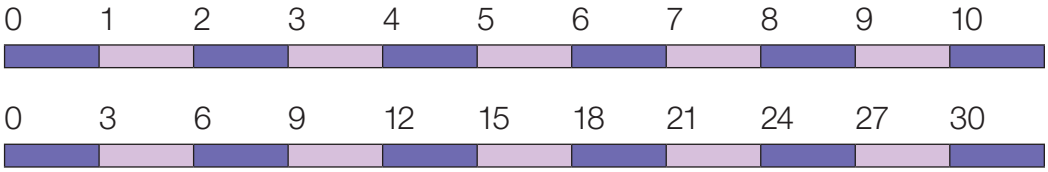
	0	x	4	=		0
	1	x	4	=		4
	2	x	4	=		8
	3	x	4	=	1	2
	4	x	4	=	1	6
	5	x	4	=	2	0
	6	x	4	=	2	4
	7	x	4	=	2	8
	8	x	4	=	3	2
	9	x	4	=	3	6
1	0	x	4	=		0



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	1	0	=		0
	1	x	1	0	=	1	0
	2	x	1	0	=	2	0
	3	x	1	0	=	3	0
	4	x	1	0	=	4	0
	5	x	1	0	=	5	0
	6	x	1	0	=	6	0
	7	x	1	0	=	7	0
	8	x	1	0	=	8	0
	9	x	1	0	=	9	0
1	0	x	1	0	=	1	0	0

	0	x	5	=		0
	1	x	5	=		5
	2	x	5	=	1	0
	3	x	5	=	1	5
	4	x	5	=	2	0
	5	x	5	=	2	5
	6	x	5	=	3	0
	7	x	5	=	3	5
	8	x	5	=	4	0
	9	x	5	=	4	5
1	0	x	5	=	5	0



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

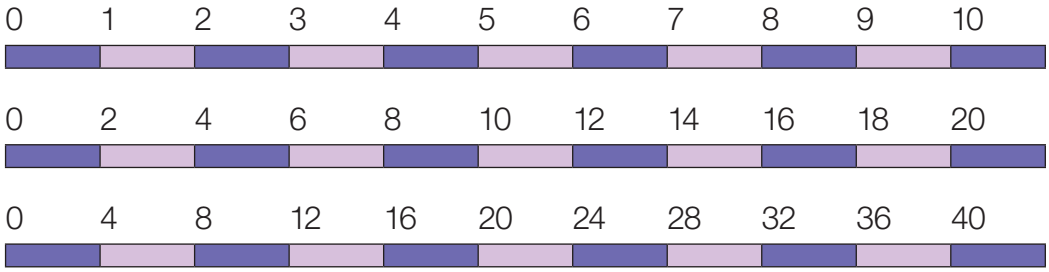
	0	x	3	=		0
	1	x	3	=		3
	2	x	3	=		6
	3	x	3	=		9
	4	x	3	=	1	2
	5	x	3	=	1	5
	6	x	3	=	1	8
	7	x	3	=	2	1
	8	x	3	=	2	4
	9	x	3	=	2	7
1	0	x	3	=	3	0

Step 4

1 times table
2 times table
4 times table

1 times table
10 times table
5 times table

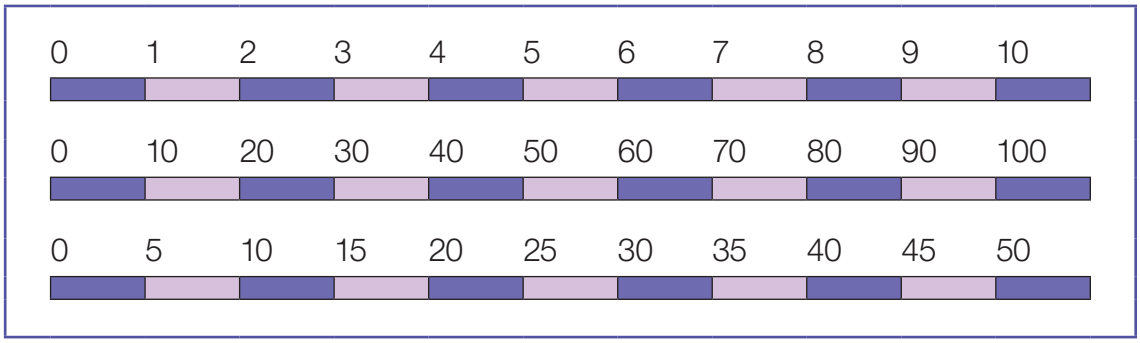
1 times table
3 times table
6 times table



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	2	=		0
	1	x	2	=		2
	2	x	2	=		4
	3	x	2	=		6
	4	x	2	=		8
	5	x	2	=	1	0
	6	x	2	=	1	2
	7	x	2	=	1	4
	8	x	2	=	1	6
	9	x	2	=	1	8
1	0	x	2	=	2	0

	0	x	4	=		0
	1	x	4	=		4
	2	x	4	=		8
	3	x	4	=	1	2
	4	x	4	=	1	6
	5	x	4	=	2	0
	6	x	4	=	2	4
	7	x	4	=	2	8
	8	x	4	=	3	2
	9	x	4	=	3	6
1	0	x	4	=	4	0



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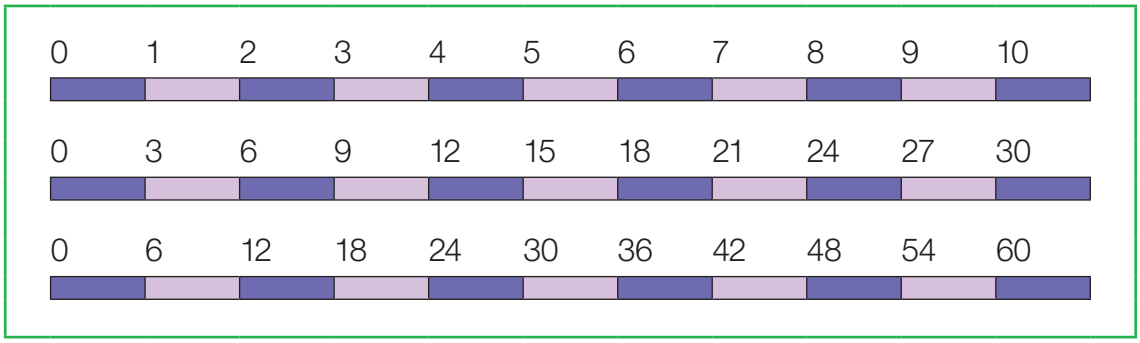
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	1	0	=		0
	1	x	1	0	=	1	0
	2	x	1	0	=	2	0
	3	x	1	0	=	3	0
	4	x	1	0	=	4	0
	5	x	1	0	=	5	0
	6	x	1	0	=	6	0
	7	x	1	0	=	7	0
	8	x	1	0	=	8	0
	9	x	1	0	=	9	0
1	0	x	1	0	=	1	0	0

	0	x	5	=		0
	1	x	5	=		5
	2	x	5	=	1	0
	3	x	5	=	1	5
	4	x	5	=	2	0
	5	x	5	=	2	5
	6	x	5	=	3	0
	7	x	5	=	3	5
	8	x	5	=	4	0
	9	x	5	=	4	5
1	0	x	5	=	5	0



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	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	3	=		0
	1	x	3	=		3
	2	x	3	=		6
	3	x	3	=		9
	4	x	3	=	1	2
	5	x	3	=	1	5
	6	x	3	=	1	8
	7	x	3	=	2	1
	8	x	3	=	2	4
	9	x	3	=	2	7
1	0	x	3	=	3	0

	0	x	6	=		0
	1	x	6	=		6
	2	x	6	=	1	2
	3	x	6	=	1	8
	4	x	6	=	2	4
	5	x	6	=	3	0
	6	x	6	=	3	6
	7	x	6	=	4	2
	8	x	6	=	4	8
	9	x	6	=	5	4
1	0	x	6	=	6	0

Step 5

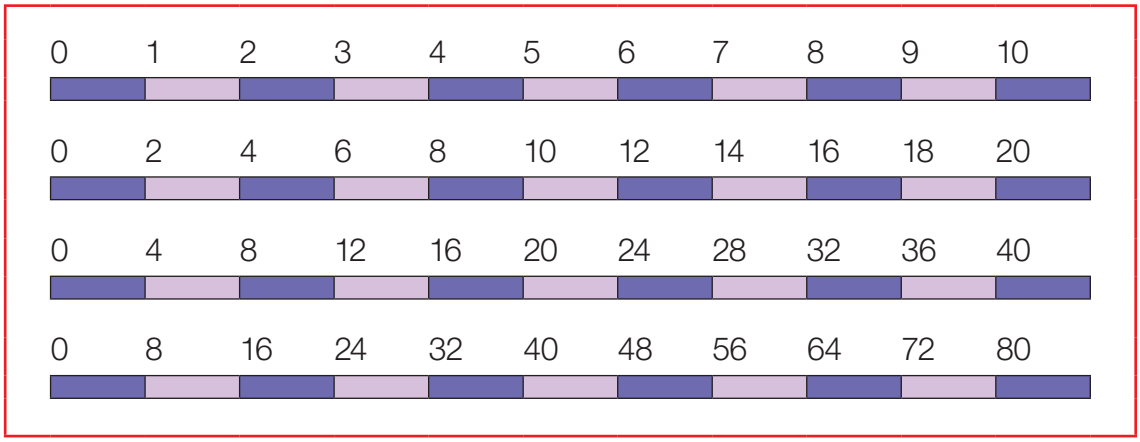
Derive and recall quickly all multiplication facts up to 10 x 10.

1 times table
2 times table
4 times table
8 times table

1 times table
10 times table
5 times table

1 times table
3 times table
6 times table
9 times table

1 times table
7 times table



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	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
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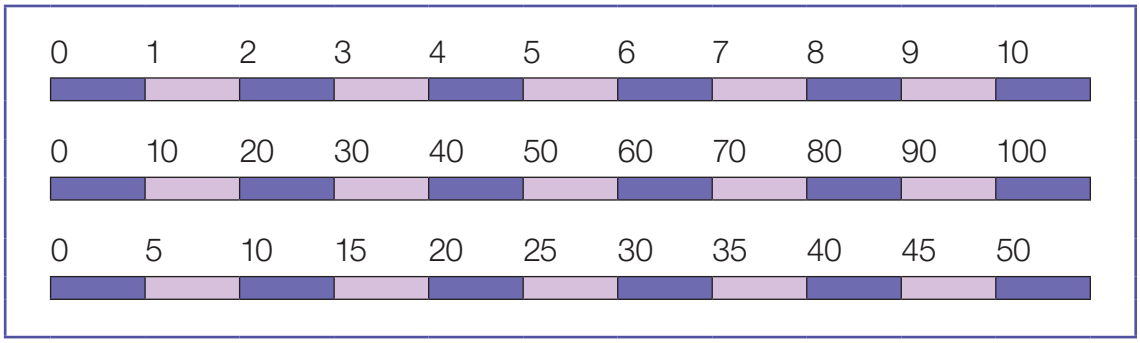
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	2	=		0
	1	x	2	=		2
	2	x	2	=		4
	3	x	2	=		6
	4	x	2	=		8
	5	x	2	=	1	0
	6	x	2	=	1	2
	7	x	2	=	1	4
	8	x	2	=	1	6
	9	x	2	=	1	8
1	0	x	2	=	2	0

	0	x	4	=		0
	1	x	4	=		4
	2	x	4	=		8
	3	x	4	=	1	2
	4	x	4	=	1	6
	5	x	4	=	2	0
	6	x	4	=	2	4
	7	x	4	=	2	8
	8	x	4	=	3	2
	9	x	4	=	3	6
1	0	x	4	=	4	0

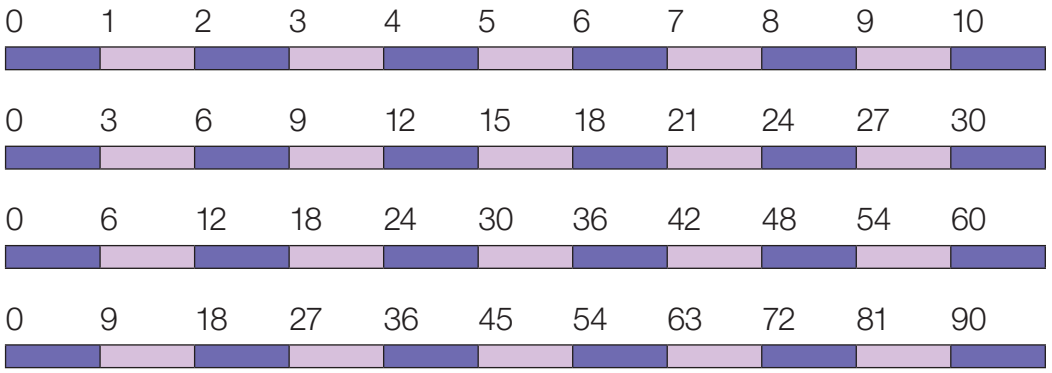
	0	x	8	=		0
	1	x	8	=		8
	2	x	8	=	1	6
	3	x	8	=	2	4
	4	x	8	=	3	2
	5	x	8	=	4	0
	6	x	8	=	4	8
	7	x	8	=	5	6
	8	x	8	=	6	4
	9	x	8	=	7	2
1	0	x	8	=	8	0



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	1	0	=			0
	1	x	1	0	=		1	0
	2	x	1	0	=		2	0
	3	x	1	0	=		3	0
	4	x	1	0	=		4	0
	5	x	1	0	=		5	0
	6	x	1	0	=		6	0
	7	x	1	0	=		7	0
	8	x	1	0	=		8	0
	9	x	1	0	=		9	0
1	0	x	1	0	=	1	0	0

	0	x	5	=		0
	1	x	5	=		5
	2	x	5	=	1	0
	3	x	5	=	1	5
	4	x	5	=	2	0
	5	x	5	=	2	5
	6	x	5	=	3	0
	7	x	5	=	3	5
	8	x	5	=	4	0
	9	x	5	=	4	5
1	0	x	5	=	5	0



	0	x	1	=		0
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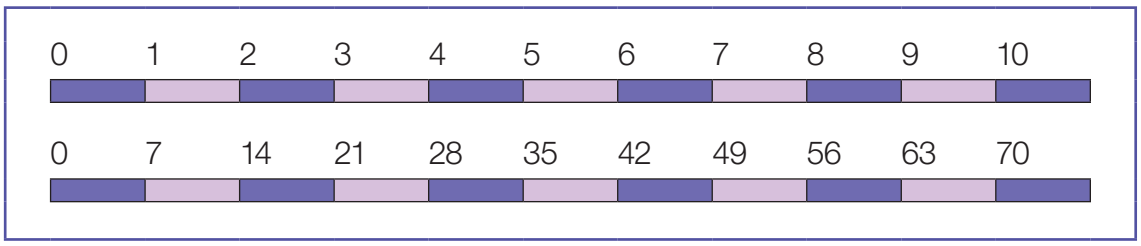
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	3	=		0
	1	x	3	=		3
	2	x	3	=		6
	3	x	3	=		9
	4	x	3	=	1	2
	5	x	3	=	1	5
	6	x	3	=	1	8
	7	x	3	=	2	1
	8	x	3	=	2	4
	9	x	3	=	2	7
1	0	x	3	=	3	0

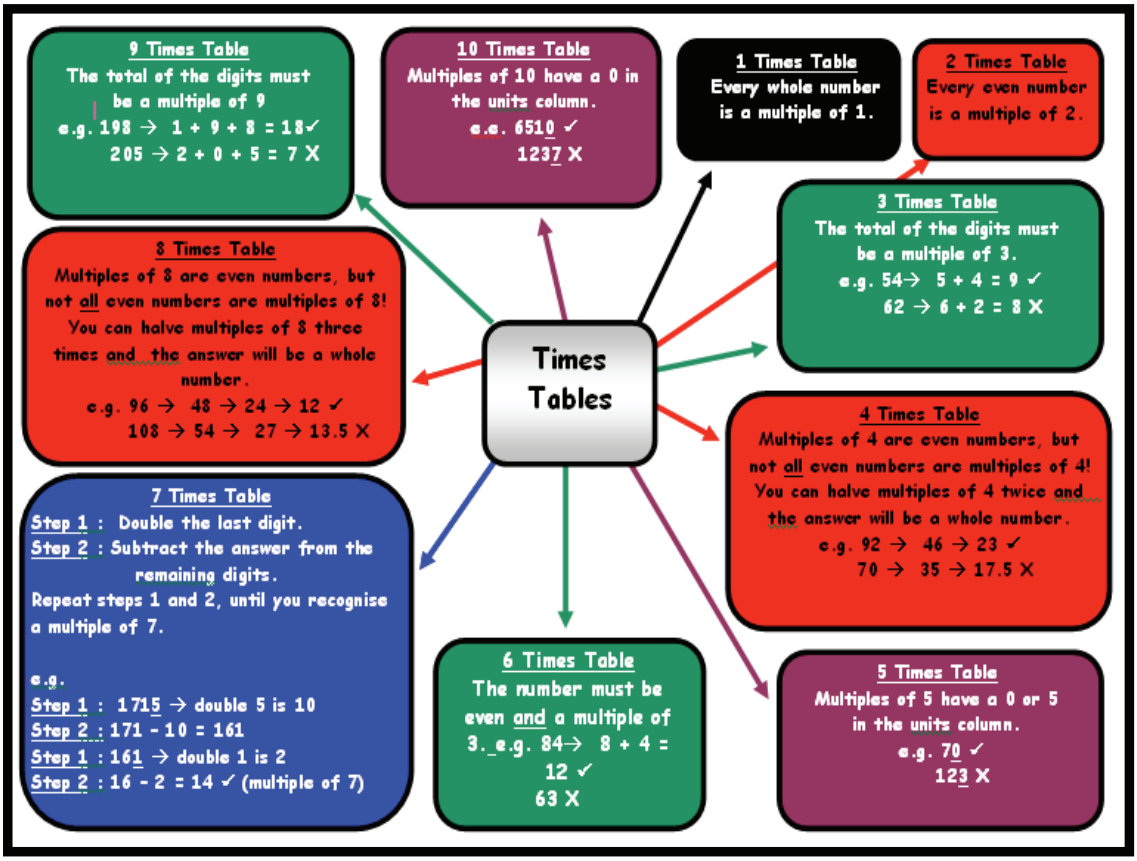
	0	x	6	=		0
	1	x	6	=		6
	2	x	6	=	1	2
	3	x	6	=	1	8
	4	x	6	=	2	4
	5	x	6	=	3	0
	6	x	6	=	3	6
	7	x	6	=	4	2
	8	x	6	=	4	8
	9	x	6	=	5	4
1	0	x	6	=	6	0

	0	x	9	=		0
	1	x	9	=		9
	2	x	9	=	1	8
	3	x	9	=	2	7
	4	x	9	=	3	6
	5	x	9	=	4	5
	6	x	9	=	5	4
	7	x	9	=	6	3
	8	x	9	=	7	2
	9	x	9	=	8	1
1	0	x	9	=	9	0



	0	x	1	=		0
	1	x	1	=		1
	2	x	1	=		2
	3	x	1	=		3
	4	x	1	=		4
	5	x	1	=		5
	6	x	1	=		6
	7	x	1	=		7
	8	x	1	=		8
	9	x	1	=		9
1	0	x	1	=	1	0

	0	x	7	=		0
	1	x	7	=		7
	2	x	7	=	1	4
	3	x	7	=	2	1
	4	x	7	=	2	8
	5	x	7	=	3	5
	6	x	7	=	4	2
	7	x	7	=	4	9
	8	x	7	=	5	6
	9	x	7	=	6	3
1	0	x	7	=	7	0



Step 6

Learners should be able to use their knowledge of tables to derive other facts; e.g. If I know 3 x 7 = 21, what else do I know?

$30 \times 7 = 3 \times 7 \times 10 = 210$

$300 \times 7 = 3 \times 7 \times 10 \times 10 = 3 \times 7 \times 100 = 2100$

$3000 \times 7 = 3 \times 7 \times 10 \times 10 \times 10 = 3 \times 7 \times 10 \times 100 = 3 \times 7 \times 1000 = 21\,000$

$0.3 \times 7 = (3 \times 7) \div 10 = 2.1$

Use closely related facts already known

13 x 11 = (13 x 10) + (13 x 1)

= 130 + 13

= 143

Multiply by 10 or 100

Knowing that the effect of multiplying by 10 is the digit(s) moving one place to the left.

Knowing that the effect of multiplying by 100 is the digit(s) moving two places to the left.

Multiplying by 10

Th	H	T	U		.1	.01	.001
	3	4	0	.			
		3	4				

When multiplying a whole number by 10 the digits move one column to the left, and a 0 is placed in the units column

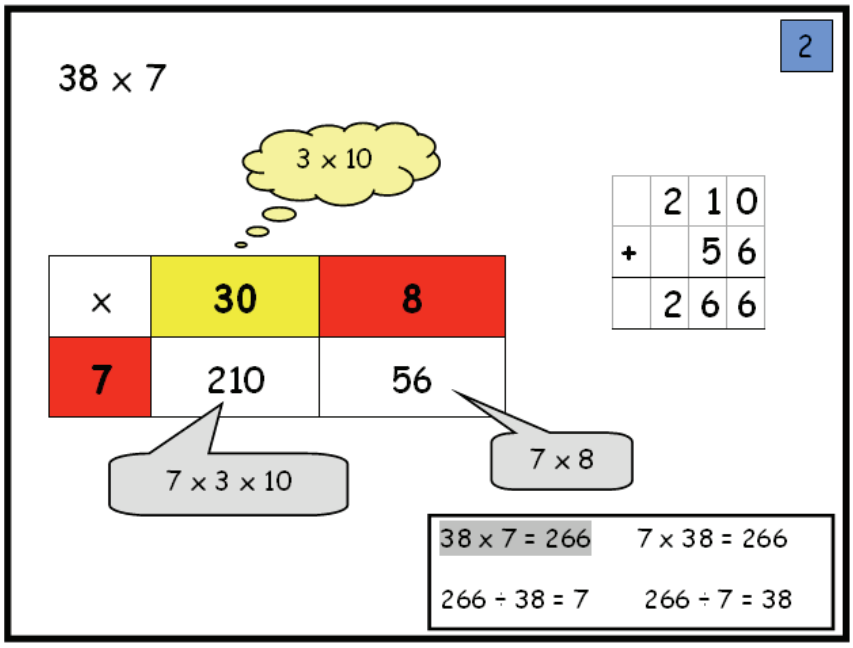
Multiplying by 100

Th	H	T	U		.1	.01	.001
3	4	0	0	.			
		3	4				

When multiplying a whole number by 100 the digits move two columns to the left, and a 0 is placed in the tens and units columns

Partitioning

38 x 7 = (30 x 7) + (8 x 7)
= 210 + 56
= 266



Use of factors

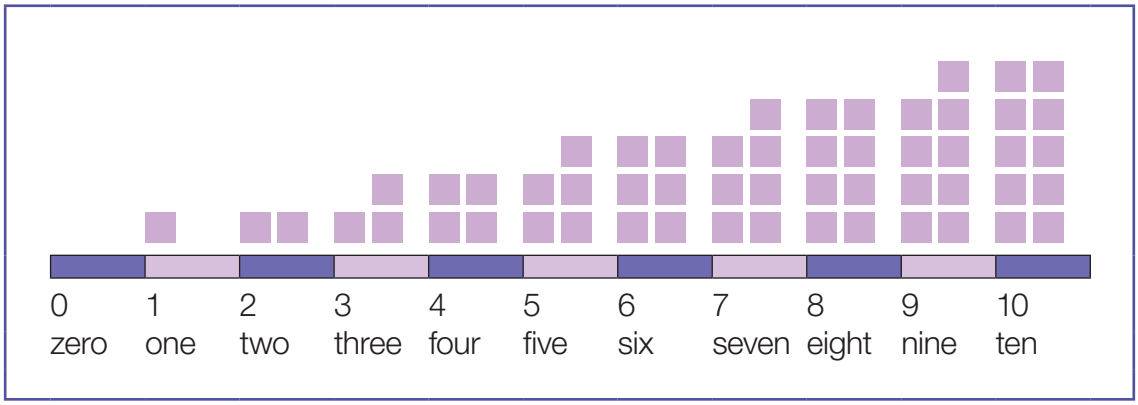
21 x 18 = 21 x 9 x 2
21 x 9 = 189
189 x 2 = 378

11 x 50 = 11 x 5 x 10
11 x 5 = 55
55 x 10 = 550

Many mental calculation strategies will continue to be used. They are not replaced by written methods.

Step 1

Learners will experience equal groups of objects and will count in 1s, 2s and 10s and begin to count in 5s. They will work on practical problem-solving activities involving equal sets or groups.



Step 2

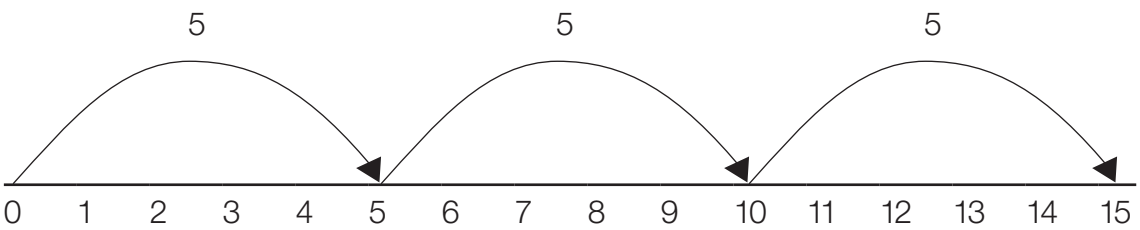
Learners will develop their understanding of multiplication and use jottings to support calculation:

Repeated addition

3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

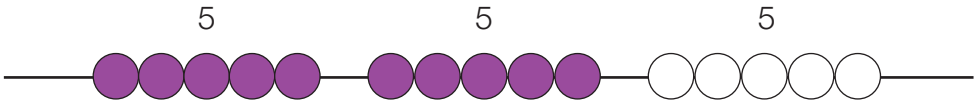
Repeated addition can be shown easily on a number line:

5 x 3 = 5 + 5 + 5



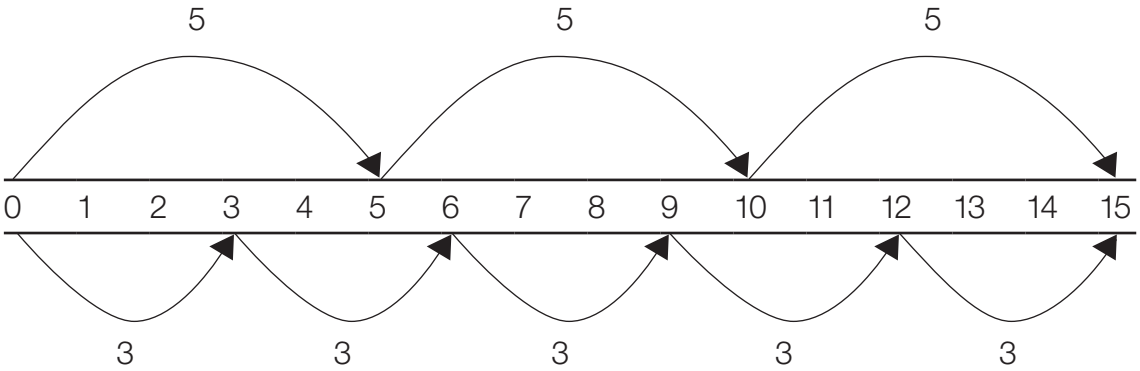
and on a bead bar:

5 x 3 = 5 + 5 + 5



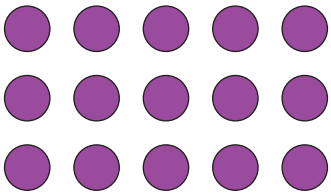
Commutative

Learners should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.



Arrays

Learners should be able to model a multiplication calculation using an array. This knowledge will support the development of the grid method.



5 x 3 = 15

3 x 5 = 15

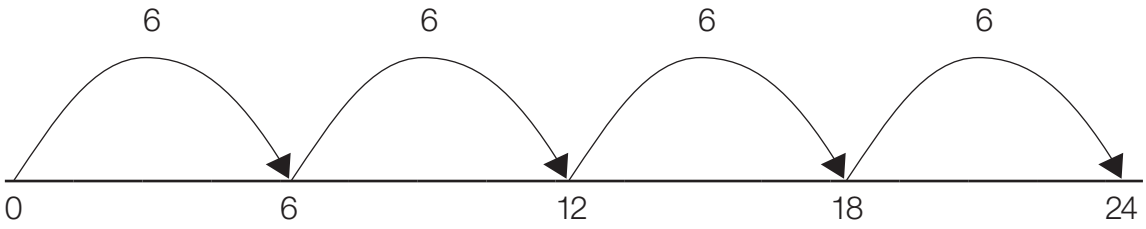
Step 3

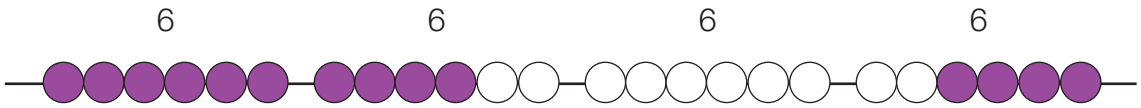
Learners will continue to use:

Repeated addition

4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4

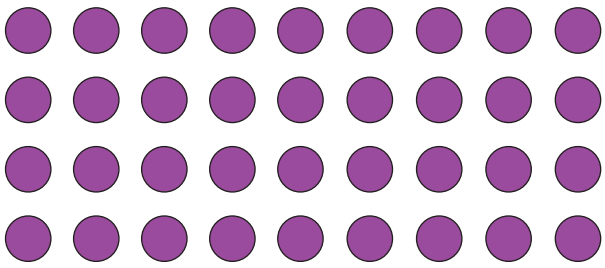
Learners should use number lines or bead bars to support their understanding.





Arrays

Learners should be able to model a multiplication calculation using an array. This knowledge will support the development of the grid method.



9 x 4 = 36

4 x 9 = 36

Learners will also develop an understanding of:

Scaling

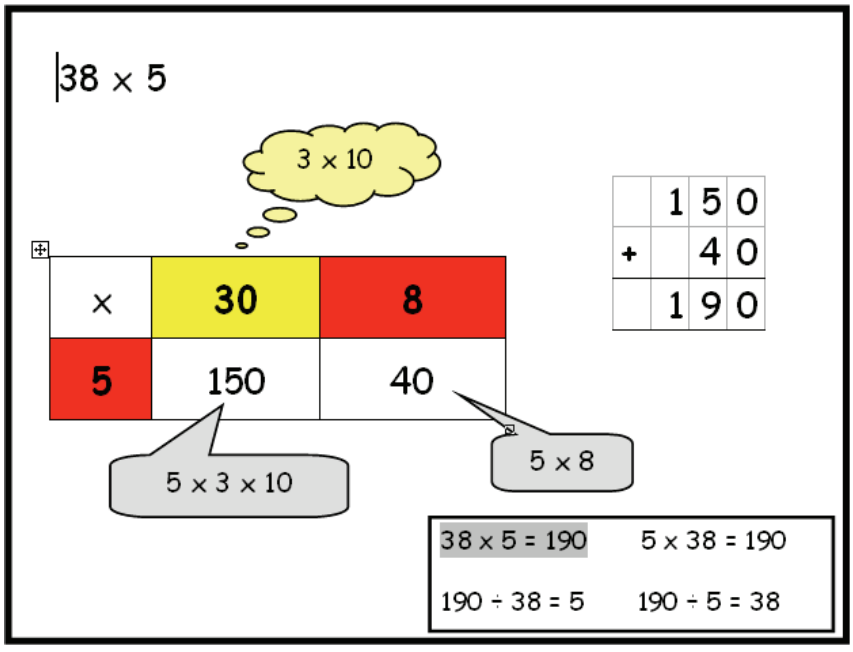
For example: find a ribbon that is 4 times as long as the purple ribbon.

- Using symbols to stand for unknown numbers to complete equations using inverse operations

□ x 5 = 20 3 x △ = 18 □ x ○ = 32

Partitioning

38 x 5 = (30 x 5) + (8 x 5)
= 150 + 40
= 190



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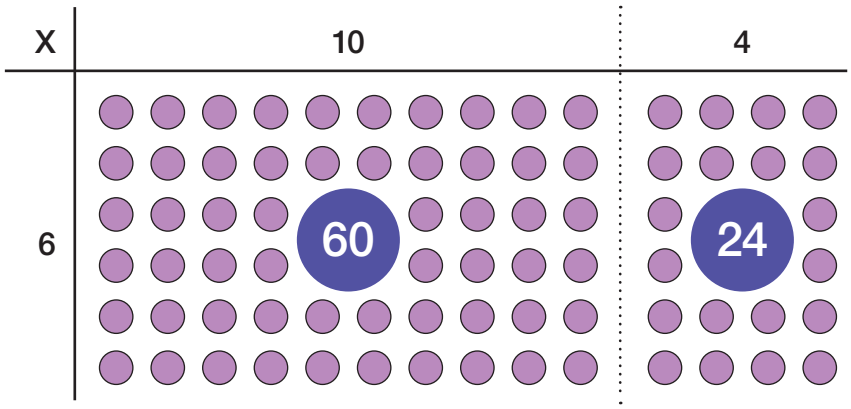
Step 4

Learners will continue to use arrays where appropriate leading into the grid method of multiplication.



(6 x 10) + (6 x 4)

60 + 24

84

Grid method

TU x U

(Short multiplication – multiplication by a single digit)

23 x 8

Learners will approximate first:

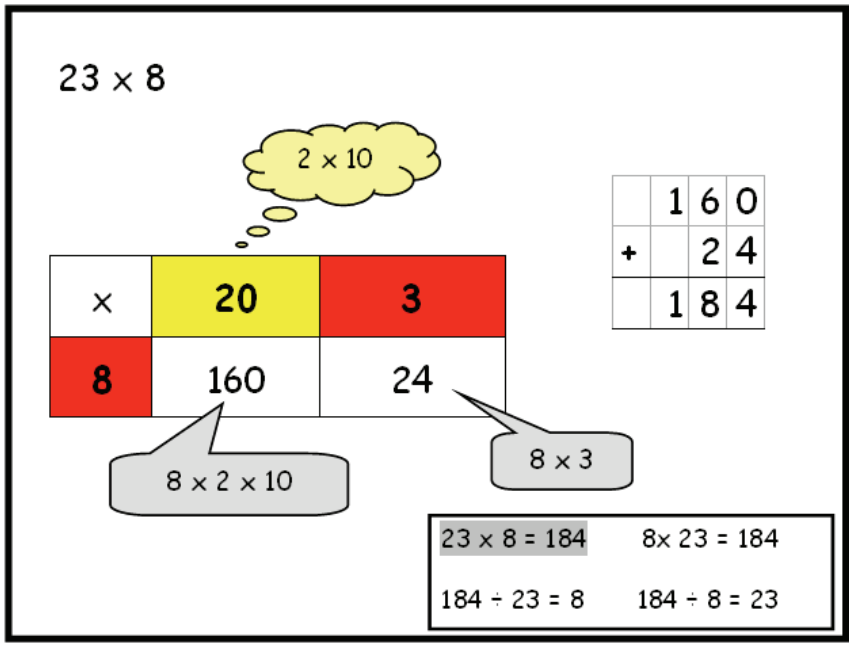
23 x 8 is approximately 25 x 8 = 200

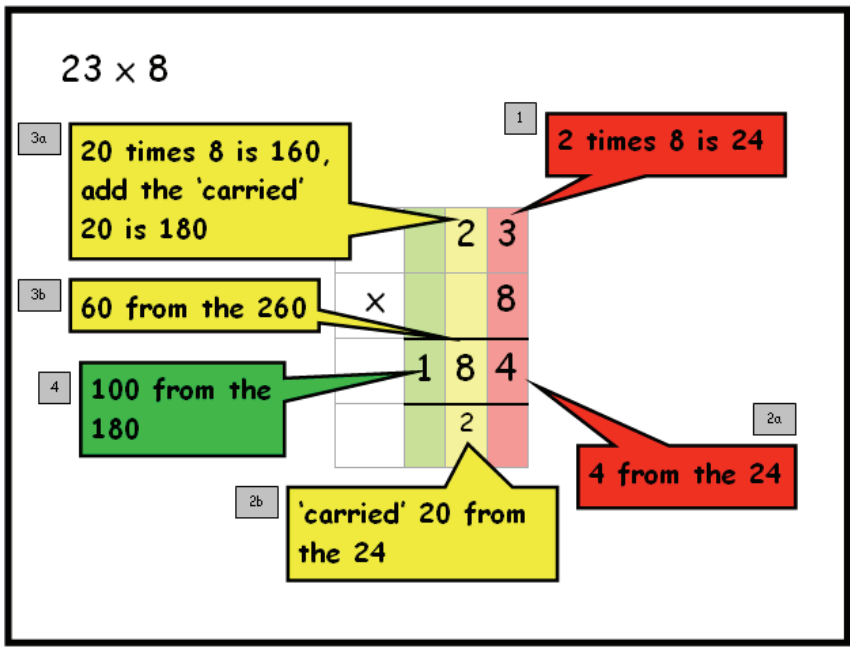
x	20	3
8	160	24

160

+ 24

184


23 x 8
2 x 10
x
20
3
8
160
24
8 x 2 x 10
8 x 3
1 6 0
+ 2 4
1 8 4
23 x 8 = 184
8x 23 = 184
184 ÷ 23 = 8
184 ÷ 8 = 23


23 x 8
3a
20 times 8 is 160, add the 'carried' 20 is 180
1
2 times 8 is 24
3b
60 from the 260
4
100 from the 180
2a
4 from the 24
2b
'carried' 20 from the 24
2 3
x 8
1 8 4
2

Step 5
Grid method

HTU x U

(Short multiplication – multiplication by a single digit)

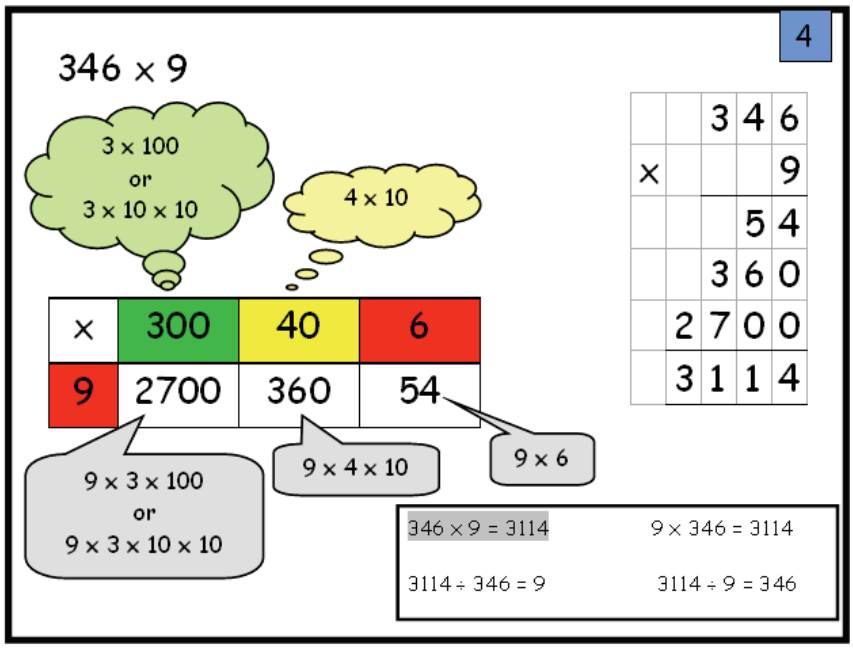
346 x 9

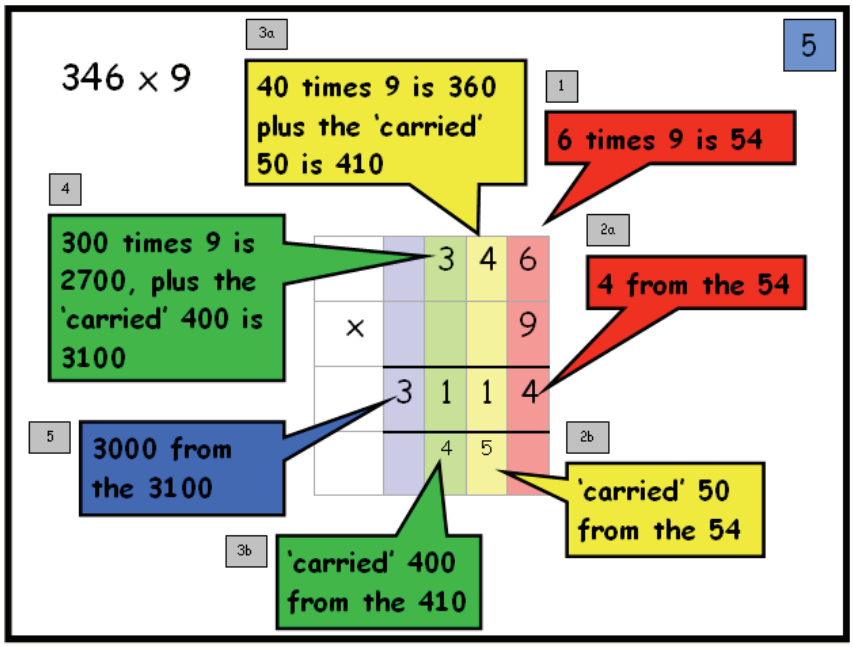
Learners will approximate first:

346 x 9 is approximately 350 x 10 = 3500

x	300	40	6
9	2700	360	54

2700
+ 360
+ 54
3114
11





TU x TU

(Long multiplication – multiplication by more than a single digit)

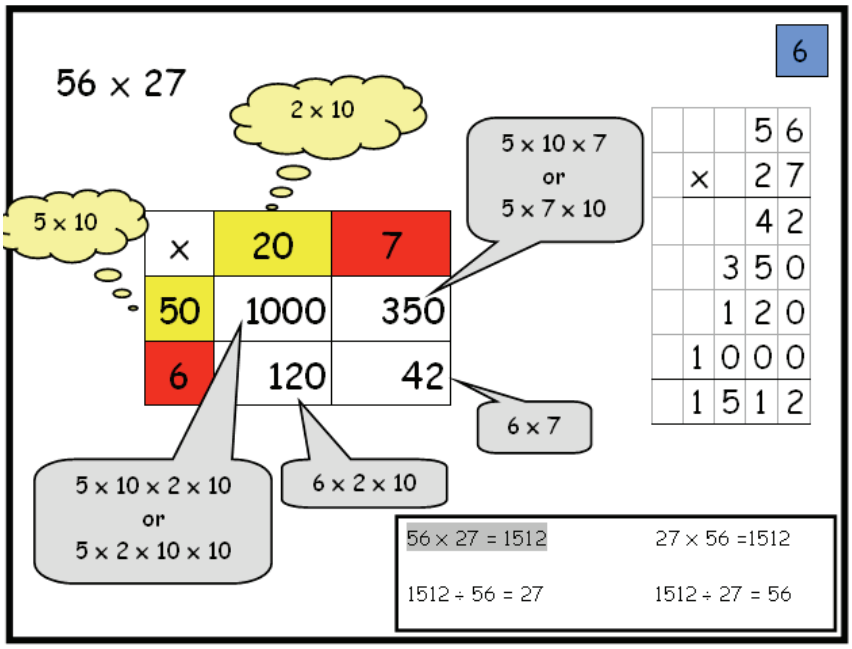
56 x 27

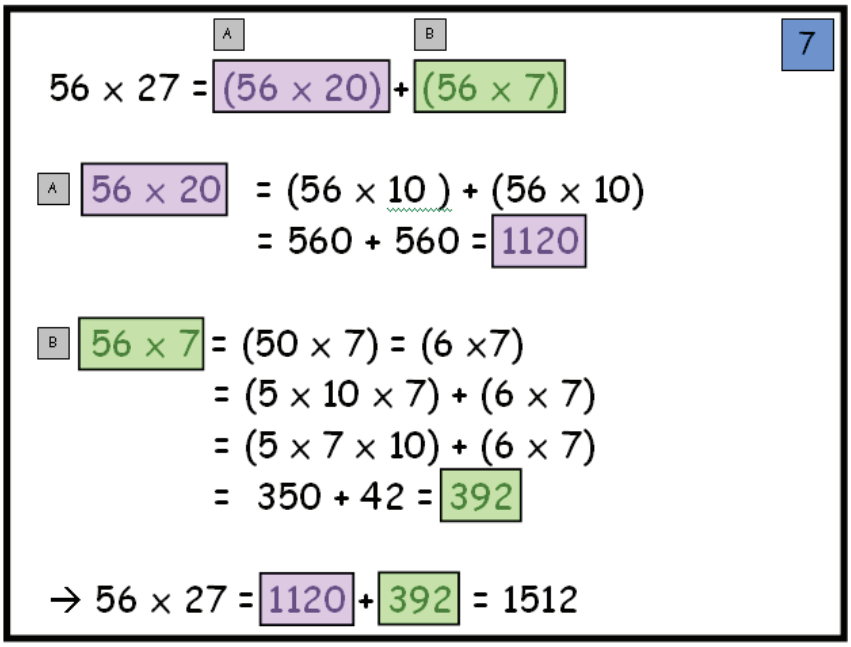
Learners will approximate first:

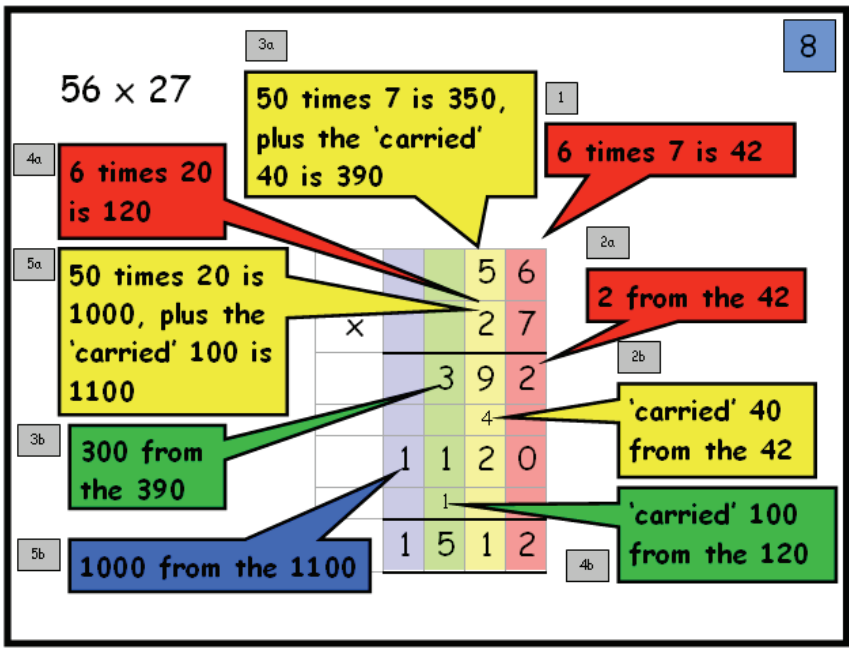
56 x 27 is approximately 60 x 30 = 1800

x	20	7
50	1000	350
6	120	42

1000
+ 350
+ 120
+ 42
1512


6
56 x 27
2 x 10
5 x 10
5 x 10 x 7
or
5 x 7 x 10
x
20
7
50
1000
350
6
120
42
6 x 7
5 x 10 x 2 x 10
or
5 x 2 x 10 x 10
6 x 2 x 10
56 x 27 = 1512
27 x 56 =1512
1512 ÷ 56 = 27
1512 ÷ 27 = 56
5 6
x 2 7
4 2
3 5 0
1 2 0
1 0 0 0
1 5 1 2


7
A
B
56 x 27 = (56 x 20) + (56 x 7)
A 56 x 20 = (56 x 10) + (56 x 10)
= 560 + 560 = 1120
B 56 x 7 = (50 x 7) = (6 x7)
= (5 x 10 x 7) + (6 x 7)
= (5 x 7 x 10) + (6 x 7)
= 350 + 42 = 392
→ 56 x 27 = 1120 + 392 = 1512



Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.

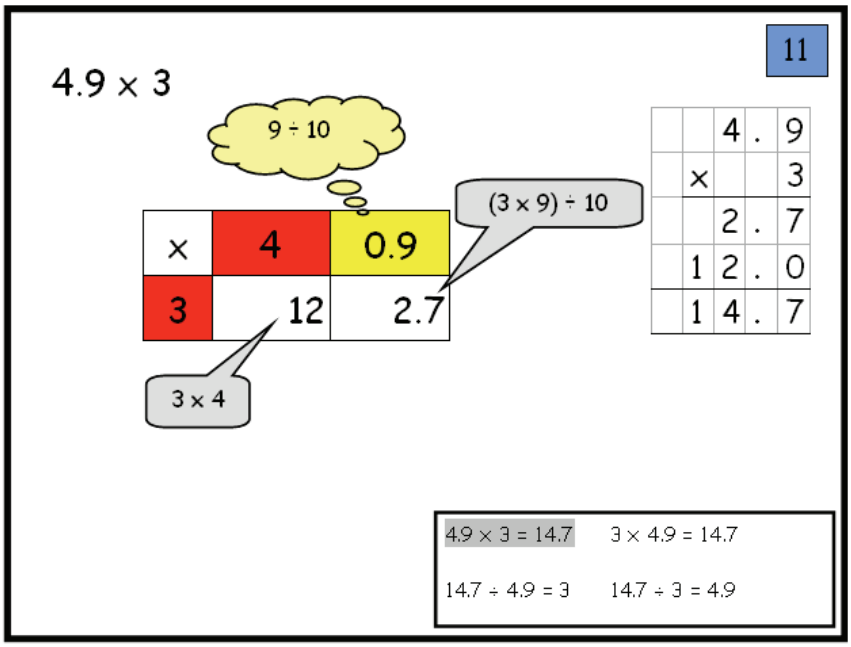
e.g. 4.9 x 3

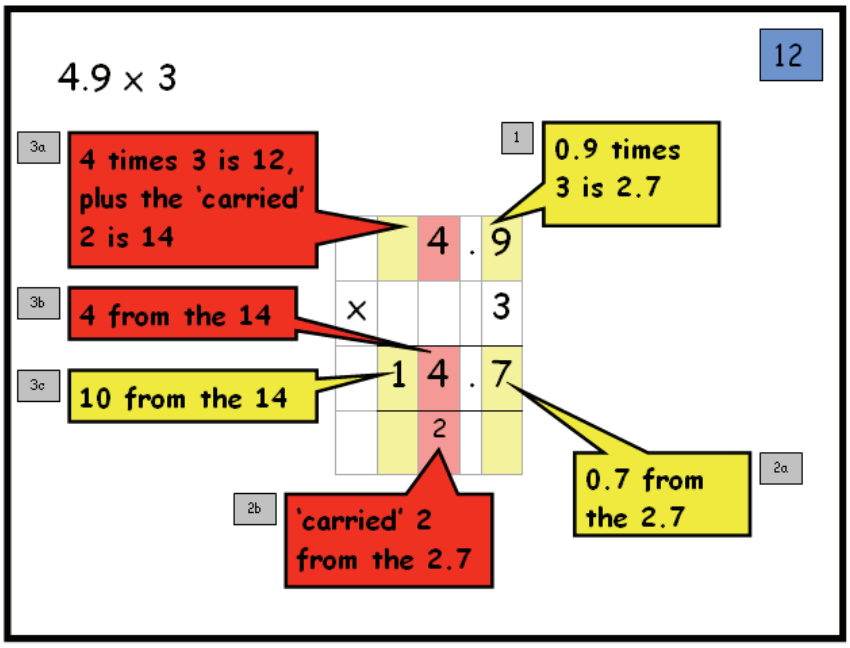
Learners will approximate first:

4.9 x 3 is approximately 5 x 3 = 15

x	4	0.9
3	12	2.7

12
+ 2.7
14.7


11
4.9 × 3
9 ÷ 10
(3 × 9) ÷ 10
×
4
0.9
3
12
2.7
3 × 4
4 . 9
× 3
2 . 7
1 2 . 0
1 4 . 7
4.9 × 3 = 14.7
3 × 4.9 = 14.7
14.7 ÷ 4.9 = 3
14.7 ÷ 3 = 4.9


12
4.9 × 3
3a
4 times 3 is 12, plus the 'carried' 2 is 14
1
0.9 times 3 is 2.7
3b
4 from the 14
3c
10 from the 14
2a
0.7 from the 2.7
2b
'carried' 2 from the 2.7
4 . 9
× 3
1 4 . 7
2

Step 6

ThHTU x U

(Short multiplication – multiplication by a single digit)

4346 x 8

Learners will approximate first:

4346 x 8 is approximately 4346 x 10 = 43460

x	4000	300	40	6
8	32000	2400	320	48

32000
+ 2400
+ 320
+ 48
34768

HTU x TU

(Long multiplication – multiplication by more than a single digit)

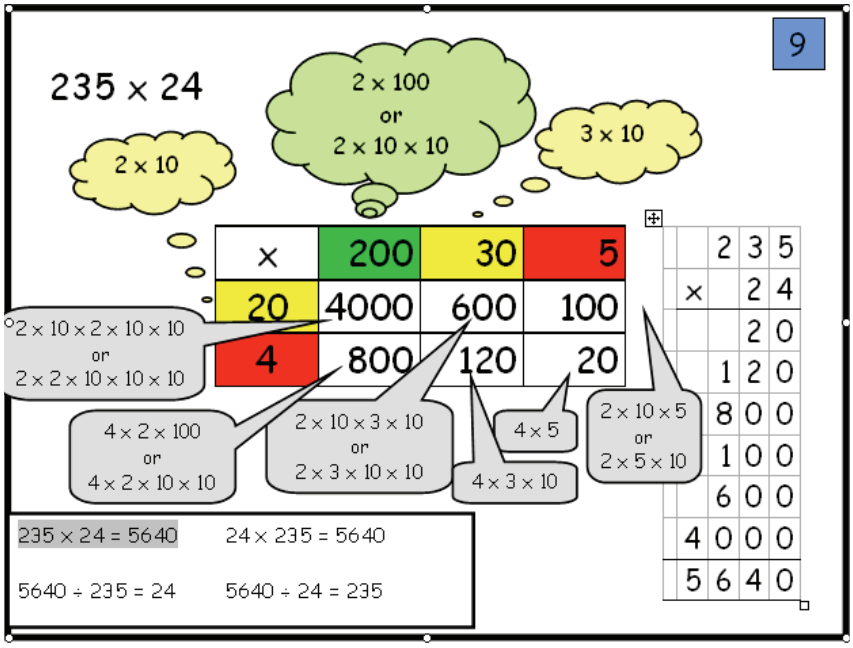
235 x 24

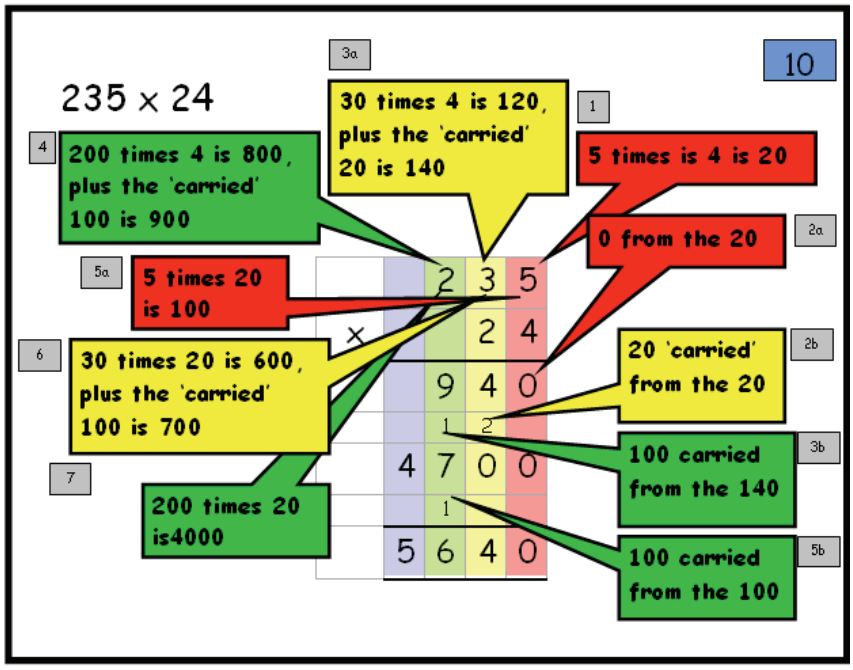
Learners will approximate first:

235 x 24 is approximately 240 x 20 = 4800

x	200	30	5
20	4000	600	100
4	800	120	20

4000
+ 600
+ 100
+ 800
120
20
5640
1





Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.

For example:

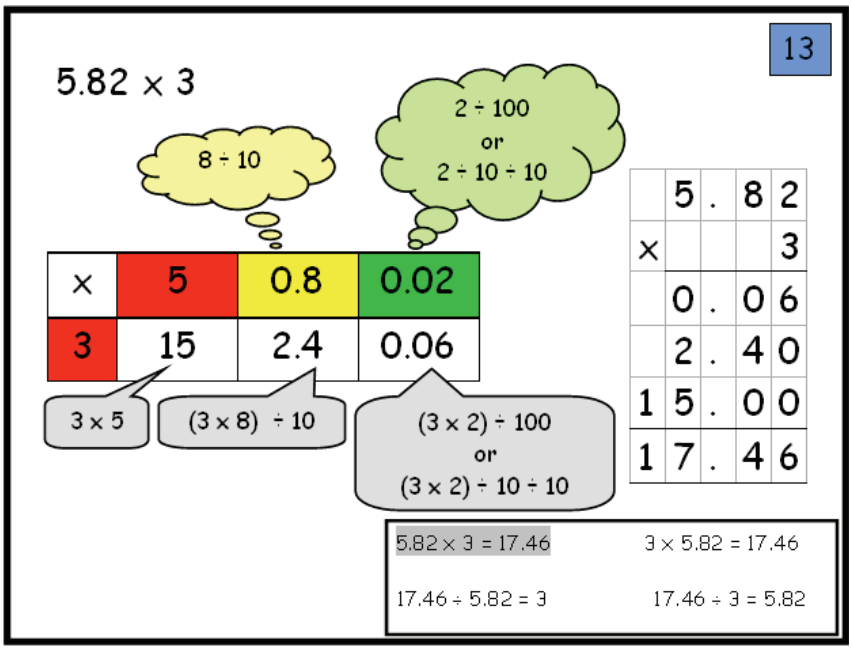
5.82 x 3

Learners will approximate first:

5.82 x 3 is approximately 6 x 3 = 18

x	5	0.8	0.02
3	15	2.4	0.06

15
+ 2.4
+ 0.06
17.46



Learners need to have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Learners should not be made to go on to the next stage if:

1) they are not ready;

2) they are not confident.

Learners should be encouraged to approximate their answers before calculating.

Learners should be encouraged to check their answers after calculation using an appropriate strategy.

Learners should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Division

This annex includes:

1. a brief overview of the most common mental calculation strategies for division including:

- doubling and halving;
- understanding division as repeated subtraction;
- multiplication facts up to 10 x 10;
- recognising the relationship between division and fractions;
- deriving and recalling division facts;
- using and applying division facts;
- dividing by 10 and 100;
- using factors;
- using related facts.

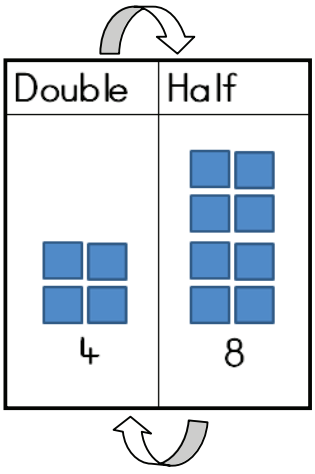
2. a step by step overview of one route by which division can be learnt from sharing equal groups of objects through to using the written vertical method for short and long division of numbers and decimals. The Grid method of division is also introduced.

Mental calculations

Step 1

Doubling and halving

Knowing that halving is dividing by 2



Step 2

Deriving and recalling division facts

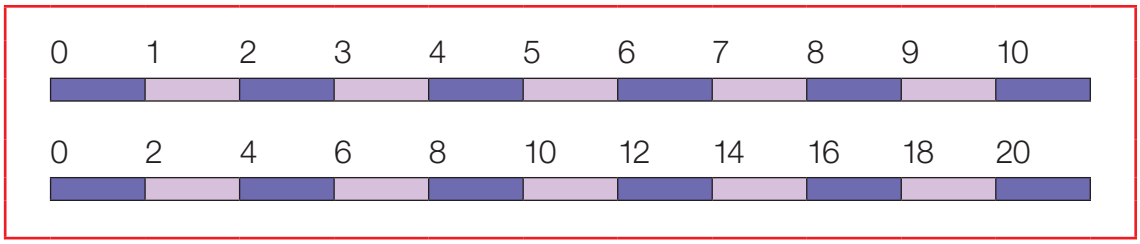
1 times table

2 times table

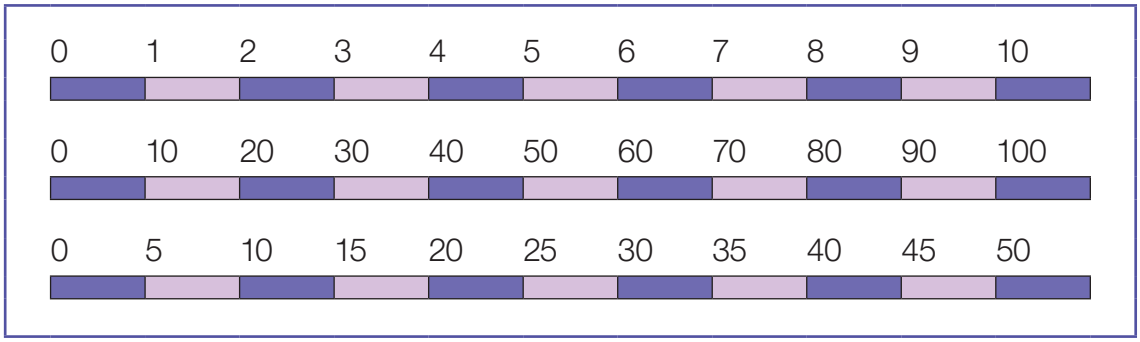
1 times table

10 times table

5 times table



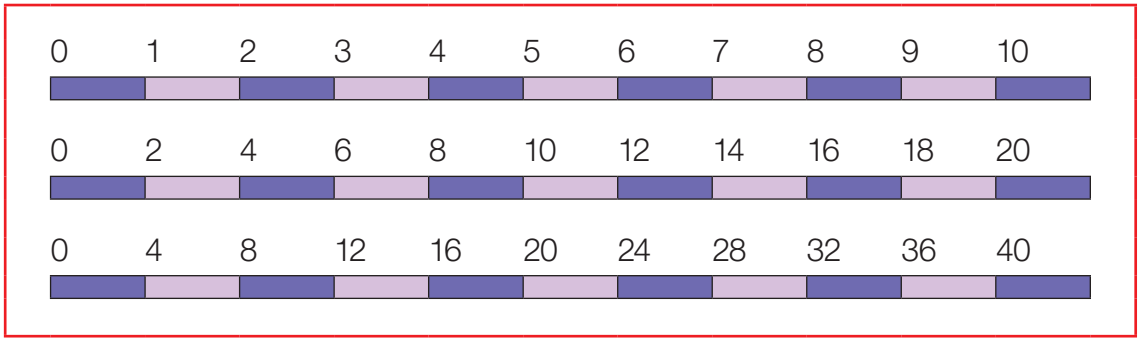
- How many 1s are there in 8?
- How many 1s are there in 3?
- How many 2s are there in 14?
- How many 2s are there in 8?



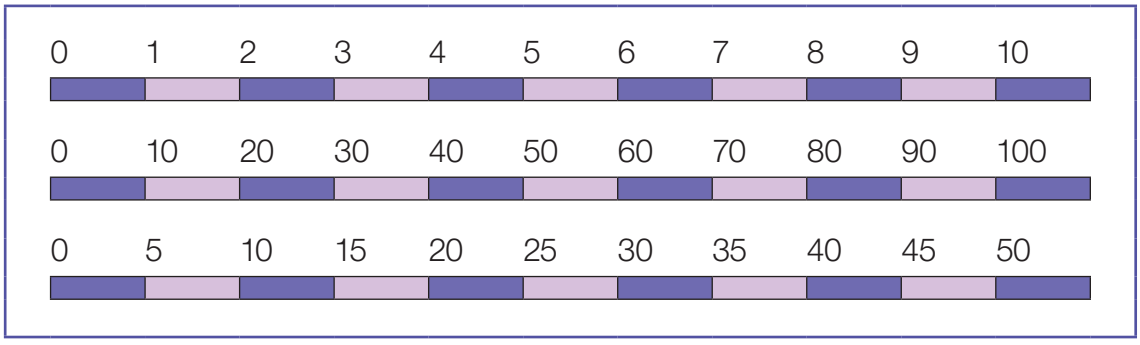
- How many 1s are there in 9?
- How many 1s are there in 5?
- How many 10s are there in 70?
- How many 10s are there in 40?
- How many 5s are there in 25?
- How many 5s are there in 50?

Step 3

1 times table	1 times table	1 times table
2 times table	10 times table	3 times table
4 times table	5 times table	

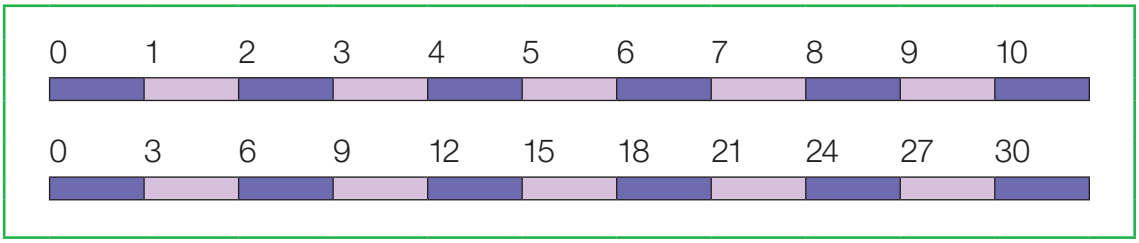


- How many 1s are there in 6?
- How many 1s are there in 8?
- How many 2s are there in 12?
- How many 2s are there in 20?
- How many 4s are there in 16?
- How many 4s are there in 32?



- How many 1s are there in 9?
- How many 1s are there in 5?
- How many 10s are there in 70?
- How many 10s are there in 40?

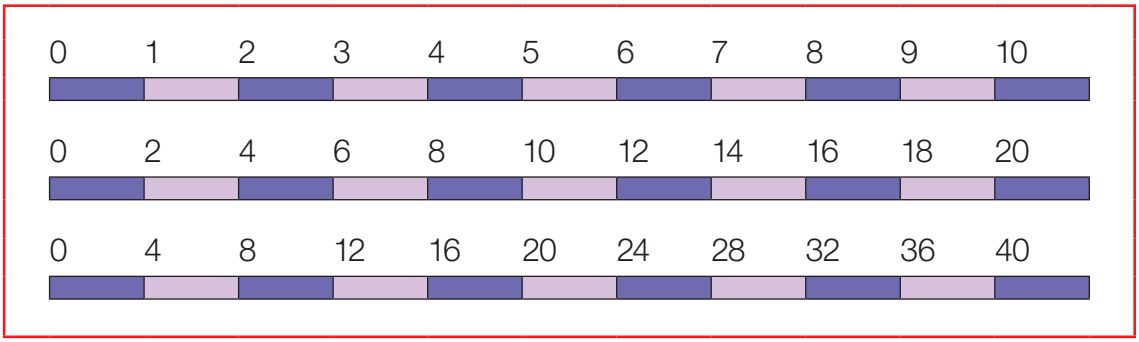
- How many 5s are there in 25?
- How many 5s are there in 50?



- How many 1s are there in 8?
- How many 1s are there in 2?
- How many 3s are there in 21?
- How many 3s are there in 9?

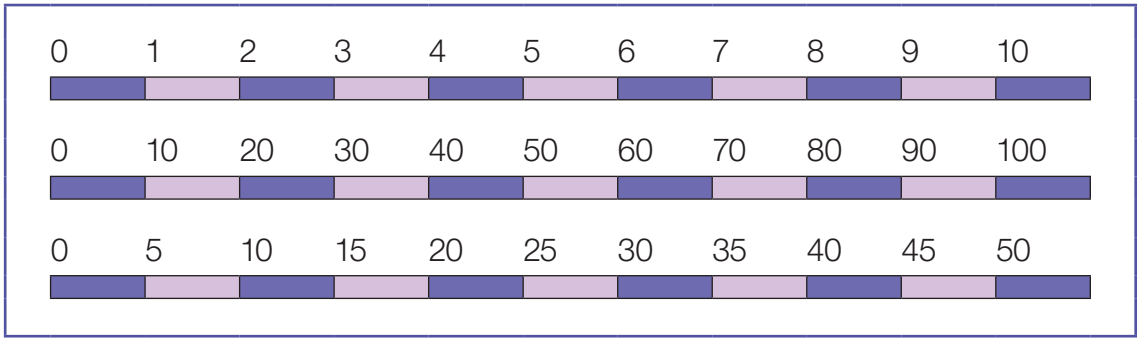
Step 4

1 times table	1 times table	1 times table
2 times table	10 times table	3 times table
4 times table	5 times table	6 times table

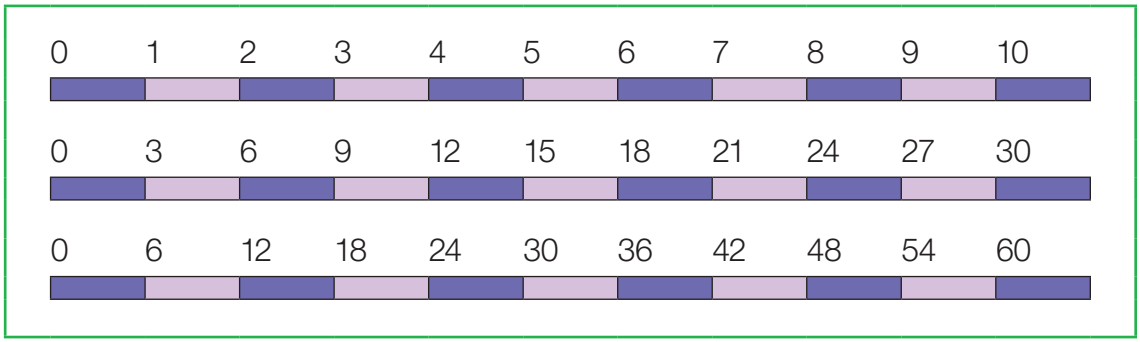


- How many 1s are there in 6?
- How many 1s are there in 8?
- How many 2s are there in 12?
- How many 2s are there in 20?
- How many 4s are there in 16?

- How many 4s are there in 32?



- How many 1s are there in 9?
- How many 1s are there in 5?
- How many 10s are there in 70?
- How many 10s are there in 40?
- How many 5s are there in 25?
- How many 5s are there in 50?

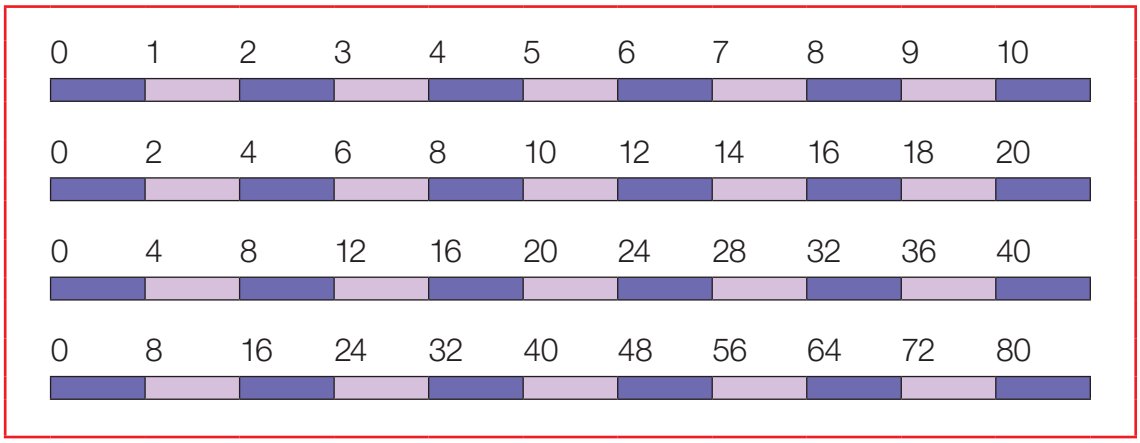


- How many 1s are there in 8?
- How many 1s are there in 2?
- How many 3s are there in 21?
- How many 3s are there in 9?
- How many 6s are there in 36?
- How many 6s are there in 54?

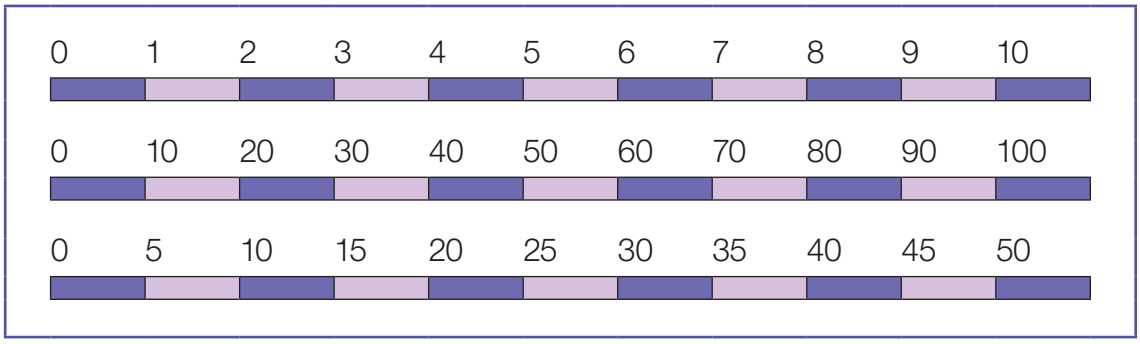
Step 5

Derive and recall quickly division facts for all tables up to 10 x 10

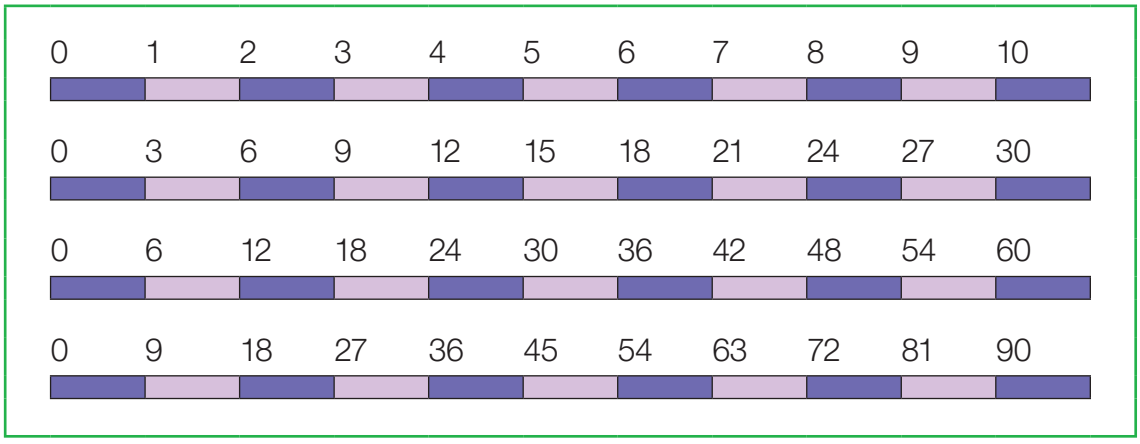
1 times table	1 times table	1 times table	1 times table
2 times table	10 times table	3 times table	7 times table
4 times table	5 times table	6 times table	
8 times table		9 times table	



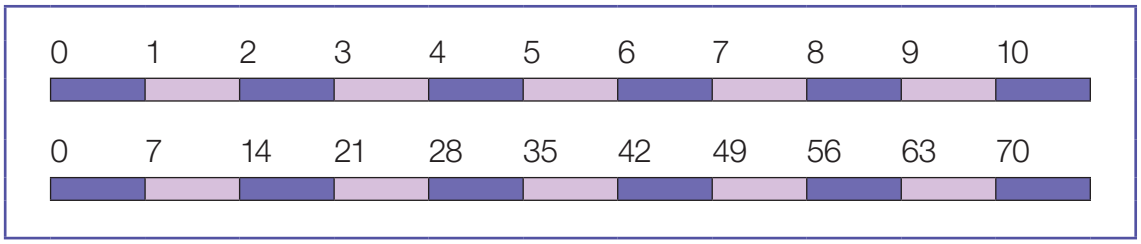
- How many 1s are there in 6?
- How many 1s are there in 8?
- How many 2s are there in 12?
- How many 2s are there in 20?
- How many 4s are there in 16?
- How many 4s are there in 32?
- How many 8s are there in 48?
- How many 8s are there in 72?



- How many 1s are there in 9?
- How many 1s are there in 5?
- How many 10s are there in 70?
- How many 10s are there in 40?
- How many 5s are there in 25?
- How many 5s are there in 50?

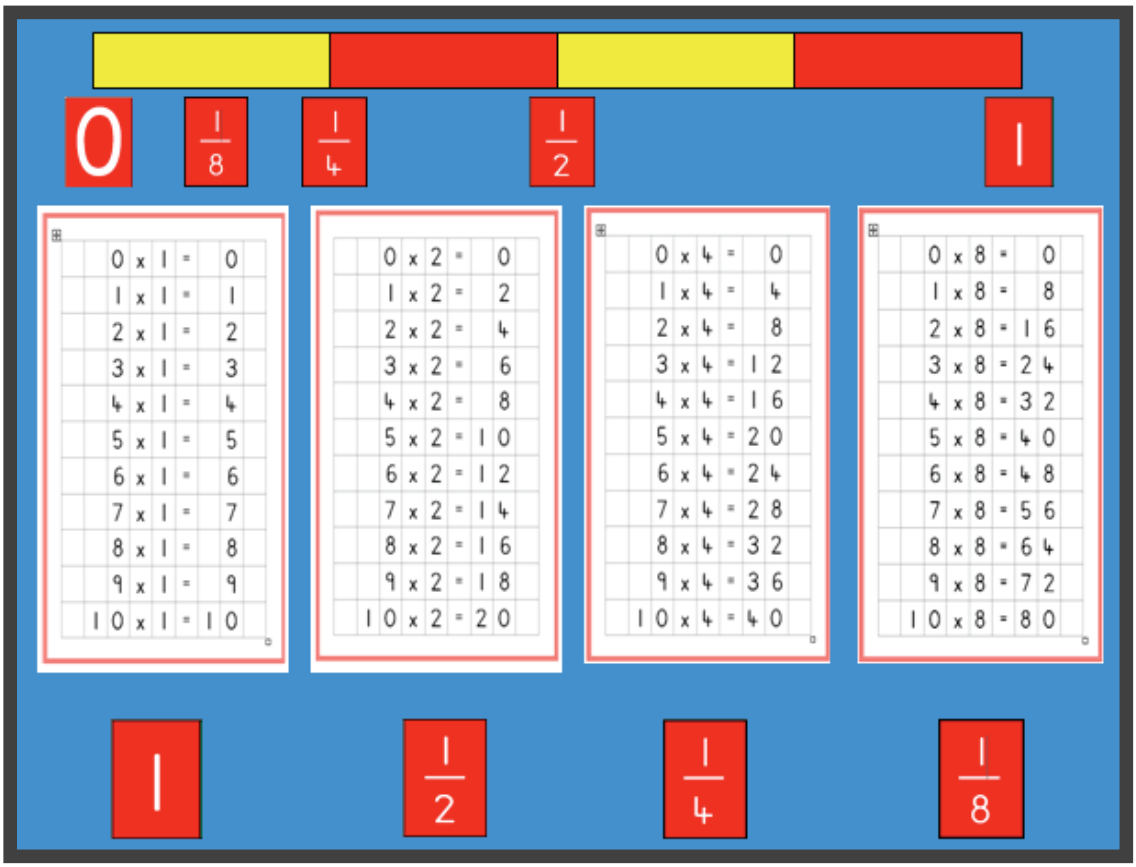


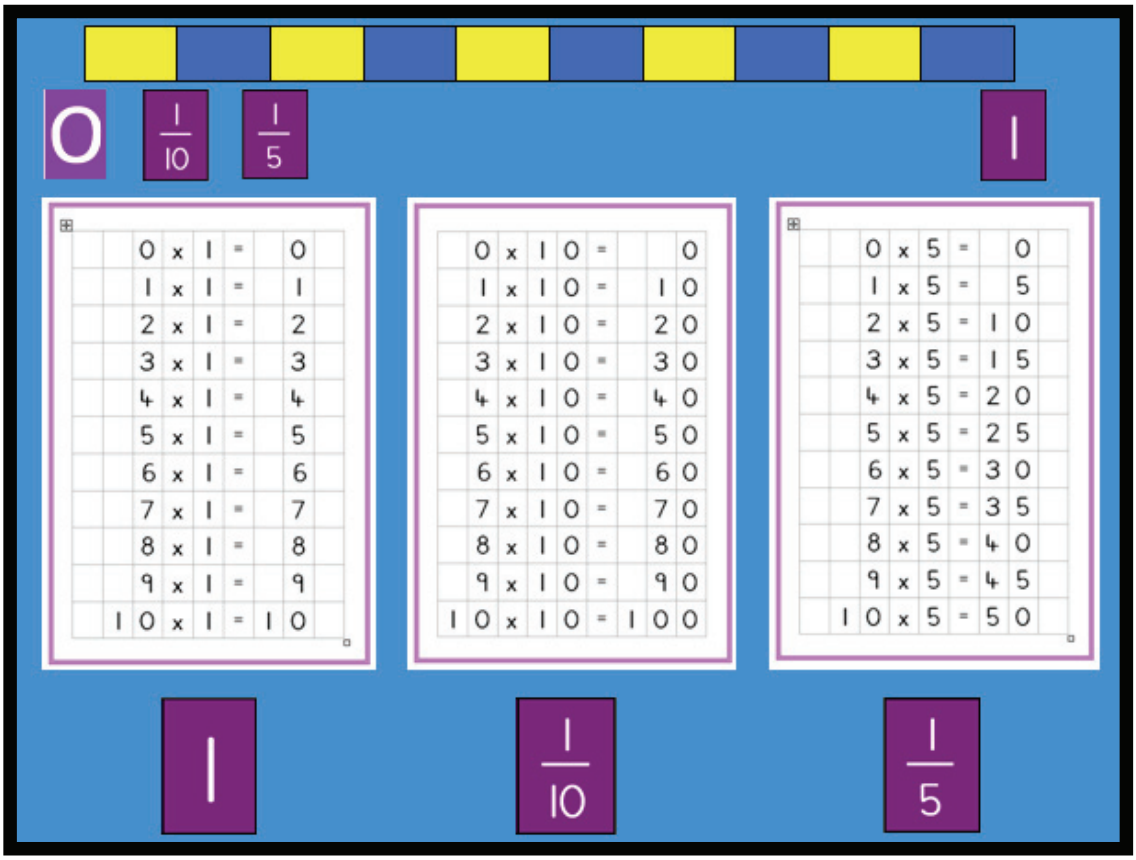
- How many 1s are there in 8?
- How many 1s are there in 2?
- How many 3s are there in 21?
- How many 3s are there in 9?
- How many 6s are there in 36?
- How many 6s are there in 54?
- How many 9s are there in 45?
- How many 9s are there in 90?

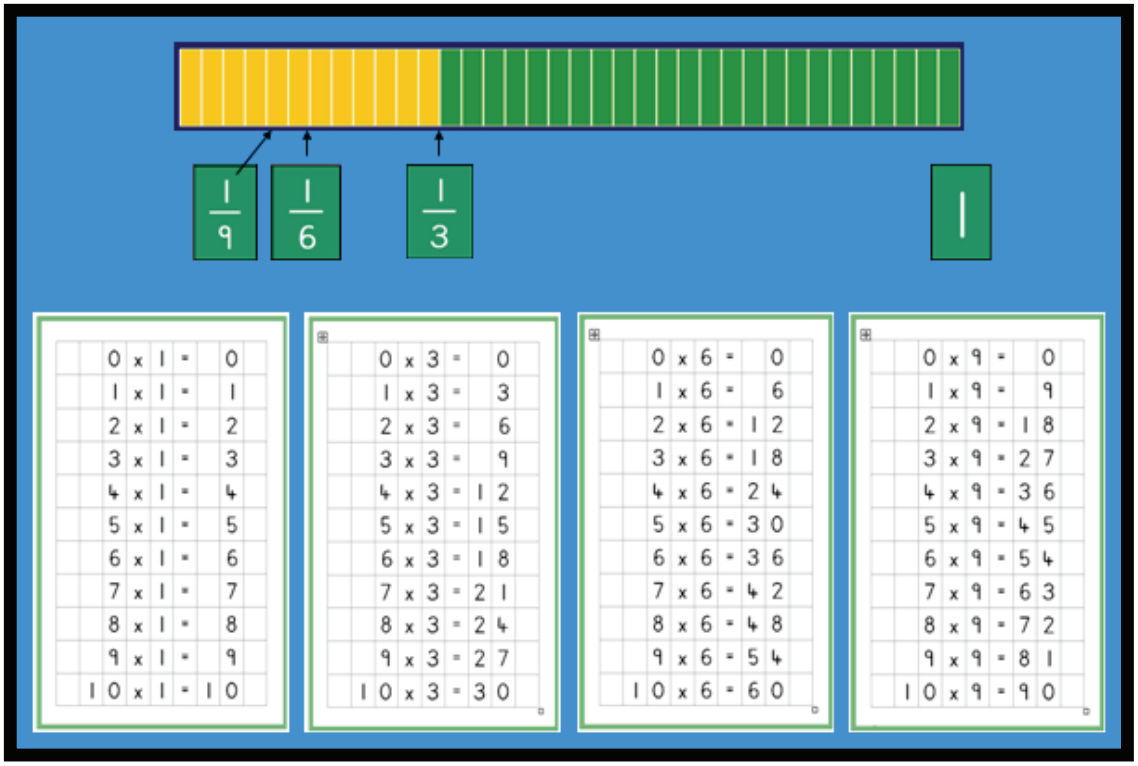


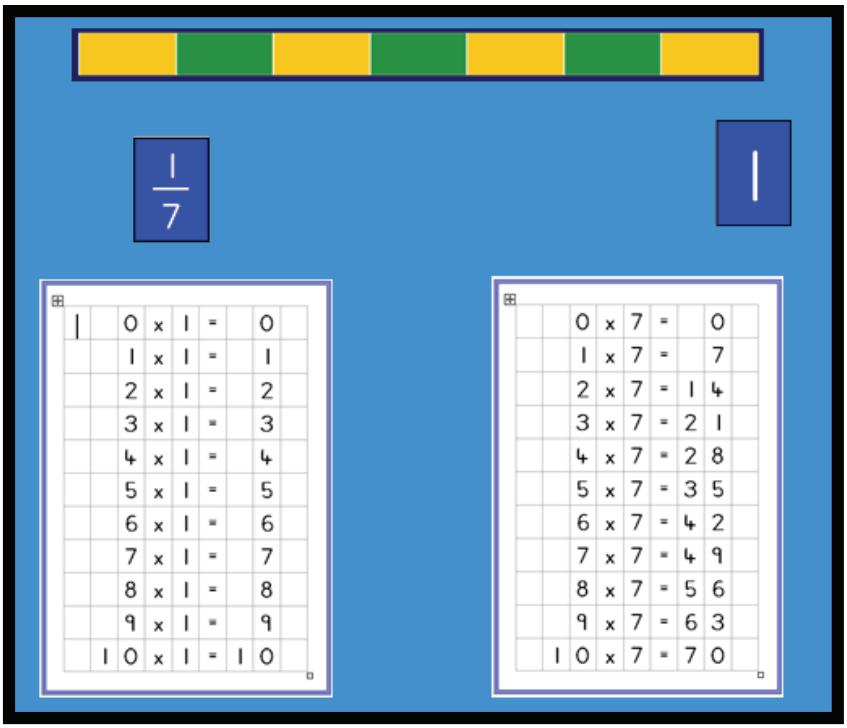
- How many 1s are there in 8?
- How many 1s are there in 2?
- How many 7s are there in 21?
- How many 7s are there in 49?

N.B. Learners should understand the close relationship between multiplication, division and fractions.




0
1/10
1/5
1
0 x 1 = 0
1 x 1 = 1
2 x 1 = 2
3 x 1 = 3
4 x 1 = 4
5 x 1 = 5
6 x 1 = 6
7 x 1 = 7
8 x 1 = 8
9 x 1 = 9
10 x 1 = 10
0 x 10 = 0
1 x 10 = 10
2 x 10 = 20
3 x 10 = 30
4 x 10 = 40
5 x 10 = 50
6 x 10 = 60
7 x 10 = 70
8 x 10 = 80
9 x 10 = 90
10 x 10 = 100
0 x 5 = 0
1 x 5 = 5
2 x 5 = 10
3 x 5 = 15
4 x 5 = 20
5 x 5 = 25
6 x 5 = 30
7 x 5 = 35
8 x 5 = 40
9 x 5 = 45
10 x 5 = 50
1
1/10
1/5


1/9
1/6
1/3
1
0 x 1 = 0
1 x 1 = 1
2 x 1 = 2
3 x 1 = 3
4 x 1 = 4
5 x 1 = 5
6 x 1 = 6
7 x 1 = 7
8 x 1 = 8
9 x 1 = 9
10 x 1 = 10
0 x 3 = 0
1 x 3 = 3
2 x 3 = 6
3 x 3 = 9
4 x 3 = 12
5 x 3 = 15
6 x 3 = 18
7 x 3 = 21
8 x 3 = 24
9 x 3 = 27
10 x 3 = 30
0 x 6 = 0
1 x 6 = 6
2 x 6 = 12
3 x 6 = 18
4 x 6 = 24
5 x 6 = 30
6 x 6 = 36
7 x 6 = 42
8 x 6 = 48
9 x 6 = 54
10 x 6 = 60
0 x 9 = 0
1 x 9 = 9
2 x 9 = 18
3 x 9 = 27
4 x 9 = 36
5 x 9 = 45
6 x 9 = 54
7 x 9 = 63
8 x 9 = 72
9 x 9 = 81
10 x 9 = 90



Step 6

Using and applying division facts

Learners should be able to use their knowledge of tables to derive other facts; e.g. If I know 3 x 7 = 21, what else do I know?

30 x 7 = 3 x 7 x 10 = 210

300 x 7 = 3 x 7 x 10 x 10 = 3 x 7 x 100 = 2100

3000 x 7 = 3 x 7 x 10 x 10 x 10 = 3 x 7 x 10 x 100 = 3 x 7 x 1000 = 21 000

0.3 x 7 = (3 x 7) ÷ 10 = 2.1

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is moving the digits one place to the right.

Knowing that the effect of dividing by 100 is moving the digits two places to the right.

Dividing by 10

Th	H	T	U		.1	.01	.001
		4	3	.	4		
		3	4				

When dividing a whole number by 10 the digits move one column to the right

Dividing by 100

Th	H	T	U		.1	.01	.001
			0	.	3	4	
		3	4				

When dividing a whole number by 100 the digits move two columns to the right

Use of factors

550 ÷ 50 =
550 ÷ 10 = 55
55 ÷ 5 = 11
550 ÷ 50 = 11

378 ÷ 21 =
378 ÷ 3 = 126
126 ÷ 7 = 18
378 ÷ 21 = 18

Use of factors

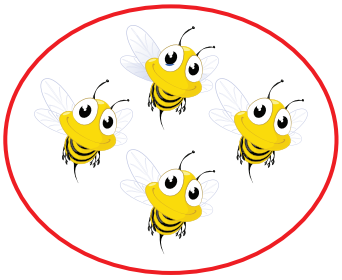
Given that 124 x 8 = 992 What is 992 ÷ 8? or 992 ÷ 124?	Given that 1.4 x 1.1 = 1.54 What is 1.54 ÷ 1.4? or 1.54 ÷ 1.1?

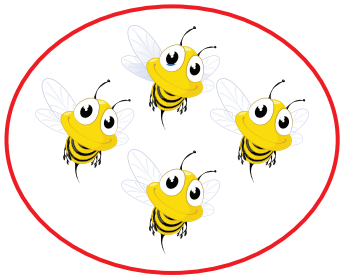
Many mental calculation strategies will continue to be used. They are not replaced by written methods.

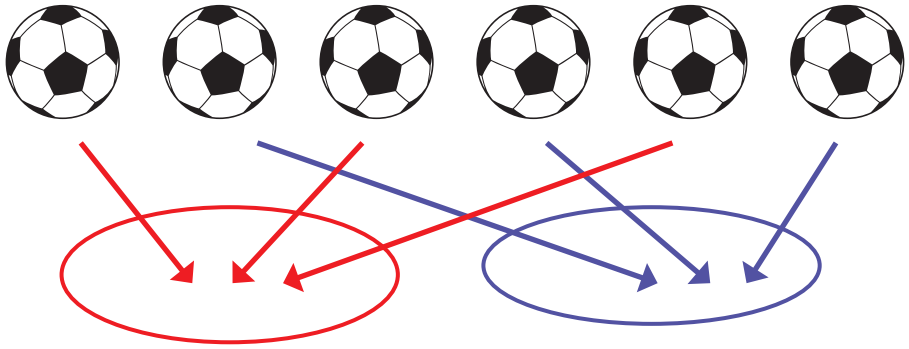
Step 1

Learners will understand **equal groups**, and **share items** out in play and problem solving.

They will count in 2s and 10s and later in 5s.





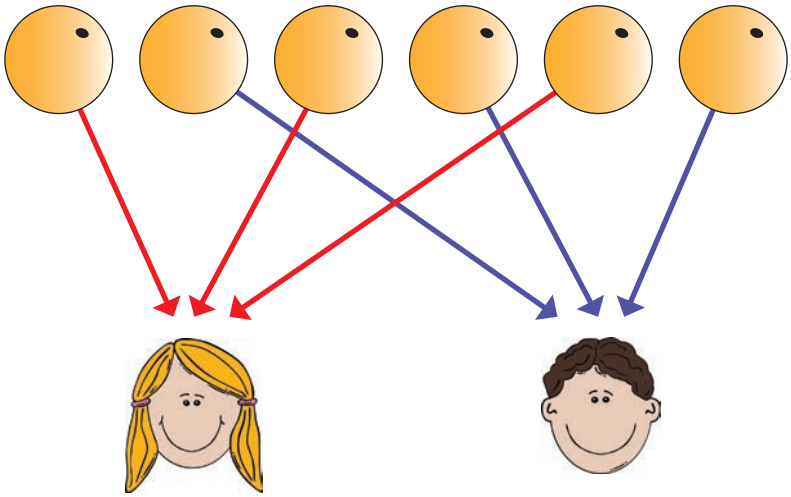


Step 2

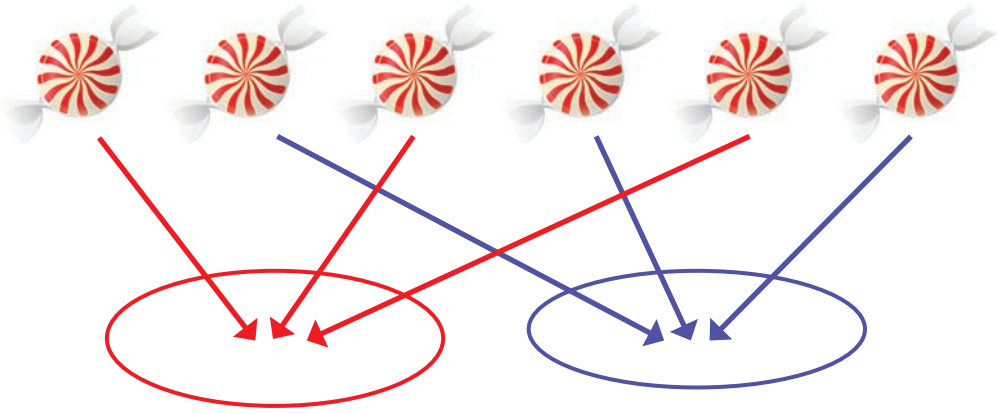
Learners will develop their understanding of division and use jottings to support calculation.

Sharing equally

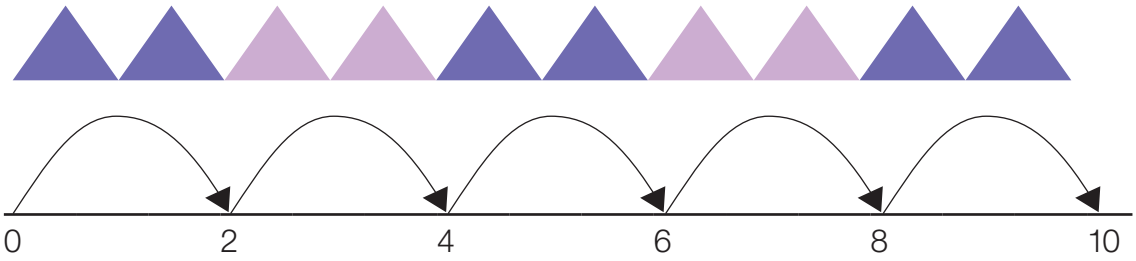
6 oranges shared between 2 people, how many do they each get?



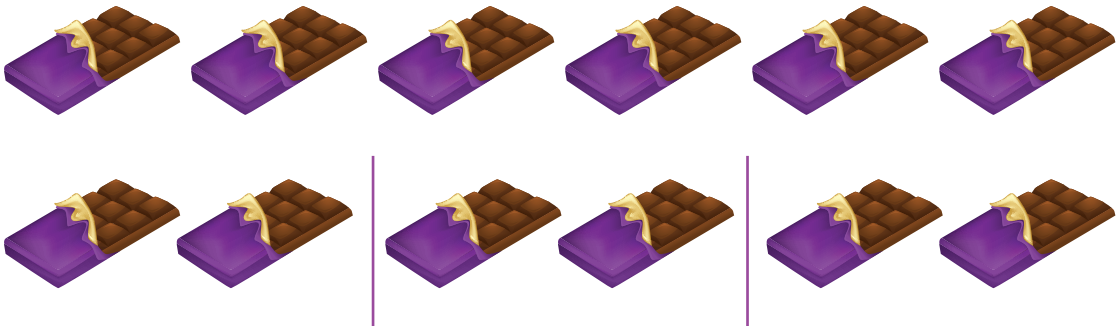
6 sweets shared between 2 people, how many do they each get?



Grouping or repeated subtraction

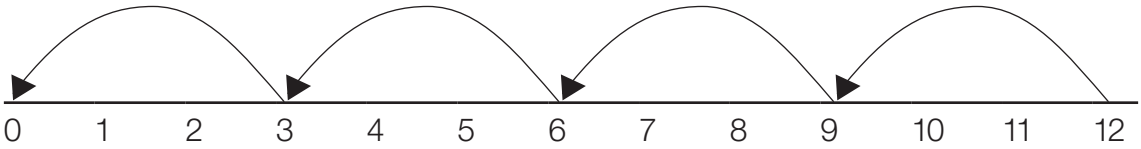


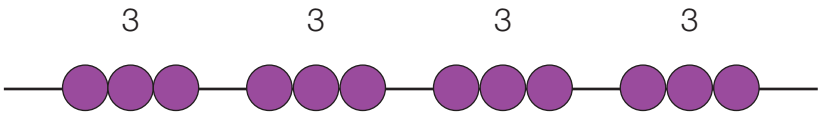
There are 6 bars of chocolate, how many people can have 2 chocolate bars each?



Repeated subtraction using a number line or bead bar

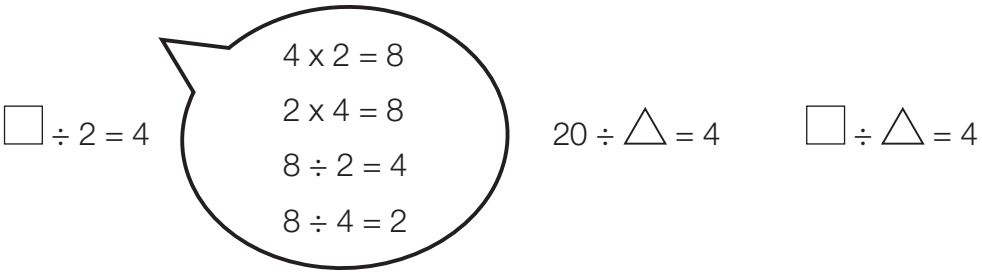
12 ÷ 3 = 4





A bead bar will help learners with interpreting division calculations such as 10 ÷ 5 as 'how many 5s make 10?'

Using symbols to stand for unknown numbers to complete equations using inverse operations



Step 3

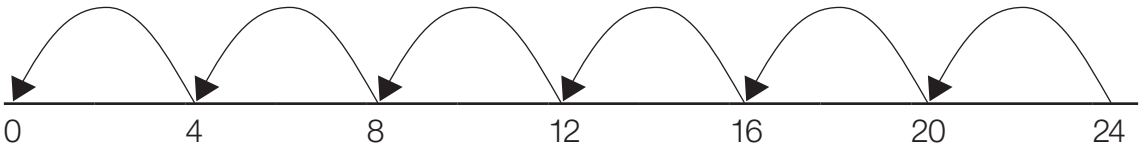
Ensure that the emphasis in Year 3 is on grouping rather than sharing.

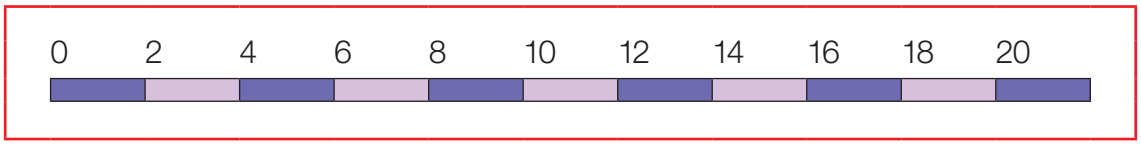
Learners will continue to use:

Repeated subtraction using a number line

Learners will use an empty number line to support their calculation.

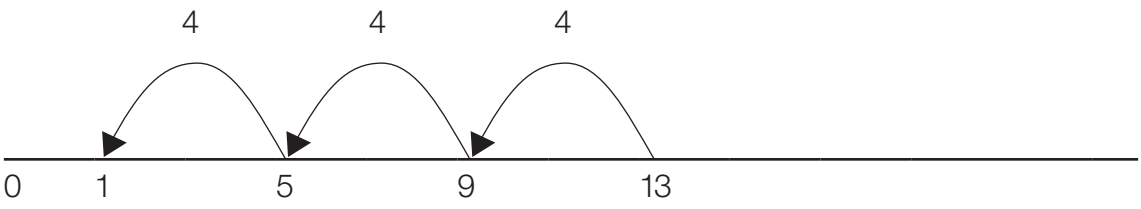
24 ÷ 4 = 6





Learners should also move onto calculations involving remainders.

13 ÷ 4 = 3 r 1



Using symbols to stand for unknown numbers to complete equations using inverse operations

26 ÷ 2 = □ 24 ÷ △ = 8

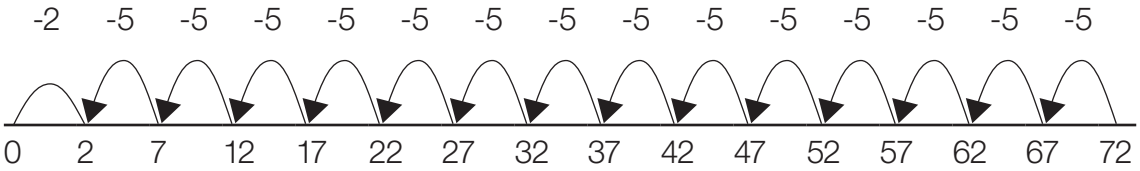
8 x 3 = 24
3 x 8 = 24
24 ÷ 3 = 8
24 ÷ 8 = 3

□ ÷ 10 = 8

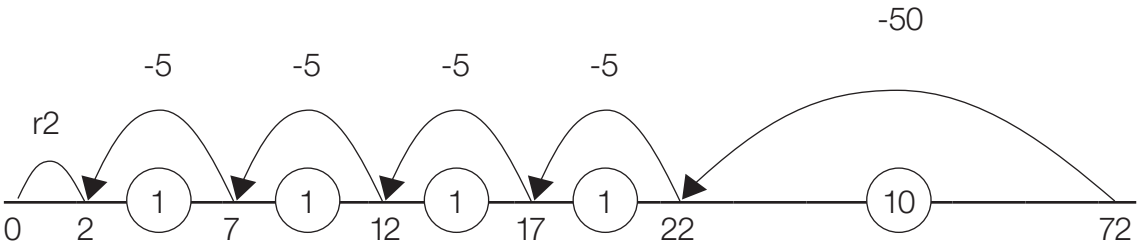
Step 4

Learners will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the learners are more familiar.

72 ÷ 5



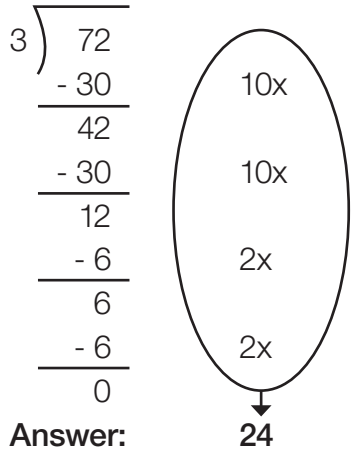
Moving onto:

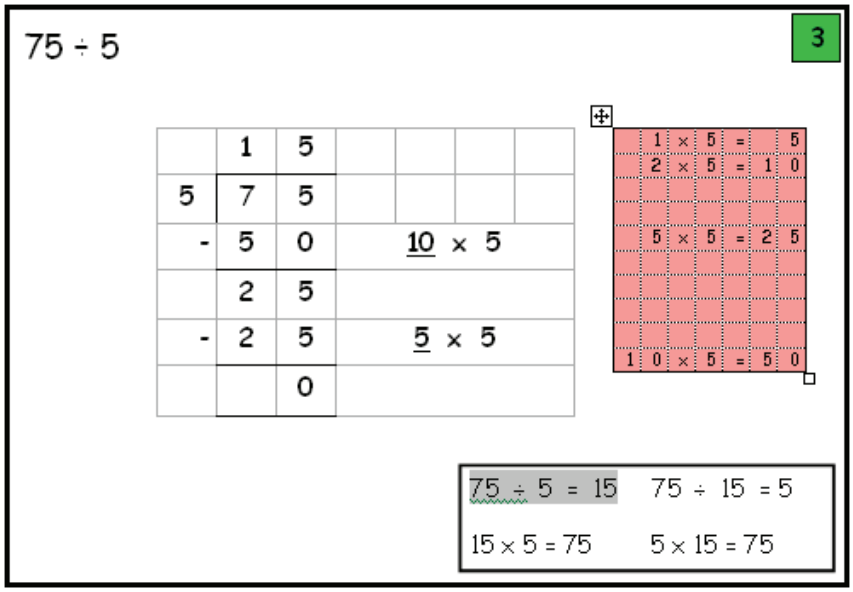


Then onto the vertical method:

Short division TU ÷ U

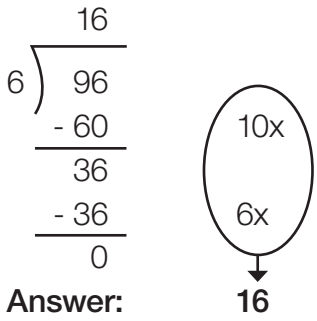
72 ÷ 3



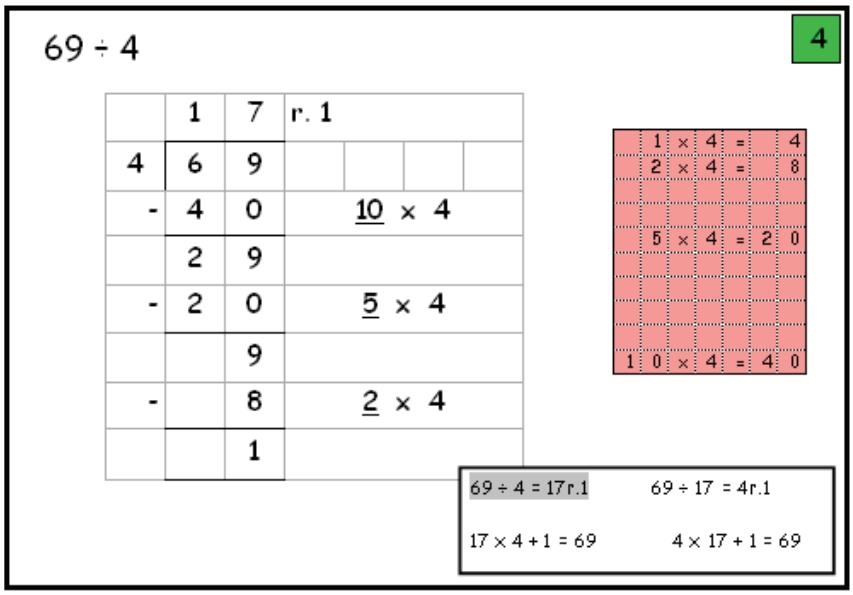


Leading to subtraction of other multiples:

96 ÷ 6



Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.



Learners need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

For example: I have 62p. Sweets are 8p each. How many can I buy?

Answer: 7 (the remaining 6p is not enough to buy another sweet).

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?

Answer: 8 (the remaining 6 apples still need to be placed into a box).

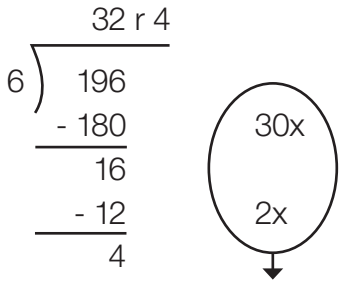
Step 5

Learners will continue to use written methods to solve short division TU ÷ U.

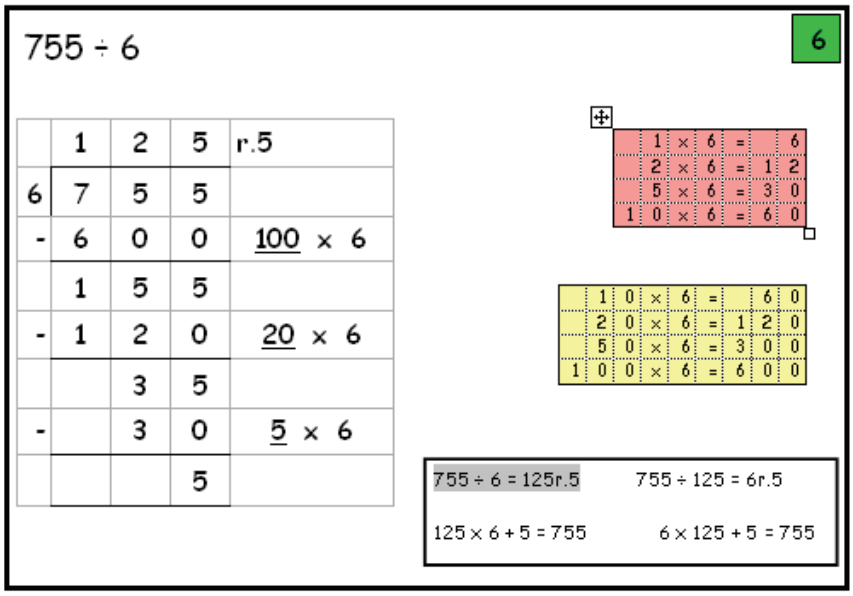
Learners can start to subtract larger multiples of the divisor, e.g. 30x

Short division HTU ÷ U

196 ÷ 6



Answer: **32 remainder 4 or 32 r 4**



Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Learners need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example, 240 ÷ 52 is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context.

Division Grid

Stage 1

755 ÷ 6

	100		
6	700	50	5
	600		
	100		

Stage 2

	100	20	
6	700	150	5
	600	120	
		30	

Stage 3

	100	20	5
6	700	150	35
	600	120	30
			5

Stage 4

	100	20	5	r5
6	700	150	35	
	600	120	30	

Stage 5

	100	20	5	5/6
6	700	150	35	
	600	120	30	

Stage 6

	100	20	5	.833
6	700	150	35	
	600	120	30	

Step 6

Learners will continue to use written methods to solve short division TU ÷ U and HTU ÷ U.

Long division HTU ÷ TU

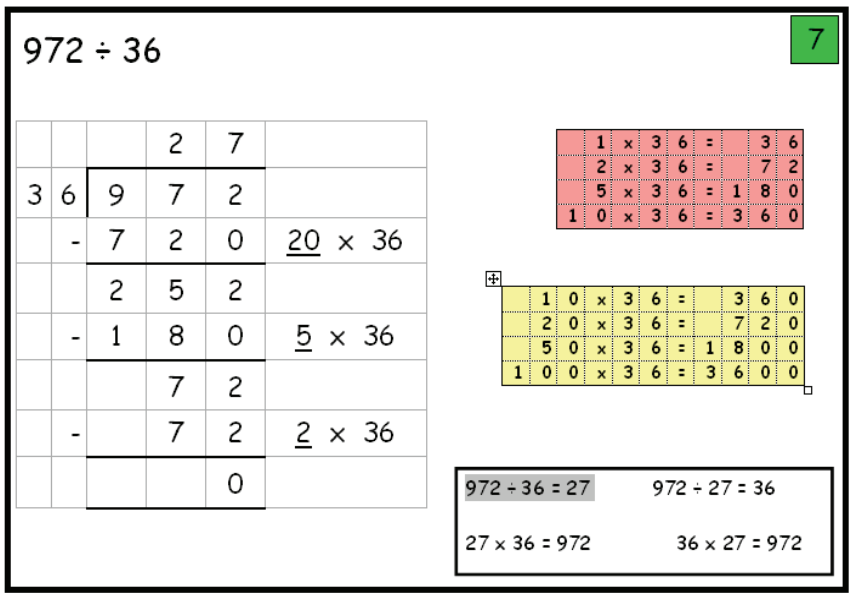
972 ÷ 36

```
      27
36 ) 972
   - 720
     252
   - 252
       0
```

20x

7x

Answer: **27**



Any remainders should be shown as fractions, i.e. if the learners were dividing 32 by 10, the answer should be shown as $3\ ^{2}/_{10}$ which could then be written as $3\ ^{1}/_{5}$ in its lowest terms.

Extend to decimals with up to two decimal places. Learners should know that decimal points line up under each other.

87.5 ÷ 7

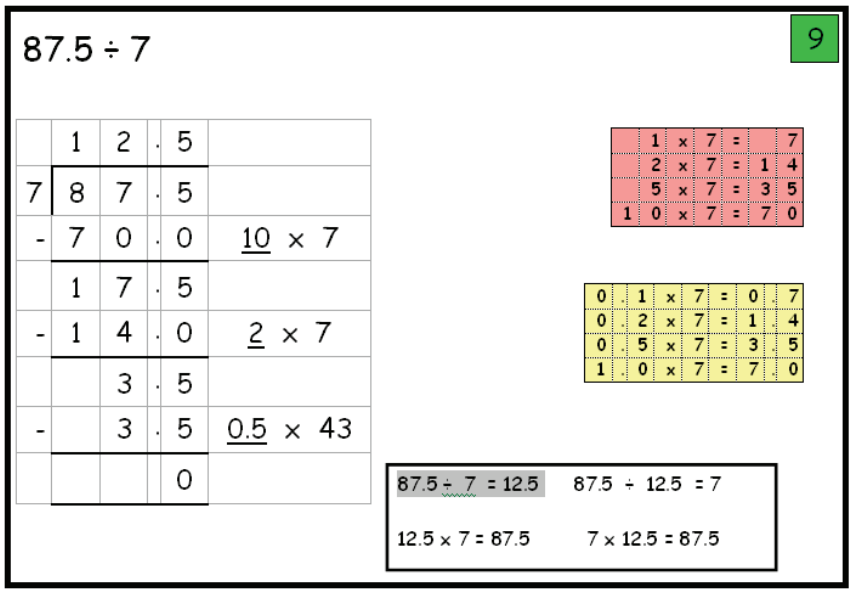
```
      12.5
   ┌──────
 7 │ 87.5
   - 70.0
   ------
     17.5
   - 14.0
   ------
      3.5
   -  3.5
   ------
         0
```

10x

2x

0.5x

Answer: **12.5**



Learners need to have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Learners should not be made to go on to the next stage if:

1) they are not ready;

2) they are not confident.

Learners should be encouraged to approximate their answers before calculating.

Learners should be encouraged to check their answers after calculation using an appropriate strategy.

Learners should be encouraged to consider if a mental calculation would be appropriate before using written methods.