Stimulus question 81

Another Dimension

Teaching notes

Key question/task

Learners are presented with diagrams of a pyramid and a cube, with some lengths marked with a symbol.
They are then given a number of formulae, and using the diagram they must identify which of the formulae give lengths, areas or volumes. Learners should try to show what lengths, areas or volumes of the diagrams are represented by these formulae.

Supplementary questions:

- Some side lengths are not marked, but could you work out what they should be?
- When you add lengths, what will you get as a result?
- When you multiply lengths, what will you get? Does it matter how many lengths are involved in the multiplication?

Resources:

- Question sheet;
- Paper, pencils and/or mini-whiteboards and pens to explore ideas.

Reasoning: questions to discuss and explore

- What would the formula $6a^3$ give you?
- Would the formula f^2 give you an area that is on the diagram of the cube?
- (Pyramid)? What would the formula a^3 give you? Is it on the diagram?

Possible extension

- Ask learners to try to develop a similar question of their own.

Commentary/notes:

This activity is suitable for Intermediate (using the cube) and Higher (additionally, using the pyramid). This does not mean that the pyramid question cannot be used for learners at Intermediate tier – this will be at the teacher's discretion.

This activity builds on question 80: 'A Different Dimension', by extending the idea to involve volumes. The intention behind both these questions is to form a conceptual bridge between handling formulae to calculate areas and volumes, and the more difficult skill of abstracting meaning from expressions that may or may not represent any areas and volumes, which are often unsupported visually.

Here the expressions only refer to what can be seen in the diagram: it is of course possible to generate expressions from the symbols that may hypothetically form areas or volumes, but that is left to the reasoning questions – for the dialogue in class.

It also gives an opportunity to review and use the formulae for the surface areas and volumes of a pyramid and a cube. The two parts of the question may be used independently of each other.

Solution: Cube

Formula	Length	Area	Volume	Length, area or volume of what?
a^2		✓		Area of any of the faces of the cube
$\frac{1}{2}af$		✓		Area of the triangle, base a, height f, d is the length of the hypotenuse
$\frac{1}{2}a^2$		✓		Half of the area of any face of the cube, e.g a triangle base length a, height a, hypotenuse length f
$6a$	✓			6 edges of the cube, with side length 'a'
$6a^2$		✓		Surface area of the cube
$2(a + f)$	✓			Perimeter of rectangle, width a and length f
$a + d + f$	✓			The perimeter of a triangle with these side lengths
a^3			✓	The volume of the cube
$\sqrt{(d^2 - f^2)}$	✓			The length a, given by Pythagoras' theorem

Solution: Pyramid

Formula	Length	Area	Volume	Length, area or volume of what?
ab		✓		The area of one triangular face of the pyramid
$4b^2$		✓		The area of the square base
$2a$	✓			The heights of two triangular faces
$8b$	✓			The perimeter of the square base
$a + b + h$	✓			The perimeter of the marked triangle
$(4b^2h)/3$			✓	The volume of the pyramid (one third of the base area × the height)
$\frac{1}{2}\ (abh)^2$	✗	✗	✗	
$\sqrt{(a^2 - b^2)}$	✓			The height, h (given by Pythagoras' theorem)
$4b(a + b)$		✓		The surface area of the pyramid

GCSE Subject Content		
Foundation	**Intermediate**	**Higher**
	Distinguishing between formulae for length, area and volume by considering dimensions. Surface area, cross-sectional area and volume of cubes. Using Pythagoras' theorem in 2-D including reverse problems.	
		Surface areas and volumes of pyramids.

Learner Outcomes and Assessment (*to aid comment-only marking*)	
Reasoning strand – **Learners are able to:**	**Assessment guidance –** **Can learners:**
• Identify, measure or obtain required information to complete the task; • Select appropriate mathematics and techniques to use; • Explain results and procedures precisely using appropriate mathematical language; • Interpret mathematical information; use diagrams to draw conclusions.	• Deduce the lengths of the sides where symbols are missing? • Use the fact that addition of lengths gives a length, and multiplication of lengths gives an area or a volume? • Identify the use of Pythagoras' theorem? • Visualise the lengths, areas and volumes expressed by the formulae, connecting these to the diagrams?