Reasoning in the Mathematics Classroom

The processes detailed in the 2015 Programme of Study for Mathematics and the Literacy and Numeracy Framework (LNF, 2013) under the strand of 'Numerical Reasoning' are clear indicators of developing learner independence and have always applied to mathematics classrooms. Actions such as:

- *Prioritise and organise the relevant steps needed to complete the task or reach a solution;*
- *Choose an appropriate mental or written strategy and know when it is appropriate to use a calculator;*
- *Identify, measure or obtain required information to complete the task,* and
- *Select appropriate mathematics and techniques to use;*

imply that the learner is making the decisions in order to solve a problem, and not just following a procedure. In order for such reasoning to take place, it is necessary for the classroom to be geared towards questions such as:

'Here is a problem. What mathematics do we use?'

The Audit of Resources within this set of materials refers to some of many resources developed by Malcolm Swan and others over the years: *The Mathematics Assessment Project, Bowland* and the *Standards' Unit: Improving Learning in Mathematics*. As Malcolm Swan[1] describes the principles behind them, he states:

> "We … reverse traditional practices by allowing students opportunities to tackle problems before offering them guidance and support".
>
> (Swan 2006)

This is quite different in nature from a mathematics lesson characterised by:

'Here is a method, and here are some problems to which it applies.'

In this type of lesson, much decision-making is removed from the learner: having just studied a chapter on a topic, there is no decision to be made about what mathematics to use when answering the subsequent end-of-chapter word problems.

Skemp (1976) suggests that there are two types of meaning of the word 'understanding': relational understanding, where the learners know what to do and why, and instrumental understanding, where the learners understand how to follow a given procedure or rule, but not necessarily how or why it works. These examples illustrate a difference between teaching learners to *become* mathematicians (where the goal of learning is a change of identity (Lave and Wenger, 1991)) and just teaching learners *about* mathematics (where the goal is more tightly focused on the acquisition of skills). The new curriculum for mathematics clearly emphasises the need for our learners to understand how and why their methods work and where to apply them, as well as having the mathematical skills.

Questioning to encourage learner independence

Probably the simplest and most powerful question we can ask in a classroom is:

'How did you work that out?'

It assumes that a learner has ventured an idea or a solution to a problem – whether or not that solution is correct – and rests on a principle[2] that emphasising methods rather than answers generates the important conversations through which we think like mathematicians. Further questions could follow, e.g.

'Is there an alternative method?' and
'Does your method always work?'

Through these questions we can diagnose misconceptions, and learners find new, creative solutions, and compare and evaluate different methods. It assumes however that learners simply haven't been required to follow a given method to solve them – the question 'How did you work that out?' followed by 'I did what you said, sir' doesn't lead to great mathematical dialogue and devalues the latter two questions.

Effective teacher questioning to support learner decision-making is essential, but not easy.

> "There is a fine line between a question that encourages the student to think and one that provides the student with too much information or inadvertently solves the problem for the student. Being able to straddle this fine line comes with reflective practice."
>
> (Ontario Ministry of Education, 2006a, p. 32)

Doing too much for learners is common. Taking the decision away from learners is also common. Consider this question:

'Which is the better buy – three 375 g boxes of cereal for £5 or two 750 g boxes for £6.50?'

If the learners are stuck, often we try to help them by stating a first step, and then engaging the learner through a low-level question, e.g.

> 'The first thing to do might be to find out how much cereal is in three 375 g boxes, and in two 750 g boxes. So let's multiply 375 by 3 and 750 by 2.
> What's 750 × 2?'

Here, we are not asking the learners to think about *how* they solve the problem, merely to conduct a simple calculation. This does not help them become independent. Therefore, we may need a set of questions and prompts for teachers that do not give the game away, and which reduce the learner's role to passive bystander but which instead help learners develop and use their own approaches to solving the problems.

As they read through problems, the questions that arise in learners' minds might be 'What does this mean?', 'Where do I start?', 'What information do I need?' and 'How do I work that out?' Polya (1945) identifies four basic principles of problem solving: understand the problem, devise a plan, carry out the plan and look back. During an examination our learners have to do this independently, and so during our day-to-day lessons we must help them build this experience.

Polya's four principles were used in building the cycle of problem-solving used in the resources here. So, for 'understand the problem' we have two lead questions: 'What is the question?' and 'What information do we have, and what do we need?' The full set of lead questions is shown in the table below.

Polya's Four Principles	Lead questions/actions
Understand the problem	1) What is the question? 2) What information do we have, and what do we need?
Devise a plan	3a) What mathematics will we use?
Carry out the plan	3b) Try out our ideas 3c) What do our results tell us? 4) How should we present our findings?
Look back	5) Evaluate our work 6) Extend the question

This cycle shows that problem-solving is not a linear process, that there are times when we go back, and reset our strategy, collect more information, undertake different calculations, and evaluate our presentations. The numbering of the lead questions and actions tries to indicate this.

Alongside the lead questions, there are several supporting questions, offering greater detail about the process. A list of these questions is given in the A3 sheet: 'Problem-solving poster'. They are not prescriptive, they may not paint the whole picture, and they will need adjusting to different situations, but may act as suitable prompts to enable learners to start and to keep going in solving challenging problems.

The questions are presented in a variety of ways in these resources:

- Two PowerPoint slide sequences, both using a display representing a cycle of thinking in solving problems. One just shows how the cycle may work, including how we might have to go back on our work as we collect more information and results. The other displays the supporting questions alongside the lead question;
- An A3 sheet 'Problem-solving poster' showing the whole cycle and the full set of questions;
- Sheets of the lead questions on large print, to use as headings for wall displays;
- Problem-solving writing frame/graphic;
- 'Steps' guidance: using the questions on two example exam-style questions, 'The Water bill' and 'The Circle'. These are also supported with PowerPoint presentations.

[1] *Mathematics Assessment Project*: http://map.mathshell.org/ 2007-2015 Mathematics Assessment Resource Service, University of Nottingham.

[2] *Standards Unit 'Improving learning in mathematics': challenges and strategies*, Malcolm Swan, University of Nottingham

References

Ontario Ministry of Education (2006a), *A guide to effective literacy instruction, Grades 4 to 6 – Volume 1*, Toronto, ON: Queen's Printer for Ontario.

Lave, J. and Wenger, E. (1991), *Situated Learning: Legitimate Peripheral Participation*, Cambridge University Press

Polya G. (1945), *How to Solve It: A New Aspect of Mathematical Method,* Princeton Science Library

Skemp, R. (1976), *Relational Understanding and Instrumental Understanding,* Mathematics Teaching 77: 20-6

Swan, M. (2006), *Collaborative learning in mathematics: A challenge to our beliefs and practices,* National Research and Development Centre for adult literacy and numeracy (NRDC) and the National Institute of Adult Continuing Education (NIACE). A CD of resources and video clips is available with the book.